Estimating Length: Differentiating Within a Unit

A Lesson with Second Graders

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All teachers have students with a range of mathematical abilities and understandings in their classrooms. In this lesson on estimation and measurement, the teacher differentiates three aspects of the curriculum—content, process, and products. This lesson is excerpted from Math for All: Differentiating Instruction, Grades K–2, *by Linda Dacey and Rebeka Eston Salemi (Math Solutions Publications, 2007).*

This second-grade teacher knows that a big idea in measurement is the relationship between the number of units and the relative size of those units. Understanding that 1 foot is longer than 8 inches is confusing or counterintuitive for many children. They are swayed by the greater number and lose sight of the size of the unit. The teacher recognizes that young children need considerable exposure to different units of measure to comprehend this inverse relationship. She wants all of her students to build preliminary concepts related to this idea, even if it's as simple as recognizing that it will take fewer giant steps than baby steps to walk the length of the classroom. The teacher decides to incorporate this big idea into the estimation process, but first, she wants to informally pre-assess her students. She wants to capture an initial perspective on their thinking, a benchmark with which she can compare later.

She'll launch the unit by asking students to estimate the length of an object using different units of measure. The activity will allow her to get a feel for her students' common understanding and to identify those students who may need more or less support in this area. She'll observe her students carefully as they work and make anecdotal records. She'll be able to use these data to inform her planning of the subsequent lessons.

She asks students to get out their math journals and pencils and gather around the art table. She shows them a craft stick and asks, "How could we measure the length of this table in craft sticks?"

"We could get a bunch and line them up," offers Sandra.

"We could use one and keep putting it down," suggests Mark.

"We could measure the stick with a ruler. You use a ruler to measure length," explains Lynne.

Already, the teacher is seeing variation in the students' thinking. Sandra's suggestion of repeated units is at a more concrete level than Mark's notion of iteration. Lynne is already thinking about the use of a measurement tool, though the teacher wonders what Lynne really means. Does Lynne realize that she can measure without a ruler as long as she uses a uniform unit? Might Lynne realize that if she finds that the craft stick is about 4 inches long, she can count by fours as she places the sticks end-to-end?

Next the teacher asks the children to imagine using one of these methods to estimate how many craft sticks it will take to measure the length of the table. She tells the students, "Look at the craft stick on the table and think about how many you would need to measure this whole length. Without talking to anyone, I want you to write that number in your journals."

Once the students have recorded their estimates, she puts six craft sticks end-to-end along a portion of the table's edge. Some of the children express surprise at how much of the table's length these sticks cover. Others look pleased, as if the placement of the sticks affirms their thinking. "Now," says the teacher, "I want you to look at these sticks and think about your estimate again. How many craft sticks do you think it will take to measure this whole length? Write your answer below your first estimate. Think, do you want to change your number or keep the same estimate?" Once the estimates are recorded, the teacher places craft sticks end-to-end until the entire length is measured. The children count together to find that the table is almost fifteen craft sticks long.

Next, the teacher asks the children to draw a line that they think is 1 inch long in their journals. After noting the different lengths the students have drawn, the teacher represents a length of 1 inch by holding up her thumb and pointer finger about an inch apart. "This is about an inch and now I want you to think about measuring the table in inches. How many inches long do you think this table is?" she queries. She asks them to record this answer in their journals as well. Once again a variety of answers are provided. (See Figure 1.)

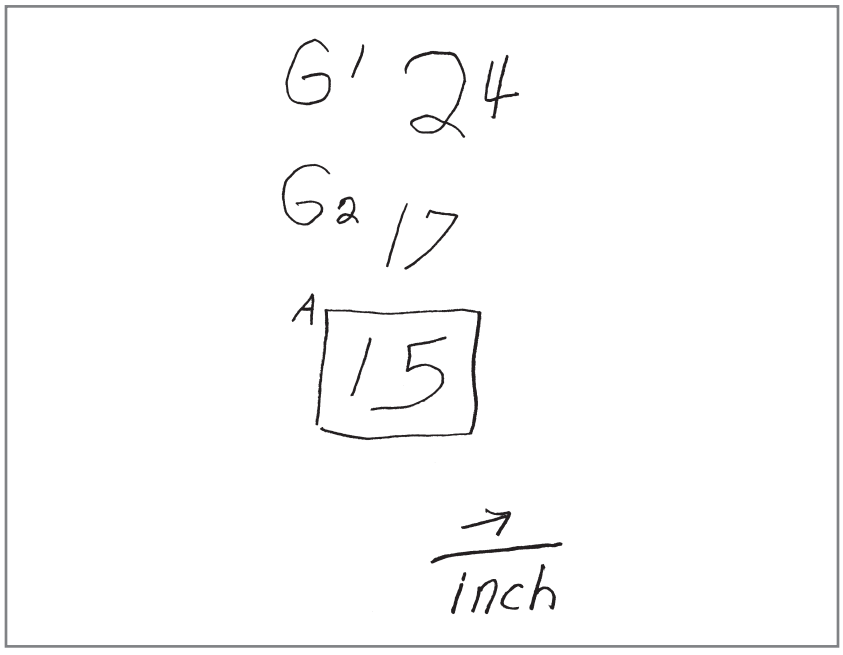

Figure 1. Terrance's initial journal response.

Abby makes a drawing to decide the relationship between the length of an inch and the length of the craft stick. (See Figure 2.) She traces a stick, estimates the distance of 1 inch from the bottom of the stick, and places a mark there. Based on this visual model, she decides that 3 inches is the same length as one craft stick and begins to make a list of numbers, counting by threes. She makes a couple of false starts, becomes frustrated, erases her work, and begins again. This time, she makes a list of the numbers one through fifteen. Her teacher overhears her say, "I know I need fifteen sticks." Next she draws a line to the left of her list and proceeds to count by twos to thirty. Finally, she makes a third column to the right by adding the two numbers to the left, for example, in the first line she adds two and one to get a sum of three. When the teacher asks about her thinking, Abby replies, "I don't know how to count by threes, so I counted by ones, and counted by twos, and put them together. So I got forty-five inches." The teacher is impressed with Abby's strategy for listing multiples of three. She also recognizes Abby's ability to use her understanding of the number of inches in each stick to estimate the length of the table in inches. Though the craft stick is closer to 4 inches long, 3 is a reasonable choice, particularly given that Abby overestimated the length of an inch in her drawing.

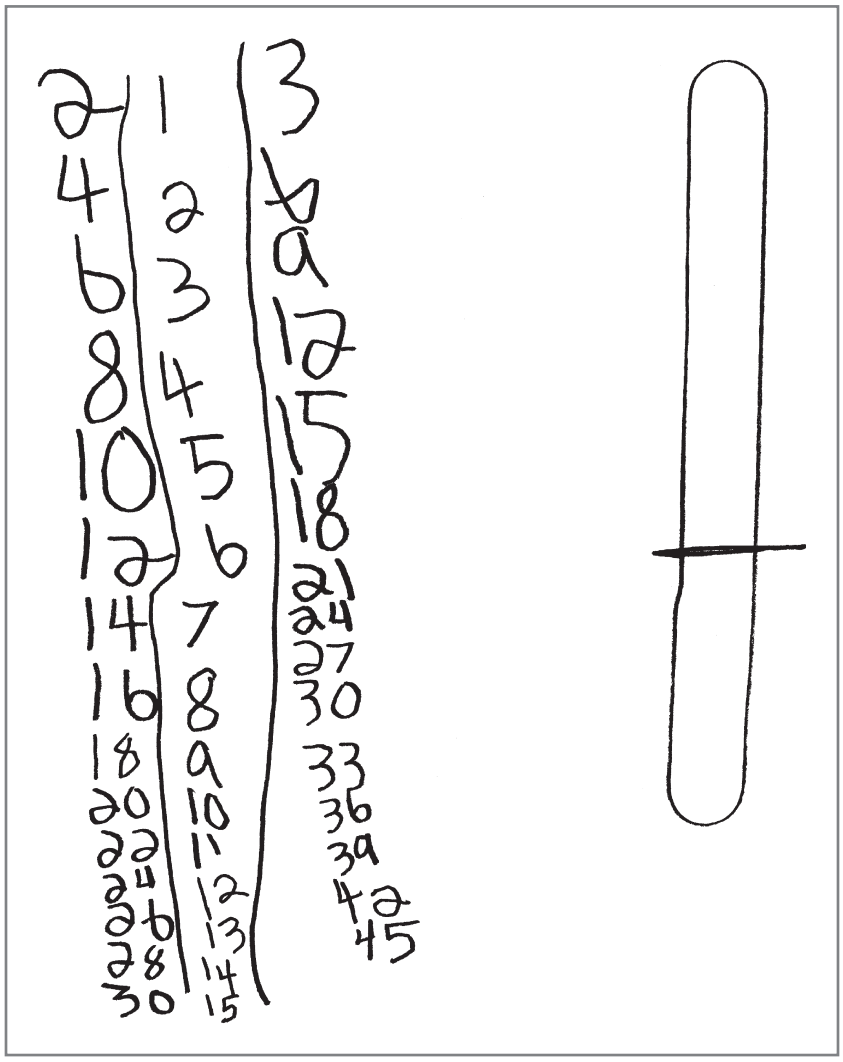

Figure 2. Abby's chart and drawing.

This activity quickly engages children in the measurement process and it gives the teacher some important information. Based on observations and student recordings, she can determine students' abilities to make initial estimates and whether their second estimates become more accurate. She has an indication of their abilities to estimate an inch. She can gain preliminary ideas about which students recognize the significance of comparing the length of an inch to the length of the craft stick, whether they recognize that an inch is shorter than the length of the craft stick, and if they use the relationship between a craft stick and an inch to estimate the table's length in inches. While keeping the mathematical standards in mind, she can use this cursory data to make initial decisions about adapting the *content* for different students. The lengths of items to be measured can vary. Some students can measure straight lengths while others can measure "crooked paths." She can have some students measure more lengths in order to better conceptualize the relative length of an inch while others work with different traditional units. Comparisons of *unit lengths* and *number of units needed* can involve informal or traditional units of measure.

Once content variations are determined, *process* is considered. The teacher has some students make "inch and foot strips" so that they can have a single model of each unit with them when they make their estimates. She posts some length measures around the room so that students can choose to use these visual images of length. She also makes inch and foot strips of Velcro® and puts them on the wall near the door. She has some of the students close their eyes and run their fingers along these strips whenever they enter or leave the classroom. She encourages these same students to pass their fingers over the lengths of the objects they are estimating before they make their estimates. Initially, she lets students choose the lengths they want to estimate, so that they can begin with items that are of interest. She provides some students with measuring strips cut to 1-inch lengths and they are encouraged to use as many as needed, while others have only one strip and must reuse it to measure. She thinks about pairs of children that will work well together during this unit and subsets of students that she wants to bring together for some focused instruction.

Then the teacher must think about *product*—how her students can demonstrate their ability to estimate lengths at the end of the unit. For example, students might write an explanation of the estimation process, pretend they are interviewing for an estimation job and explain how their skills and experiences will help them to be an effective estimator, teach a kindergarten student something about estimation, estimate ribbon lengths needed for an art project, or participate in an estimation Olympics.

It's not necessary, or even possible, to always differentiate these three aspects of curriculum, but thinking about differentiating content, process, and products prompts teachers to

- identify the mathematical skills and abilities that students should gain and connect them to big ideas;
- pre-assess readiness levels to determine specific mathematical strengths and weaknesses;
- develop mathematical ideas through a variety of learning modalities and preferences;

- provide choices for students to make during mathematical instruction;
- make connections among mathematics, other subject areas, and students' interests; and
- provide a variety of ways in which students can demonstrate their understanding of mathematical concepts and acquisition of mathematical skills.

It is also not likely that all attempts to differentiate will be successful, but keeping differentiation in mind as we plan and reflect on our mathematics instruction is important and can transform teaching in important ways. It reminds us of the constant need to fine-tune, adjust, redirect, and evaluate learning in our classrooms