Uncovering Misconceptions and Errors

Classroom talk not only helps students develop conceptual understanding, but also is effective for revealing and clarifying students' partial understandings and misconceptions. In the process of making sense of experiences, students often generalize ideas in incomplete ways. For example, as seen in Chapter 2, if young students' experiences with triangles involve only viewing pictures of equilateral triangles oriented along a base, their generalization of the concept "triangle" may be very limited—they might conclude that scalene triangles are not triangles or that a triangle that is resting on a vertex is not a triangle. Talk is a powerful tool for revealing students' partial understandings of a concept or their misconceptions about that concept.

NCTM Standard:
Number and Operations

Grade 5

Mr. Lyman used talk to learn about his fifth-grade students' generalizations (and misconceptions). One topic that Mr. Lyman felt was not too difficult for his students was decimal addition and subtraction. Yet he noticed that when he didn't list the numbers vertically, one under the other with the decimal points lined up, many students made errors. Because of comments by a few students, he began to suspect that their understanding of decimals was procedural and not very complete. One day, when students were working in small groups on decimal word problems, he noticed that one group of four students incorrectly solved the computation 25 − 17.7 by setting up the exercise with the 2 and 5 underneath the two 7s:

$$\begin{array}{r} 17.7 \\ -\,2\;5 \\ \hline \end{array}$$

Mr. Lyman decided to probe the understanding of the students in the small group.

1. Mr. L: I noticed that you have been adding and subtracting decimal numbers to solve these word problems. How are decimals the same or different from whole numbers? How is adding and subtracting decimals the same or different from adding and subtracting whole numbers? Take a minute to think about what you know about decimal numbers and how we add and subtract them. Then I'll talk with your group about this.

After giving them a couple of minute to talk among themselves, Mr. Lyman listened carefully to his students' ideas. At first he allowed them to make their points without much commentary from him, but then he began to ask clarifying questions as alternative understandings emerged.

2. Sarah: Decimals are like regular numbers; you just line them up and add them.
3. Corey: Yeah, it's easy because we all know how to add and subtract.
4. Kei: Decimals are numbers with a dot in them.
5. Mr. L: Sarah, what do you mean by "like regular numbers"?
6. Sarah: You know, like the numbers we use to count with—tens, hundreds, thousands.
7. Mr. L: So decimals are exactly the same as the counting numbers?
8. Sarah: Yes.
9. Mr. L: What do other people think? Are decimals the same or different from our regular counting numbers?
10. Kei: They are the same. You just line everything up on the right side and then add or subtract.
11. Mr. L: Could you give us an example, Kei?
12. Kei: Sure. Eight point two plus seven point nine. [Kei records and solves the addition problem correctly.] I lined up the two and the nine on the right and then just added. You bring the decimal point straight down.

$$\begin{array}{r} 8.2 \\ +\ 7.9 \\ \hline 16.1 \end{array}$$

13. Bob: I think you have to line up the decimal points.
14. Mr. L: What do you mean?
15. Bob: Well, if the problem was eight point twenty-five plus seven point nine, you don't line up the five and the nine. [Bob writes the following on his paper as he speaks to the group.]

$$\begin{array}{r} 8.25 \\ +\ 7.9 \\ \hline \end{array}$$

16. Mr. L: Bob, would you please say those numbers again but read them so we hear the place values?

17. Bob: Sure. Let's add eight and twenty-five hundredths and seven and nine tenths. I think we should line up the decimal points, not the last two numbers. It should look like this. [Bob records the addition on his paper.]

$$\begin{array}{r} 8.25 \\ +\ 7.9 \\ \hline \end{array}$$

Teachers regularly must decide how to respond to an interaction similar to this one. One response is to tell students how to line up the numbers and to offer some explanation as to why this is the case. Another possible teacher response is to ask Bob to explain his reasoning in order to establish for the group the correct procedure. This has the benefit of another student providing the justification for the choice. We have found that a third alternative that uses talk productively is for the teacher to give no indication of the right or wrong answer—in this case how to line up the decimal numbers—and instead to send the question back to the students.

18. Mr. L: Would you all discuss what Bob just said? Do we line up decimal numbers using the last digit or the decimal point? Also be ready to explain why you think your choice is correct.

Why use talk in this way? What are the benefits? Both the teacher and Bob understand the mathematics in this situation. But it is unclear exactly what the other three students in the group do and don't understand. By forcing them to engage with the ideas—namely, to take a position on how to line up the numbers—the teacher has prompted them to think more deeply about the mathematics. They will be listening more closely to each other now to see if their ideas are the same. They will be working to come up with a way to explain why their response is correct. If the reasoning behind their ideas is faulty (line up the last number on the right), other students will usually offer explanations, evidence, or counterexamples in their own words, words that often resonate more effectively with the other students. Bob will be able to add his ideas to the group in response to the other students, which will help move them all forward to understanding they need to line up the decimal points. But the main advantage of talking in this way is that the three students who will benefit the most mathematically from the exchange are being forced to talk and reason about the ideas.

19. Sarah: I don't think it matters which way you do it.

20. Corey: Let's use one of the word problems to see if it makes a difference. [She reads] "Gus put four gallons of gasoline into the gas can. He kept filling and adding another seven-tenths of a gallon. How much gas is now in the can?"

21. Kei: I think maybe Bob is right. I thought you just lined up the numbers but if you add four and point seven like this [Kei writes the problem down], the answer is wrong; it's too small. Like four gallons of gas plus seven-tenths more is more than four.

$$\begin{array}{r} 4 \\ +\ 0.7 \\ \hline \end{array}$$

22. Bob: I think it is because we have to add the same things—like we add hundreds and hundreds with big numbers so now we have to add tenths and tenths or ones and ones.

23. Corey: But where are the tenths in four? It kinda makes sense but not completely.

Mr. Lyman has already learned a lot using the small-group format for this short discussion. First, three of the students in the group did not think there was a difference between whole numbers and decimal numbers. He will need to probe further to really understand what they think. However, he doubts that many students in his class think of a decimal number as the sum of a whole number and a part of a whole. He now suspects that many students' conceptions of decimals are unrelated to quantity and that they instead think that a decimal is a number with a "dot" in it. When adding decimals, they do not consider the size of the numbers or the place values of the digits. By listening to students as they discussed their ideas in a small-group format, Mr. Lyman was able to hear from each student and learn about their superficial understanding. Sometimes talking with a small group can clarify and extend concepts for students, but in this case, Mr. Lyman realized that small-group talk was not enough and additional instruction would be necessary. Mr. Lyman used productive talk to get mathematical ideas and concepts visibly out on the table. Both he and the students benefited from this interaction; Mr. Lyman gained valuable information on which to build future lessons, and these students had an opportunity to start making sense of the addition algorithm for decimals.