Measuring Angles

A Lesson for Seventh and Eighth Graders

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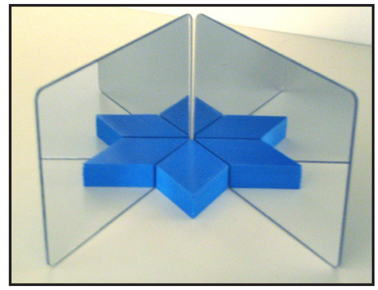


In this lesson, the first day of a six-day sequence of activities presented in chapter 6 of Marilyn Burns and Cathy Humphreys's A Collection of Math Lessons, From Grades 6 Through 8 *(Math Solutions Publications, 1990), students use hinged mirrors to figure out the number of degrees in each of the angles of pattern blocks. The activity gives students a valuable concrete experience with measuring angles that is effective for extending into further investigations of angles and later helping students learn to use protractors.*

"I'm interested in finding out what you know about angles," I said to the class.

"They have something to do with triangles and rectangles," Jose said.

"They have degrees," Erica said.

Patty asked, "Isn't an angle like not straight up and down? Slanted?" I asked Patty to draw on the overhead what she meant. She drew an acute angle.

"That's an acute angle," Paul said.

"What do you know about acute angles?" I asked Paul.

"They're pointy," he answered, "not real open." I wrote *acute angle* next to the angle Patty had drawn.

"Acute angles have less than 90 degrees," Jeremia added.

"I know about right angles," Russell said. "They're like Ls." I drew a right angle on the overhead and labeled it.

"You know when you're skiing, and you turn all the way around?" Cheryl asked. "Isn't that like three hundred sixty degrees?"

"Yes," I answered.

"It's the same on a skateboard," Herman said. "You do three-sixty turns, and one-eighties, too."

"Does anyone know anything else about angles or have another question?" I asked.

"Does a right angle have 90 degrees?" Jennie asked.

"Yes," I answered. "Can anyone explain why a right angle has ninety degrees?"

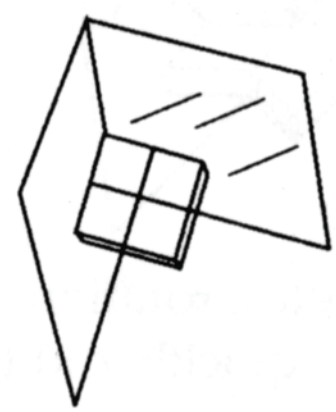
Patty said, "I think it has to do with what Cheryl said about three hundred sixty degrees and dividing it up." Though others knew that a right angle measured 90 degrees, no one else had any idea about why.

The students brought a variety of ideas to this discussion. Most seemed to have the general understanding that an angle is the shape of a corner. Fewer, however, seemed to know about measuring the size of an angle.

I then told the students what they were to do. "Today you're going to be exploring angles," I said. "I'll show you how to measure all the different-size angles of the pattern blocks using a pair of hinged mirrors. For this activity, you'll be using the information that three hundred sixty degrees makes a complete circle, as Cheryl and Herman said." If Cheryl and Herman hadn't come forth with their examples, I would have told the students that there are 360 degrees in one complete rotation. This is a fact, an arbitrary piece of information that students need to know.

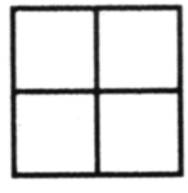
I gathered the students around one group's table to show them how to use the hinged mirrors to measure the angles of the pattern blocks. The students had had enough previous experience with pattern blocks that they didn't need time to explore them.

I placed a corner of an orange square into the corner of a pair of hinged mirrors and closed the sides so that they were touching the sides of the square and the square was firmly nestled. I had students do the same with three other pairs of mirrors so that more of the class could see.



"What do you see when you look into the hinged mirrors?" I asked.

"There are four squares," Alisa said, "three in the mirrors and the one on the table." There was much interest among the students to explore with the mirrors. I put four squares on the desk, and Alisa arranged them to show what she saw.



"As Cheryl and Herman said," I continued, "it takes three hundred sixty degrees to make a complete circle. Putting the square in the mirrors shows that it takes the corners of four squares to fill a complete rotation. So one square corner is one-fourth of three hundred sixty degrees."

Some of the students understood this immediately; others looked perplexed.

"How much is one-fourth of three hundred sixty degrees?" I asked. Some students figured quickly that it was 90 degrees.

"So a square corner, which is called a *right angle,* equals ninety degrees," I said. "Notice that all four angles of the squares are the same size. They're congruent. Some of the pattern blocks have all congruent angles and some don't."

Some students called out what they noticed. "They're all the same on the green triangle."

"On the yellow block, too."

"They're different on the blue diamond."

I had the students seated at the table take one of each of the six blocks and sort them into two groups: those with all angles congruent and those with different-size angles.

I then used the blue rhombus to model what to do with blocks with angles of more than one size. "You can place the blue block in the hinged mirrors two different ways," I said. "Which vertex you put in the corner of the mirrors determines the reflection you see."

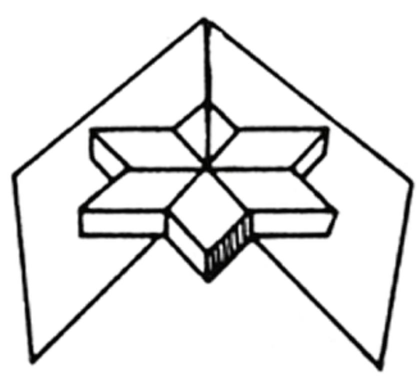
"What's vertex?" Jennie asked.

"It's another word for the corner of a polygon or the point of an angle," I said.

There were pleased reactions when the students saw the pattern made by putting the smaller angle into the mirror. "Ooh, that's pretty."

"It's like a star."

Kristin, seated at the table, built what she saw.



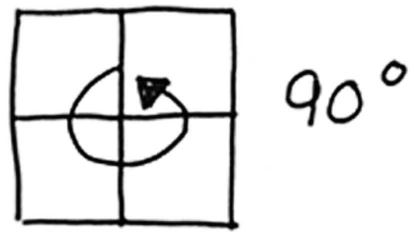
"Since it takes six blocks to complete a rotation," I said, "each of those angles is one-sixth of three hundred sixty degrees." Some students quickly did the division in their heads.

"But then you need to figure out the size of the other angle as well. To do this, position the block with the wide corner nestled in the hinged mirrors," I said. I did this with the block.

"Go back to your seats now," I then said, "and I'll give you specific directions about what you're to do."

When the students were seated, I told them they were to figure out how many degrees there were in each angle of the blocks. "Use the hinged mirrors as I showed you," I said. "Build what you see and then draw it." I showed on the overhead what they should draw for the square.

"Then draw a circle to show the complete rotation," I continued, "and figure out how much the angle measures. To write degrees, you use a little zero." I showed them how to do this.



"You may find it easier to trace the pattern blocks than to make a sketch," I added.

I then gave directions about the materials. "You and your partner will each need pattern blocks, a pair of hinged mirrors, and a piece of newsprint for recording," I said, holding up a piece of 12-by-18-inch newsprint for them to see. I always kept the pattern blocks for the class in small plastic baggies and I had distributed the contents of five complete sets into eight baggies. Also, I had hinged small plastic mirrors (each about 2 by 3 inches) with strapping tape, so there was a pair for every two students.

"Send someone to get supplies for both pairs at your table," I said. "Because you've never explored with the hinged mirrors before, take some time to do so. But be sure to start on the assignment in five minutes." I've learned that students need time to satisfy their curiosity about a new material.

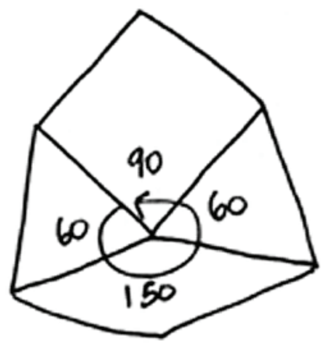
Once the students began working, some confusion surfaced. I helped several pairs who hadn't been able to see very well during the demonstration. A few others could figure how many blocks it took to make 360 degrees but couldn't remember what to do then. Gradually, all of the groups figured out what to do.

The tan rhombus posed a problem for the students. Placing the small angle in the hinged mirrors produces a star cluster of 12 blocks. This delighted students, and most were able to figure that the angle measured 30 degrees. However, placing the larger angle in the hinged mirrors produces a reflection that's frustrating because it can't be built. Students had to find other ways to figure out its size.

Kristy and Erika found that the larger angle of the tan rhombus was equal to two angles of green triangles and the smaller angle of the tan rhombus. They added 120 degrees and 30 degrees to get 150 degrees. They wrote: *Two of the triangles and one tan rhombus fit in the angle.*

Jennifer and Sarah proved that the angle measured 150 degrees by seeing that the angle was the same size as the 30 degree angle and the large 120 degree angle of the red trapezoid combined.

Tami and Kelly showed that the orange square, two green triangles, and the large angle of the tan rhombus combined to make 360 degrees.



The hinged mirrors aren’t essential for this activity. It’s possible to use just the blocks and see how many of each angle are needed to complete a 360 degree rotation. However, using mirrors with the blocks is both aesthetically pleasing and exciting for the students.