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PROFESSIONAL DEVELOPMENT

A Measurement Problem

by Marilyn Burns

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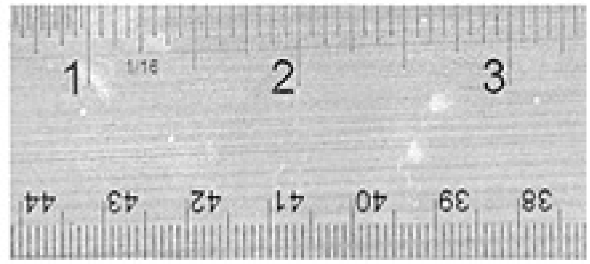
The following lesson for sixth- and seventh-grade students is excerpted from Marilyn Burns's Writing in Math Class *(Math Solutions Publications, 1995). In introducing the lesson, Marilyn explains that she learns "a great deal from lessons that don't go well. As difficult as they are, lessons that are problematic usually push me to think more about my teaching practice." The lesson was originally taught with students working alone and then retaught with students working in small groups.*



For this lesson, I planned to have the students work individually to solve a measurement problem involving fractions. Before the period began, I drew on the chalkboard a line segment that measured $22\frac{1}{2}$ inches. Also, I put thirty-three Unifix cubes into a plastic bag.

"When I snap these cubes together," I said, "do you think the train will be longer, shorter, or about the same length as the line segment I drew?" The students had no way of making a reasonable prediction by looking at the bag of cubes, but they were willing to guess.

After all who wanted to had voiced an opinion, I asked, "How could we find out?"

"Do it!" they answered in unison.

Two students snapped the cubes together. They matched the train to the line segment and found that the train was about 2 inches longer than the line segment. We discussed what "about the same length" means when measuring. We had a lively conversation about when it was necessary to be accurate (building shelves for a bookcase, measuring fabric for a dress, or timing a soft-boiled egg) and when approximations would suffice (cutting paper and ribbon to wrap a gift, measuring water for cooking spaghetti, or dishing out equal portions of mashed potatoes).

Then I asked them to make a different estimate. "How long do you think the line segment is?" I asked.

"It's shorter than the yardstick," Mark said. The yardstick was resting on the chalkboard tray.

"Maybe thirty inches," Marcie said.

"How many cubes long is it?" Peter asked. Peter's question led us in the direction I had planned.

I held the train up to the line segment and removed three cubes so that their lengths matched. Then I split the train into tens. There were thirty cubes in all.

"So the line segment is as long as a train of thirty cubes," I said. "Can that information help you figure out the length of the line segment?"

"How big are the cubes?" Amy asked.

"They're three-quarters of an inch on each side," I said. "What else do you need to know?"

There were no more questions. I then said, "I'd like you to figure out how long the line segment is. When you record your answer, be sure to explain why it makes sense." The students got to work.

The room became quiet with the kind of quiet that test taking often produces. Some students started to write about their ideas; some did calculations on their papers; others gazed into the distance, apparently thinking.

The students' papers gave me much to think about. Scott's paper was representative of many of the students who couldn't make any headway. He wrote: *I have not figured this out because I don't know how. I'm stuck!*

Some students made some progress, but ran into snags. Jonathan, for example, wrote: $\frac{3}{4} \times 30 = \frac{37}{4} = 9\frac{1}{4}$. *First I multiplied* $\frac{3}{4}$ *x 30. I think this is a good way of doing this because all you have to do is multiply the numbers and you have your answer.* Jonathan didn't look at the line segment on the chalkboard to notice that an answer of less than 10 inches made no sense.

Karine came up with an interesting beginning. She wrote: *I know its less than 30 inches because the cubes are smaller than 1 inch. Its more than 15 because that would be half and* $\frac{3}{4}$ *is more.* She was then stumped and had no place to turn.

Mark made a good start but then took a false turn. He wrote: *Two cubes make* $1\frac{1}{2}$ *inch. 4 cubes make 3 inches. So 8 x 3 makes 24 inches.*

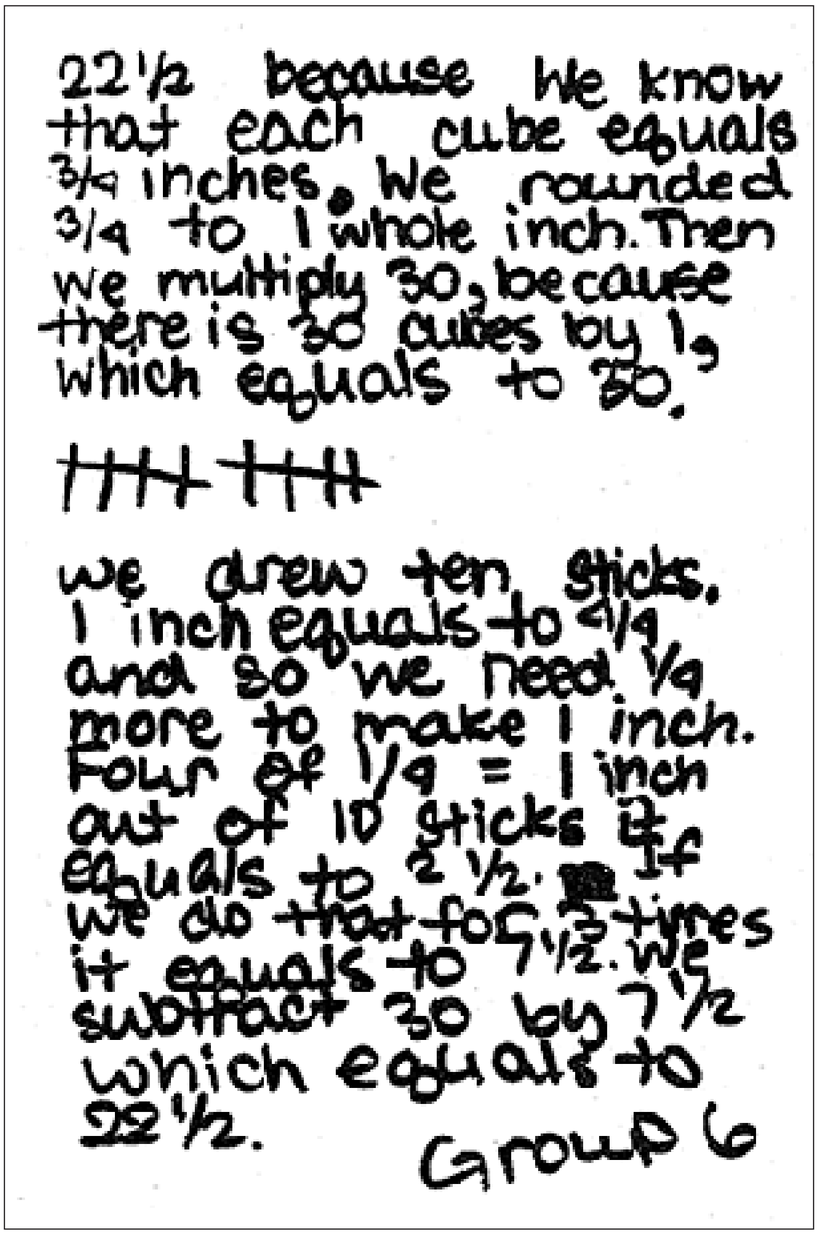
Jessica was one of three students for whom the problem was easy, even trivial. She wrote: $22\frac{1}{2}$ *I multiplied* $\frac{3}{4}$ *x 30. 30 is equal to* $\frac{30}{1}$. *I multiplied and then I reduced to get my answer. It makes sense because you're doing* $\frac{27}{4}$ *30 times. It's easy to multiply it.*

What I had done was put the students in a testing situation, not a learning situation. Dealing with fractions is difficult for many students, and they need as much support as possible to learn about them. By having students struggle individually, I didn't provide any way for them to get feedback on their thinking or hear about other students' approaches. And when they are working individually, there's no way that I can get around to help all of them.

Cathy Humphreys presented the same problem to seventh graders. She introduced it as I had. However, rather than have the students solve the problem and write individually, she had them work in groups of four. That way, the students could talk with one another and draw from their collective thinking.

To promote further communication in the class, Cathy gave each group an overhead transparency and marker. "Record your solutions and your thinking on the transparency," she said. "Then each group will present its thinking."

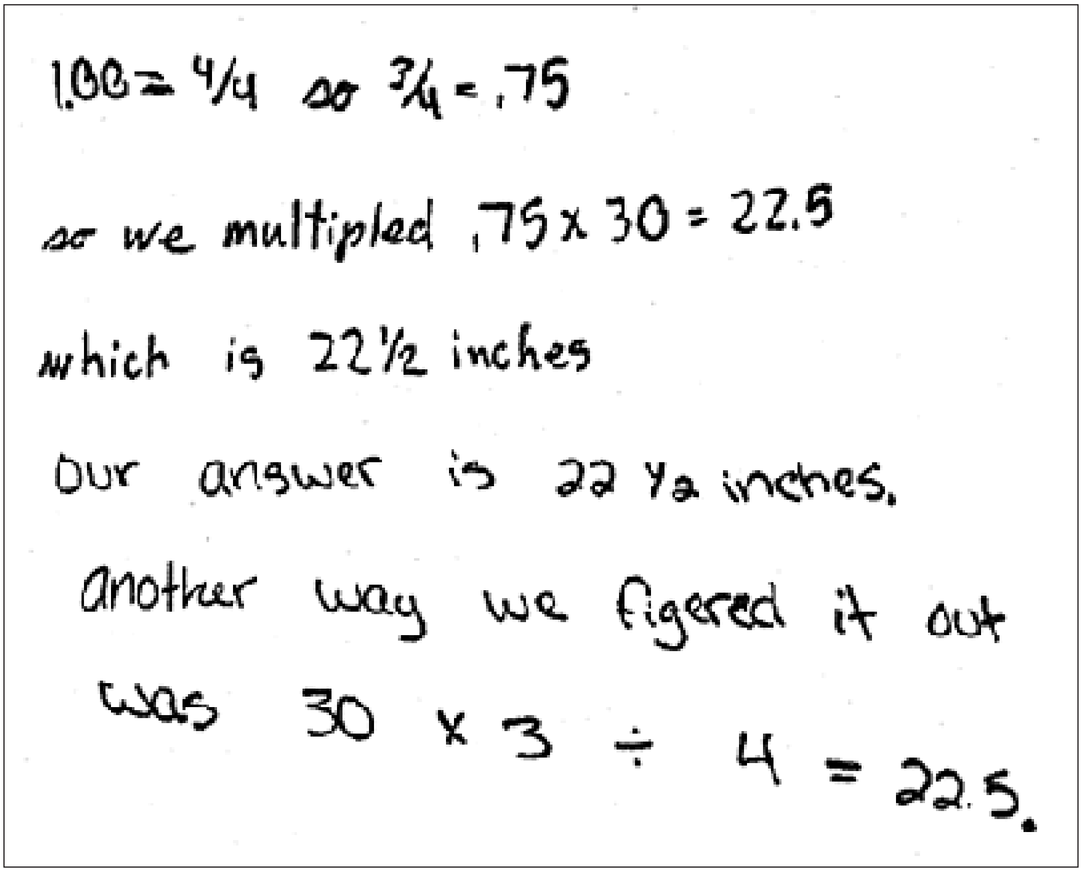
The groups' interactions were animated and their explanations revealed that the students used a variety of approaches. Group 6 wrote: *22 $\frac{1}{2}$ because we know that each cube equals $\frac{3}{4}$ inches. We rounded $\frac{3}{4}$ to 1 whole inch. Then we multiply 30, because there is 30 cubes by 1, which equals to 30. We drew ten sticks. 1 inch equals to $\frac{4}{4}$ and so we need $\frac{1}{4}$ more to make 1 inch. Four of $\frac{1}{4}$ = 1 inch out of 10 sticks it equals to 2 $\frac{1}{2}$. If we do that for 3 times it equals to 7$\frac{1}{2}$. We subtract 30 by 7 $\frac{1}{2}$ which equals to 22 $\frac{1}{2}$.* (See Figure 1.)


22½ because we know
that each cube equals
3/4 inches. We rounded
3/4 to 1 whole inch. Then
we multiply 30, because
there is 30 cubes by 1,
which equals to 30.

we drew ten sticks.
1 inch equals to 4/4
and so we need 1/4
more to make 1 inch.
Four of 1/4 = 1 inch
out of 10 sticks ~~it~~ If
equals to 2 ½.
we do that for 3 times
it equals to 7½. We
subtract 30 by 7½
which equals to
22½.
Group 6

Figure 1. Group 6 figured the length of the train if the cubes were 1 inch long and then adjusted. (Grade 7)

Group 3, however, used both decimals and fractions to figure out the problem. They wrote: *1.00 =* $\frac{4}{4}$ *so* $\frac{3}{4}$ *= .75 so we multipled .75 x 30 = 22.5 which is 22* $\frac{1}{2}$ *inches. Our answer is 22* $\frac{1}{2}$ *inches. Another way we figured it out was 30 x 3 ÷ 4 = 22.5.* (See Figure 2.)



1.00 = 4/4 so 3/4 = .75
so we multipled .75 x 30 = 22.5
which is 22½ inches
Our answer is 22 ½ inches.
another way we figered it out
was 30 x 3 ÷ 4 = 22.5.

Figure 2. Group 3 used a combination of fractions and decimals. (Grade 7)

Group 1 wrote: *The total inches are 22.5. We think its 22.5 because each cube is* $\frac{3}{4}$ *of an inch and their is 30 cubes so you split one each into 4 parts and you times 30 cubes by 3. And you get 90. And then divide 90 by 4.* They showed how they did the calculation.

Group 7 had a different approach. They wrote: *The answer is 22* $\frac{1}{2}$*. Our group figured it out by putting two cubes together. Two cubes = 1* $\frac{1}{2}$ *inches.*

Then we multiplied 1 $\frac{1}{2}$ *by 15 because we used 2 cubes to make 1* $\frac{1}{2}$ *we cut 30 in half which = 15. That's how we got our answer.*

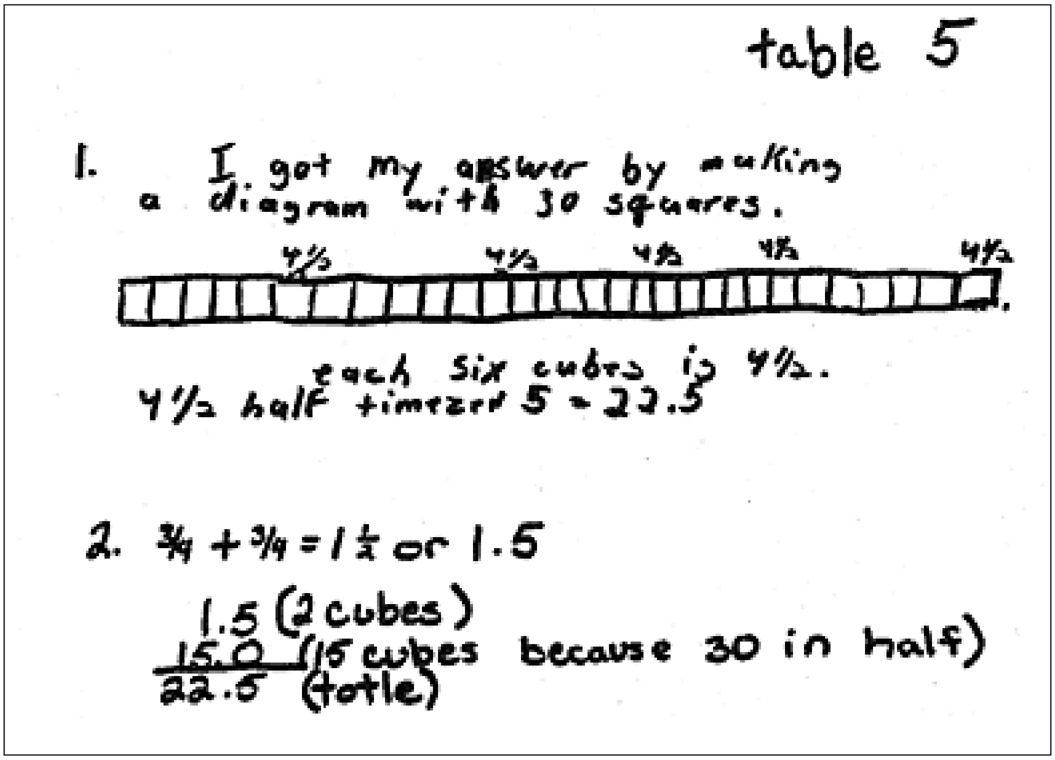


table 5

1. I got my answer by making a diagram with 30 squares.

$4\frac{1}{2}$ $4\frac{1}{2}$ $4\frac{1}{2}$ $4\frac{1}{2}$ $4\frac{1}{2}$

each six cubes is $4\frac{1}{2}$.
$4\frac{1}{2}$ half timezed $5 = 22.5$

2. $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$ or 1.5

1.5 (2 cubes)
15.0 (15 cubes because 30 in half)
22.5 (totle)

Figure 3. Group 5 gave two solutions, first figuring the length of six cubes and then figuring the length of two cubes. (Grade 7)