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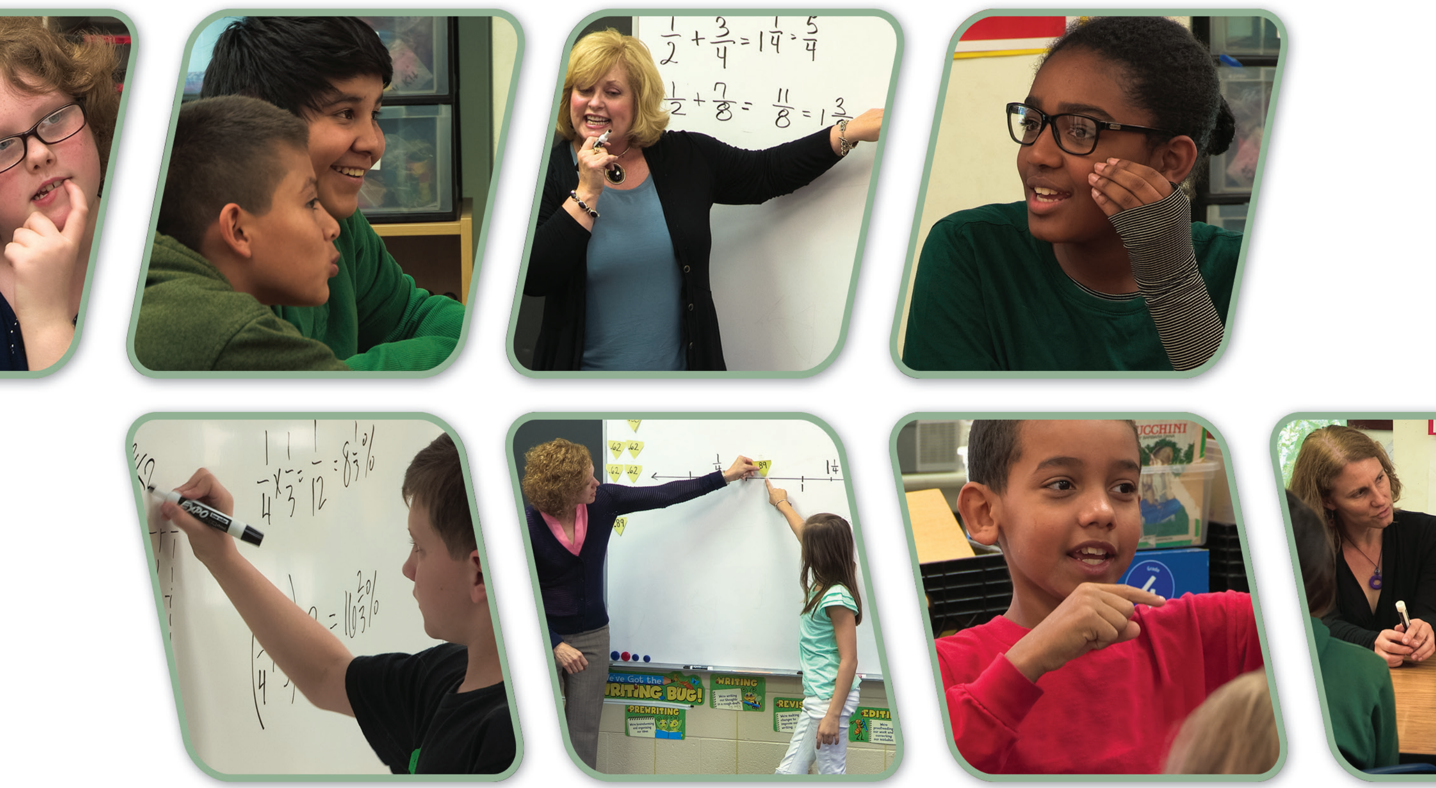
NUMBER TALKS

FRACTIONS, DECIMALS, AND PERCENTAGES



SHERRY PARRISH AND ANN DOMINICK

Foreword by Steve Leinwand

A Multimedia Professional Learning Resource

NUMBER TALKS

FRACTIONS, DECIMALS, AND PERCENTAGES

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INTRODUCTION

Why Fractions, Decimals, and Percentages?

In this section we tackle the "why"—why fractions, decimals, and percentages?—sharing a broad spectrum of compelling research and data. We open with data that show a history of the ongoing struggle students have with fractions, and we discuss the reasons this needs to change. We then take a closer look at why the process of working with fractions, decimals, and percentages can be so challenging for students who often struggle with making connections to other areas of mathematics. Finally, we explore how this struggle can be related to the way students are frequently taught procedural methods versus conceptual processes.

OVERVIEW

Fractions Are a Critical Foundation in Mathematics

In 2008, the National Mathematics Advisory Panel stated that proficiency with fractions should be a major goal of K–8 education. The panel described this area of mathematics as a Critical Foundation for Algebra and made the following recommendation:

> *The most important foundational skill not presently developed appears to be proficiency with fractions (including decimals, percent, and negative fractions). The teaching of fractions must be acknowledged as critically important and improved before an increase in student achievement in algebra can be expected. (xvii, 17)*

Many teachers can identify with the panel's finding that proficiency with fractions is underdeveloped; our own memories of learning about fractions consist primarily of dividing pizzas, pies, and candy bars and memorizing procedures for computation. When we were students, our sole access to a solution was often limited to following the rule of invert and multiply to divide, multiplying across the "top" and the "bottom" to find a product, or using cross multiplication to compare two fractions. As we progressed from grade to grade, we continued to follow these procedures to reach a short-term goal of getting a correct answer; consequently, we did not readily acquire the lifelong goal of developing fractional reasoning.

Evidence of Our Struggles (The Data from Research)

So how did the National Mathematics Advisory Panel conclude that "the most important foundational skill not presently developed appears to be proficiency with fractions"? Let's look at the data. The panel's findings were corroborated with a survey of 1,000 U.S. algebra teachers, who indicated that a lack of fractional knowledge was the second biggest problem students faced in being prepared to learn algebra. The final report also shared the following data, which indicate that difficulties with fractions, decimals, and percentages extend beyond the K–12 populations and into the general adult population:

- 78 percent of adults cannot explain how to compute the interest paid on a loan;

- 71 percent cannot calculate miles per gallon on a trip; and
- 58 percent cannot calculate a 10 percent tip for a lunch bill.

Other studies support this perspective, reinforcing the idea that fluency with fractions, decimals, and percentages is a critical gatekeeper to higher mathematics (Booth and Newton 2012; Geary et al. 2012; Siegler et al. 2012).

Data from student performance on the *National Assessment of Educational Progress* (NAEP) provides an overarching perspective on American students' difficulties with fractional reasoning from the late 1970s to the present. Consider the following examples:

1978 NAEP

When eighth-grade students were asked to estimate the sum for $\frac{12}{13} + \frac{7}{8}$:

- only 24 percent questioned could correctly answer 2;
- 28 percent chose 19;
- 27 percent chose 21;
- 14 percent indicated that they did not know; and
- 7 percent selected 1. (Carpenter et al. 1981)

2004 NAEP

- When asked to shade $\frac{1}{3}$ of a rectangle, 27 percent of the eighth graders questioned could not do this;
- 45 percent of the eighth graders could not solve a word problem that required division with fractions; and
- only 50 percent of the eighth graders could successfully order $\frac{2}{7}$, $\frac{1}{12}$, and $\frac{5}{9}$.

2009 NAEP

- Almost 75 percent of the fourth graders questioned could not choose a fraction, from four common fractions, that was closest to $\frac{1}{2}$; and
- less than 30 percent of the eleventh graders could correctly change 0.029 into $\frac{29}{1000}$.

Concern about the lack of understanding around fractional reasoning is warranted, and if a shift is not made, the impact on adult mathematical literacy as well as secondary education and mathematics in general could be far reaching (Hecht and Vagi 2010; Mazzocco and Devlin 2008).

Why Do Students Struggle with Fractions, Decimals, and Percentages?

As shared thus far, the struggles that people of all ages frequently have with understanding and operating with fractions, decimals, and percentages are well-documented, but this overwhelming evidence begs the question, "Why are they difficult?" In the following pages, we attempt to answer this through two perspectives: a look at the unique mathematical challenges that surface, and a look at procedural versus conceptual knowledge.

Mathematical Areas Linked to Students' Difficulties with Fractions

Student difficulties with fractions can often be linked to three issues: whole number reasoning, multiple interpretations, and multiple representations.

Three Issues Linked to Student Difficulties with Fractions

1. Whole Number Reasoning
2. Multiple Interpretations
3. Multiple Representations

Learn More...

Chapter 3 further discusses the idea of inappropriate whole number reasoning and how to confront this misconception when comparing and ordering fractions on a number line.

Whole Number Reasoning

A typical sequence in mathematics curricula and instruction starts with whole number reasoning and operations on whole numbers. However, as students begin to understand and make generalizations in this area of mathematics, they often make assumptions that the characteristics of and operations on whole numbers also apply to rational numbers. It is common for students to inaccurately apply what they know and understand about whole numbers and whole number operations to rational numbers.

While some characteristics of whole numbers and rational numbers are similar, there are also many characteristics that are not interchangeable (Siegler et al. 2013). For example, with whole numbers there is an exact counting sequence with an exact successor for each number counted. If a student is counting by ones and has already counted 1, 2, 3, 4, 5, the number "6" is the only correct option for the next number in the sequence. When counting by fractions, however, there are multiple equivalent representations for every possible fraction, so counting in a fraction sequence can yield multiple answers. For example, if students are counting by fourths and begin with $\frac{1}{4}$, the next numbers in the sequence could be $\frac{2}{4}$, $\frac{4}{8}$, $\frac{3}{6}$, 0.50, and so forth. If we think about counting sequences represented on a number line, only one whole number can be represented at one specific point, whereas multiple fractions with the same value can be represented at one specific point on a number line.

Learn More...

As a reminder, the term *rational numbers* comes from the word *ratio*. It means any number that can be made by dividing an integer by a nonzero integer. We first talk about this in the "How to Use This Resource" section.

Also, learners often assume that operating with fractions will result in the same outcomes as operating with whole numbers. For example, when students operate with whole numbers, they often generalize that multiplication produces a larger quantity and division results in a smaller one. The same generalizations do not hold true when operating with fractions:

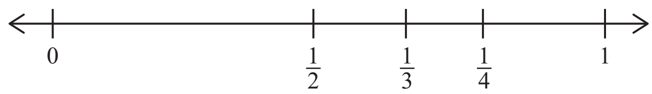
1. When we multiply a given number by a proper fraction, the result is a smaller amount (e.g., $6 \times \frac{1}{2} = 3$).
2. When we divide a given number by a nonzero proper fraction, the result is larger (e.g., $6 \div \frac{1}{2} = 12$).

Learn More...

Chapter 8 and Chapter 9 offer number talks specifically for helping students move beyond assumptions that the characteristics of and operations on whole numbers also apply to rational numbers.

From a procedural perspective, there are also differences between operating on whole numbers and fractions. Traditionally, for example, students must align whole numbers to the right when adding, but they must align the decimal points when adding decimals. In this way, the so-called rules change. Yet without conceptual understanding, students do not have a foundation for understanding these rules or a framework for remembering when to apply specific rules, such as when to cross multiply or find a common denominator. When teachers present the rules without ensuring that they are embedded in students' conceptual understanding, there are many places where confusion may arise.

An example of a situation in which students may inappropriately apply whole number reasoning to fractions is when ordering fractions from least to greatest. Consider the fractions $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$. A common error students make when placing these fractions on a number line is to focus solely on the denominators as whole numbers and then order them by those denominators (in this case, 3, 2, 4). Students who use this inappropriate whole number reasoning—that is, 2 is followed by 3, which is followed by 4—typically order the fractions as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ like this:



Learn More...

Each chapter in Section III, Number Talks to Help Students Operate with Fractions, offers specific suggestions for using contexts with fractions. See Chapters 4–9.

Multiple Interpretations

Another reason many students struggle as they work with fractions is that they can have different meanings or interpretations depending on the context or situation in which they are used (Brousseau, Brousseau, and Warfield 2004; Cramer, Post, and del Mas 2002; National Research Council [NRC] 2001). Without a context, it is difficult to determine if a fraction should be interpreted as a part–whole relationship, a measure, a ratio, a quotient, or an operator. To better understand this idea, see Figure I–1 on the next page, which looks at how context can influence the interpretation of the fraction $\frac{3}{4}$ (Wong and Evans 2007).

Multiple Representations

A third unique challenge that surfaces in understanding fractions is that there are multiple representations for the same quantity. A rational number can be represented as a common fraction, as a decimal fraction, or as a percent. To understand that $\frac{1}{2}$ can be represented and used three ways—as 50%, as an equivalent decimal fraction (0.5, 0.50, 0.500), and as an equivalent fraction ($\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$)—requires reasoning about all representations simultaneously (NRC 2001). In order for students to understand and apply fractional reasoning, they must develop an understanding of equivalence as well as the ability to use all representations interchangeably while considering the unique applications of each representation.

Interpretation	Context
Part–Whole Relationship In this situation the fraction focuses on how many parts there are in relationship to parts in the whole.	$\frac{3}{4}$ of a dozen cookies have sprinkles. $\frac{3}{4}$ of a brownie was eaten. $\frac{3}{4}$ of the jewels are red.
Measure In this situation a fraction represents the measure of a distance, an amount, or a region and focuses on the relationship between the part and the whole. This interpretation requires students to focus on the individual unit and consider how many of those units are within the whole.	A recipe requires $\frac{3}{4}$ of a cup of sugar. Miguel ran $\frac{3}{4}$ of a mile.
Ratio A fraction as a ratio represents a comparison of either a part-to-part or part-to-whole relationship.	3 girls to 4 boys (part-to-part) 3 girls to 4 students (part-to-whole)
Quotient When interpreting a fraction as a quotient, the fraction represents the result when two integers are divided.	3 cookies are shared with 4 people; each person receives $\frac{3}{4}$ of a cookie.
An Operator When a fraction is used as an operator, it is used as a function or a rule to allow the fraction to become a number that acts on another number. The situation in which this occurs is always multiplicative.	Use $\frac{3}{4}$ of 12 feet of ribbon. Sonia ate $\frac{3}{4}$ of 2 pizzas.

Figure I–1. *How Context Influences the Interpretation of the Fraction* $\frac{3}{4}$

Procedural Knowledge Linked to Students' Difficulties with Fractions

Conventional instruction with rational numbers tends to be procedure- or rule-based (NRC 2001). A typical classroom lesson begins with the teacher showing a rule or procedure, asking students to repeatedly practice until they can perform it automatically, then assessing the speed and accuracy with which the procedure or rule is executed.

The statement "Yours is not to wonder why, just invert and multiply" is characteristic of most rational number instruction. The unfortunate message that students receive, whether unintended or not, is that reasoning and conceptual understanding are not important, but memorized algorithms and rules are. In interviews with community college students, Givvin, Stigler, and Thompson (2011) found that 77 percent of the students believed that math was only about memorizing rules and procedures without understanding or sense making.

There is much evidence that teaching by telling rules—particularly before students have developed a conceptual framework and explored opportunities to build their own knowledge of rational numbers—impedes understanding (Mack 1990; NRC 2001; Peck and Jencks 1981; Wearne and Kouba 2000). Even in classrooms with a direct teaching approach, students struggle to correctly execute the algorithms or rules, and few can explain why the procedures work (NRC 2001).

The work done by Van de Walle, Karp, and Bay-Williams (2010) further supports the mounting evidence that memorization of rules hinders sense making. They indicated that teaching rational number procedures before students construct understanding of this system of numbers results in two negative outcomes:

1. Students do not develop a sense of correctness or reasonableness of an answer when they blindly follow memorized rules.

2. Students become confused about which procedures to use for different applications and often forget the memorized algorithms.

Evidence that a conventional approach to teaching fractions has led to a widespread lack of understanding has resulted in gradual shifts in the mathematics content standards toward understanding and reasoning with rational numbers. Documents such as the National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* (2000) provide an initial catalyst in this direction, and the Common Core State Standards for Mathematics lay the groundwork for moving from innumeracy in this area to accuracy, fluency, and efficiency grounded in understanding.

Providing learners with a framework for understanding existing procedures and developing conceptual understanding is a needed shift. Educators and researchers alike express their concerns regarding the premature use of algorithms before students have an opportunity to construct meaning and reason about mathematical relationships (Kamii and Dominick 1997; Mueller, Yankelewitz, and Maher 2010). Too often we place procedures first and hope understanding will follow. Research continues to confirm that conceptual understanding directly impacts students' abilities to understand and correctly apply procedures (Hallett, Nunes, and Bryant 2010; Siegler, Thompson, and Schneider 2011). This same perspective is supported in the National Council of Teachers of Mathematics (NCTM) *Principles to Actions* (2014).

This resource continues to focus on the overwhelming research that points to the importance of building conceptual understanding of fractions, decimals, and percentages before procedural knowledge. The routine of number talks is used as a vehicle to focus on the essential understandings of rational numbers and develop a robust understanding of this realm of number.

Learn More...

The chapters in Section III, Number Talks to Help Students Operate with Fractions, provide number talk strings designed to help students build strategies grounded in understanding when computing with fractions.

Looking Ahead

This introductory chapter addressed a couple of big "whys"—why fractions are critical in the study of mathematics and why fractions are so challenging for students. Chapter 1 continues to build on these ideas, addressing the important role that number talks have in teaching and learning fractions. It takes a focused look at the four foundational principles of number talks, as well as how number talks elicit the Standards for Mathematical Practice.

CHAPTER 5

Number Talks to Help Students Connect Fractions, Decimals, and Percentages

In the first section of this chapter, we explore how a focus on fractions provides a natural connection for number talks with percentages and decimals. We then offer examples of number talks that help students develop an understanding of decimals and percentages as fractions.

OVERVIEW

The Importance of Focusing on Fractions Before Decimals and Percentages

Though fractions, decimals, and percentages are all ways to represent parts of a whole, it's fractional reasoning that forms the basis of understanding for each of these rational number representations. Students often treat decimals and percentages as whole numbers in their reasoning, but in doing so they usually lose the relationship between the parts and the whole. Consider, for example when students are asked, "Which is greater: 0.25 or 0.125?" Students may ignore the decimal point and value and instead think about the decimals as whole numbers. This whole number reasoning results in an inaccurate answer of 0.125, because students view "125" as greater than "25." Likewise, this same inappropriate focus on whole number reasoning sometimes occurs when students are introduced to percentages before they develop a robust understanding of fractional relationships. For example, to add $\frac{1}{4} + \frac{1}{5}$, students may convert the fractions to percentages and change the problem to *25% + 20%*. While this may be efficient, and the answers of $\frac{9}{20}$ and 45% are correct because they are equivalent, students can easily ignore the part–whole relationship and the meaning of the fractions. These are all reasons why, as teachers, we should initially focus on fractions with learners before transitioning students to the study of decimals and percentages.

Number Talks That Connect Fractions to Percentages

Students are likely familiar with many real-world situations that refer to percentages. Here are a few examples.

- There is a 50% chance of rain.
- 85% of the test questions are correct.
- 75% of the computer program is downloaded.
- The diners left a 20% tip.
- Jeans are 40% off.

As teachers, we can use students' informal knowledge of percentages to help them make a connection to fractions. For example, we could ask students, "When have you heard the word *percent*?" Then take one of their examples, such as "I got 75% of the questions right on my test," and ask, "How could you also use a fraction to describe the percent of the test questions you answered correctly?"

The word *percent* is derived from the Latin root *percentum*, which means one part in 100. A percentage represents a quantity out of 100 and connects to fractions with a denominator of 100. For example, 52% can also be represented as $\frac{52}{100}$. It's not usually a challenge for students to figure out the percent when the fraction is already presented in hundredths, but in order for students to find an equivalent percentage with other fractions, they need to think about the ratio of the fraction to hundredths.

Let's say we ask students to solve for x in this problem:

$$\frac{1}{4} = \frac{x}{100} = x\%$$

To find this ratio, students must repartition a whole that is in fourths into one that is in hundredths. One way students have solved this problem is to draw two candy bars with both "wholes" or candy bars the same size. (See Figure 5–1.) Candy Bar A is represented using fourths, while Candy Bar B is represented using percentages.

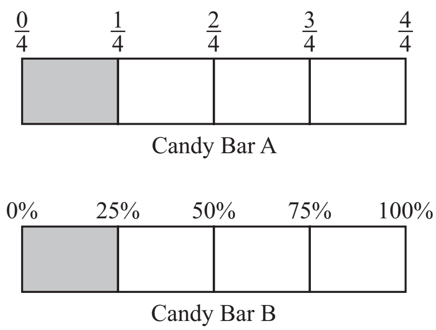


Figure 5–1. *Using Candy Bars to Represent Thinking About the Problem,* $\frac{1}{4} = \frac{x}{100} = x\%$

As teachers, we can anticipate students may use their understanding that one-half of the candy bar would be the same as 50% and half of the half would be $\frac{1}{4}$ or 25%. Halves, fourths, and tenths are important benchmarks for fractions; these benchmarks serve as anchors for students as they think about percentages. Students can use these benchmarks to make relationships with common percentages such as 10%, 25%, and 50%. It is essential that we help students develop reasoning around benchmark fractions and the fractions' percent equivalents because this is a first step for learners in developing reasoning with percentages.

Let's look at how this reasoning may apply when we ask students to find a percentage of a whole number. Let's say we ask students to consider the following story problem.

The School Dance Problem

If the school needs $300 for the school dance and your class has to raise 25% of the money, how much money would your class need to raise?

Think about how students might divide 300 into fourths. Figure 5–2 shows a model students use to show how $300 can be partitioned into fourths and how $75 is equal to 25% of the money.

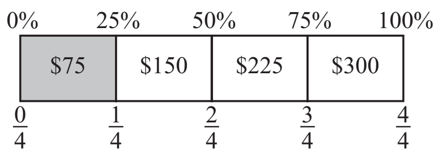


Figure 5–2. *Using an Area Model to Show Fraction and Percentage Equivalents for $300 Partitioned into Fourths*

The next example shows how benchmarks can be used to find 60% of $70. Students often reason that 50% of $70 would be $35 and 10% of $70 would be $7, so 60% of $70 would be $35 + $7 or $42. In this way, students can use what they know (benchmarks) to figure out what they do not yet know. They could also represent their thinking using an area model. (See Figure 5–3.)

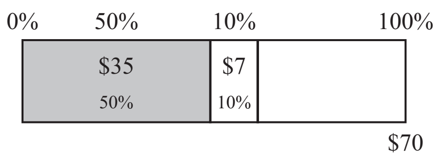


Figure 5–3. *Using an Area Model to Represent 60% of $70*

This same strategy can be captured numerically to highlight the distributive property at work. (See Figure 5–4.)

$$
\begin{aligned}
&60\% \text{ of } \$70 \\
&= (50\% + 10\%) \times \$70 \\
&= (50\% \times \$70) + (10\% \times \$70) \\
&= \$35 + \$7 \\
&= \$42
\end{aligned}
$$

Figure 5–4. *Using the Distributive Property to Solve 60% of $70*

Learn More...

Benchmark fractions and unit fractions are often entry points for students to connect fractions with percentages. See Chapter 4 for further discussion about benchmark and unit fractions.

Students also use unit fractions to find percentages. Think about how learners might write $\frac{4}{5}$ as a percentage. The standard procedure is to divide the 5 into the 4 until students have an answer in the hundredths place. By following this procedure, learners derive a quotient of 0.8, which converts into 0.80 or $\frac{80}{100}$ or 80%. In contrast, students who are fluent with unit fractions may solve this problem by thinking about how $\frac{1}{5}$ equals 20% and $\frac{4}{5}$ equals four times 20% or 80%. Figure 5–5 shows two ways students might explain their thinking for this problem.

Strategy 1	Strategy 2
$\frac{4}{5} = \_\_\_\_\%$ $\frac{1}{5} = 20\%$ $20\% \times 4 = 80\%$ *I know $\frac{1}{5}$ of 100 is 20%. Since I need four $\frac{1}{5}$s, I need to multiply 20% times 4.*	$\frac{4}{5} = \_\_\_\_\%$ $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \_\_\_\_\%$ $20\% + 20\% + 20\% + 20\% = 80\%$ *I know $\frac{4}{5}$ can be broken apart into four $\frac{1}{5}$s. Since $\frac{1}{5}$ is the same as 20%, I can add 20% four times for an answer of 80%.*

Figure 5–5. *Student Reasoning for $\frac{4}{5} = \_\_\_\_\%$*

CHAPTER 9

Number Talks for Division with Fractions

In this chapter, we offer a starting place to help in the planning of number talks around the division of fractions. In the opening sections, we discuss why the use of number talks is helpful in developing division strategies with fractions. We then explore and offer number talks that are focused on helping students develop strategies to build their understanding of dividing fractions. It is highly recommended that students should be solid and flexible in their fractional reasoning (see Chapter 4) before using the number talks in this chapter.

OVERVIEW

How Number Talks Help the Development of Division Strategies with Fractions

Division with fractions is a complex part of mathematics that requires multiplicative reasoning and a thorough understanding of what it means to divide. Many students are familiar with the interpretation of division with whole numbers as a "sharing" situation but have not developed a complete understanding of division. When division with fractions is taught from a procedural perspective without developing a conceptual understanding first, students struggle to make sense of the operation.

Learn More...

For more insights on procedural versus conceptual understanding, see the Introduction section, "Procedural Knowledge Linked to Students' Difficulties with Fractions."

If asked to solve the problem $\frac{7}{9} \div \frac{2}{3}$, do you automatically think, "keep, change, flip" or "yours is not to reason why, just invert and multiply"? When this problem was given to prospective middle school teachers, 93 percent could accurately solve this problem using procedural knowledge. However, when the same math methods students were given the problem, "How many $\frac{1}{2}$s are in $\frac{1}{3}$?" which requires conceptual understanding, only 52 percent gave a correct answer. The university students could think about division of fractions procedurally but lacked conceptual understanding (Li and Smith 2007).

This lack of conceptual understanding affects our interpretation of what it means to divide with fractions. A common misconception is that students often interpret dividing by 2 and dividing by $\frac{1}{2}$ as synonymous (Isik 2011; Mack 1995; Rizvi 2007). Consider the following two story problems; which scenario represents $1\frac{1}{4} \div \frac{1}{2}$?

> *Malika has $1\frac{1}{4}$ cakes left from the party. She shared this amount with a friend. How much cake will they each get?*
>
> *Roberto has $1\frac{1}{4}$ yards of bubble wrap. It takes $\frac{1}{2}$ of a yard to wrap a package for shipping. How many packages can Roberto wrap?*

Because Malika is sharing the cake between two people, this situation is actually about dividing by 2 rather than dividing by $\frac{1}{2}$. Many students mistakenly interpret dividing by $\frac{1}{2}$ as splitting something into two equal parts. The second scenario is asking, "How many halves are in $1\frac{1}{4}$?," which is a quotative interpretation of division and matches the expression, $1\frac{1}{4} \div \frac{1}{2}$.

This is where the importance of number talks comes into play: Number talks help students focus on developing strategies to build their conceptual understanding of dividing fractions. Specifically, there are approaches that we can take during a number talk to support students in inventing and developing strategies—and thus deepen their understanding of mathematics. These approaches include:

- Making connections to whole number computation
- Understanding the shifting whole
- Using story problems
- Using models

Each of these approaches is discussed in more detail in the sections that follow.

Making Connections to Whole Number Computation

Will the strategies used for dividing whole numbers also work with fractions? This is an important question to pose to students when transitioning into division with fractions. This idea can be tested by starting with the whole number problem, $300 \div 25$. Ask students, "What are some ways we can solve this problem without using the U.S. standard algorithm?" Once students have indicated that they have at least one way in mind, then ask, "Now consider the strategies you used and test them with the problem $\frac{3}{4} \div \frac{1}{2}$. Do your strategies still work?" Figure 9–1 shows three whole number division strategies for solving $300 \div 25$ and applies them to solving $\frac{3}{4} \div \frac{1}{2}$.

Students may assume if whole number division strategies work with fractions, then other generalizations will also hold true. For example, when dividing a positive number by a whole number greater

Division Strategy	Recording for $300 \div 25$	Recording for $\frac{3}{4} \div \frac{1}{2}$
Proportional Reasoning	$300 \div 25$ $= (300 \times 4) \div (25 \times 4)$ $= 1200 \div 100$ $= 12$	$\frac{3}{4} \div \frac{1}{2}$ $= \left(\frac{3}{4} \times 2\right) \div \left(\frac{1}{2} \times 2\right)$ $= 1\frac{1}{2} \div 1$ $= 1\frac{1}{2}$

Figure 9–1. *Three Whole Number Division Strategies for $300 \div 25$ and $\frac{3}{4} \div \frac{1}{2}$* *(continues)*

Division Strategy	Recording for $300 \div 25$	Recording for $\frac{3}{4} \div \frac{1}{2}$
Partial Quotients	$300 \div 25$ $= (100 + 100 + 100) \div 25$ $= (100 \div 25) + (100 \div 25) + (100 \div 25)$ $= 4 + 4 + 4$ $= 12$	$\frac{3}{4} \div \frac{1}{2}$ $= \left(\frac{1}{2} + \frac{1}{4}\right) \div \frac{1}{2}$ $= \left(\frac{1}{2} \div \frac{1}{2}\right) + \left(\frac{1}{4} \div \frac{1}{2}\right)$ $= 1 + \frac{1}{2}$ $= 1\frac{1}{2}$
Multiplying Up	$300 \div 25$ $25 \times \_\_\_\_ = 300$ $25 \times 4 = 100$ $25 \times 4 = 100$ $25 \times 4 = 100$ $25 \times \underline{12} = 300$	$\frac{3}{4} \div \frac{1}{2}$ $\frac{1}{2} \times \_\_\_\_ = \frac{3}{4}$ $\frac{1}{2} \times 1 = \frac{1}{2}$ $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ $\frac{1}{2} \times \underline{1\frac{1}{2}} = \frac{3}{4}$

Figure 9–1. *Continued*

than one, the quotient is always smaller than the dividend. Does this generalization hold true when dividing fractions? Figure 9–2 on the next page provides an opportunity to investigate this idea.

When dividing a positive number by a whole number greater than one, the quotient is always *smaller* than the dividend because the whole amount is being divided into groups. Using the quotative model for division, you partition the dividend into groups of a given size and determine how many groups there are. When the size of each group is greater than one, the number of groups will be smaller than the dividend. For example, if we have 12 candies and put them into equal-size groups, as long as there is more than one piece in a group, we will have fewer than 12 groups.

In contrast, when dividing a positive number by a proper fraction, the quotient is *larger* than the dividend. For example, for $\frac{4}{5} \div \frac{1}{2}$, we are asking how many groups of size $\frac{1}{2}$ are in $\frac{4}{5}$. When the size of each group is less than one, the number of groups will be larger than the dividend. Figure 9–3 provides problems to help us consider these ideas.

Division Problems with Whole Numbers	Division Problems with Fractions
$6 \div 3 = 2$ $24 \div 4 = 6$ $156 \div 3 = 52$ $300 \div 15 = 20$	$\frac{1}{5} \div 2 = \frac{1}{10}$ $\frac{1}{4} \div \frac{1}{8} = 2$ $\frac{3}{5} \div \frac{1}{4} = 2\frac{2}{5}$ $\frac{2}{3} \div \frac{4}{3} = \frac{1}{2}$

Figure 9–2. *Do the Generalizations for Dividing Whole Numbers Apply When Dividing Fractions?*

Division Structure	Examples	Generalization
Positive Number ÷ Whole Number Greater than 1	$6 \div 2 = 3$ $2 \div \frac{3}{2} = 1\frac{1}{3}$ $\frac{1}{5} \div 2 = \frac{1}{10}$ $\frac{3}{4} \div \frac{5}{4} = \frac{3}{5}$	Quotient is smaller than the dividend
Positive Number ÷ Proper Fraction	$6 \div \frac{1}{2} = 12$ $\frac{1}{5} \div \frac{1}{2} = \frac{2}{5}$ $\frac{4}{3} \div \frac{2}{3} = 2$ $\frac{7}{8} \div \frac{3}{4} = 1\frac{1}{6}$	Quotient is larger than the dividend

Figure 9–3. *Developing Generalizations with Division*

When we ask students to consider whether whole number strategies will work with rational numbers, we are purposefully providing an opportunity for students to explore regularity in repeated reasoning (MP8) and think about mathematical structures (MP7).

These are important generalizations students can construct for themselves without being told. Using number talks and purposefully crafted classroom tasks to provide opportunities for students to investigate and explore relationships between factors and products and dividends, divisors, and quotients help students to consider these important ideas.

Understanding the Shifting Whole

Just as with multiplication of fractions, the idea of the shifting whole also occurs when dividing fractions. Consider this idea through the story context and area model in Figure 9–4.

Wanda plans to make hair bows for her daughter.
She has $\frac{3}{4}$ of a yard of ribbon. Each bow requires $\frac{1}{8}$ of a yard.
How many bows can Wanda make?

Figure 9–4. *The Shifting Whole: Thinking About the Original Whole and the "New" Whole*

This question is asking how many $\frac{1}{8}$s of a yard are in $\frac{3}{4}$ of a yard. One yard of fabric is the original whole, and the $\frac{3}{4}$ of a yard shaded above becomes the "new" whole. To find how many $\frac{1}{8}$s are in $\frac{3}{4}$ of a yard, we can repartition the original whole yard into eighths. Six $\frac{1}{8}$s of a yard are in $\frac{3}{4}$ of a yard. (See Figure 9–5.)

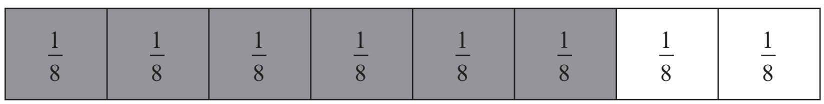


Figure 9–5. *Repartitioning the Whole to Find How Many $\frac{1}{8}$s Are in $\frac{3}{4}$ of a Yard*

Thinking about the relationship between the original whole and the "new" whole created when considering $\frac{3}{4}$ of the yard of fabric is a complex idea. It should not be surprising that this shifting whole can be problematic for many students.

Praise for *Number Talks: Fractions, Decimals, and Percentages . . .*

In this essential resource Sherry and Ann tackle one of the most important content areas in all of school mathematics—fractions. Building on the proven effectiveness of number talks, they show teachers a clear picture of what classrooms can look like and offer concrete steps to stimulate the kind of rich discourse that helps students make sense of numbers, gain confidence, and develop the critical skills they need for the future.

—Cathy L. Seeley, NCTM Past President, Author of *Faster Isn't Smarter* and *Smarter Than We Think,* and Educational Speaker

Number Talks \ˈnəm-bər\ \ˈtōks\

1. A five- to fifteen-minute classroom conversation around purposefully crafted problems that are solved mentally.
2. The best part of a teacher's day.

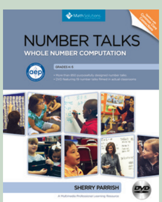
This dynamic multimedia resource was created in response to the requests of teachers—those who want to implement number talks but are unsure of how to begin and those with experience who want more guidance in crafting purposeful problems. It supports teachers in understanding:

- what a classroom number talk is;
- how to follow students' thinking and pose the right questions to build understanding;
- how to prepare for and design purposeful number talks; and
- how to develop fractional reasoning and strategies for operating with fractions, decimals, and percentages.

***Number Talks* supports the Common Core State Standards for Mathematics.** It also includes an abundance of reference tables to help you quickly and easily locate strategies, number talks, and video clips.

Video Clips

The video clips provide a visual platform for teachers to reflect on their current practices and target essential understandings from their readings. The video clips feature number talks filmed in actual classrooms grades 3-7. Clips range from five to ten minutes in length with a total viewing time of approximately two hours.



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About the Authors

Sherry Parrish has spent more than 35 years in the field of education. Sherry holds a doctorate in Educational Leadership from Samford University and is a recipient of the Presidential Award for Excellence in Mathematics Teaching. As a former classroom educator and assistant professor, she advocates for the teaching and learning of mathematics through inquiry. She is the author of the award-winning book, *Number Talks: Whole Number Computation, Grades K–5,* and currently works with teachers nationally and internationally in the area of mathematics.

Ann Dominick received her undergraduate degree in education from Auburn University, a Master's in Education from the University of Alabama at Birmingham, a doctorate from Vanderbilt University, and a Certificate of Advanced Studies in Curriculum and Instruction at Harvard University. She was awarded the National Science Foundation's Presidential Award for Excellence in Teaching Elementary Mathematics and is a former Alabama State Teacher of the Year. She is currently an Assistant Professor at the University of Alabama at Birmingham teaching mathematics and education courses and preparing undergraduates to be secondary mathematics and science teachers.

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