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Bulging Backpacks

A Lesson with Sixth Graders

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In this lesson excerpt, students estimate and find the actual weight of each of their backpacks. The class constructs a scatter plot to display some of the data, and then each student creates a personal scatter plot. The lesson gives students experience plotting points on a coordinate plane and interpreting those points in terms of a real-world situation. And, for the first time, students consider a set of points on a graph as a representation of the relationship (or lack of it) between two variables. The complete version of this lesson appears in Ann Lawrence and Charlie Hennessy's Lessons for Algebraic Thinking, Grades 6–8 *(Math Solutions Publications, 2002).*

To begin the lesson, I told the class, "Each of you will estimate the weight of each student's backpack and then we will find the actual weights, using a bathroom scale. We'll work together to create a graph known as a *scatter plot.* Then each of you will create and interpret a personal scatter plot using your own estimates and the actual weights for the backpacks."

I asked the students for suggestions about how we might estimate and find the actual weights of the backpacks, and then record the information. The students designed this plan:

- Each student rules three columns and records his or her own data. The first column has the names of all the students in the class, the second is for estimates, and the third is for actual weights.
- For a first experience, students make estimates for three backpacks.
- Students then use the bathroom scale to weigh each of the three backpacks to the nearest half-pound.
- Students continue to make estimates and subsequently find weights to the nearest half-pound for the rest of the backpacks.

I asked Daniel, Alice, and Jack to place their backpacks on the table in front of the room. All of the students lifted each of the three backpacks and recorded an estimate for the weight of each. Daniel then weighed his backpack — it weighed 10.5 pounds. (The students' estimates ranged from 4 to 26 pounds.) Then Alice and Jack weighed their backpacks — 14.5 pounds and 22.5 pounds — and the students recorded the weights.

Next, eight more students brought their backpacks to the front of the room to be estimated and weighed. Each student in the class estimated the weight of all eight backpacks. They could lift the eight backpacks or any of the first three backpacks to better estimate. When everyone had entered their estimates into their tables, each of the eight students weighed his or her own backpack and announced the results. We repeated this for the rest of the backpacks.

"Now you'll make scatter plots of the results to help you determine how well you estimated," I said.

Kevin asked, "What's a scatter plot?"

I responded, "A scatter plot is a kind of graph that shows information about two variables at the same time. It can help a person decide whether there is a relationship between the two variables. In this case, our variables are the estimate and the real weight for each backpack."

I taped a large, blank grid of one-inch squares on the board and drew two axes. I polled the class and found that all weights and estimates were less than 30 pounds, so I numbered each axis from 0 to 30. I used the horizontal, x-axis for the actual weights of the backpacks and the vertical, y-axis for the estimates. By using the same scale and numbers on each axis, the graph of $y = x$ (to be plotted later in the lesson) would bisect Quadrant I so the students would have a visual image of that benchmark function, with points on this line indicating estimates and actual weights that were the same. Then, the point for any *under* estimate would fall *below* the $y= x$ line, and the point for any *over* estimate would fall *above* the $y = x$ line.

To be sure that students would know how to create their own scatter plots, I had the class create a practice scatter plot on the chart paper. Each came up and used a yellow dot to mark a point that represented the estimate and actual weight for his or her own backpack. Daniel located the point (10.5, 10) by moving one finger up from 10.5 on the x-axis and one finger right from 10 on the y-axis. Alice next came up and explained, "My backpack weighed fourteen and a half pounds, so I need to move along the bottom line to fourteen point five. My estimate was six pounds, so I move up six and put my dot here." She correctly located (14.5, 6).

After everyone had located a point, I asked the original group of three students to circle their points in red (see Figure 1). I explained, "Because those estimates were made with no prior experience, I want to separate them from the overall impression the scatter plot might indicate about the relationship between the estimates and actual backpack weights."

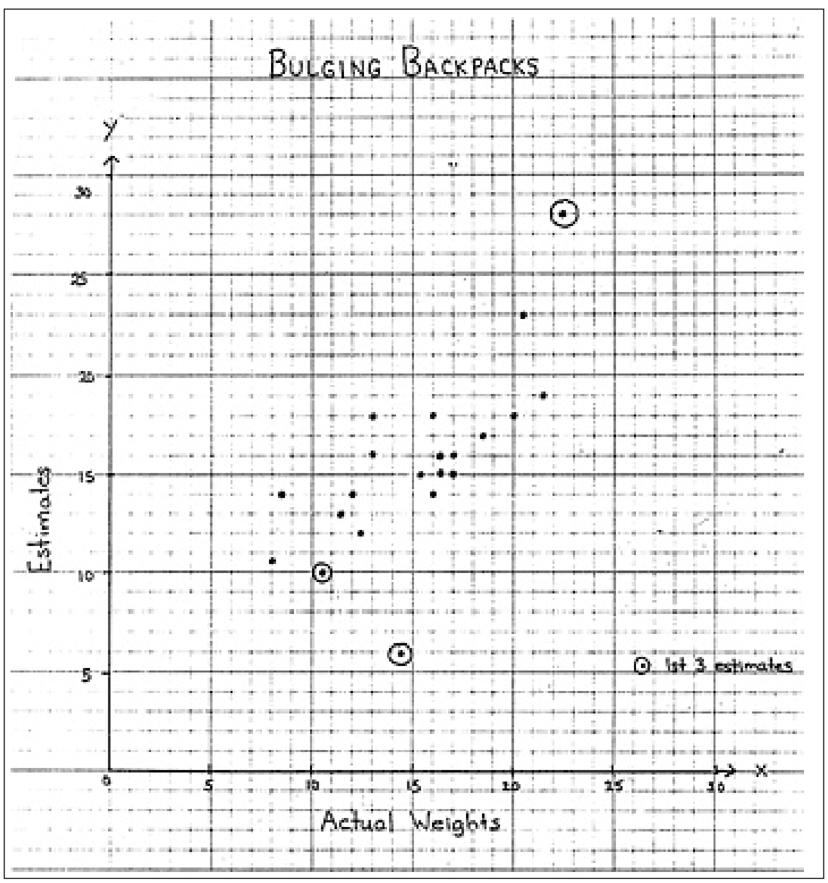

Figure 1. Class scatter plot #1.

I asked a few questions about the class scatter plot. First, pointing at the dot at (13, 16), I asked, "What does this point tell you?"

Terrence explained, "For a backpack that weighed thirteen pounds, the estimate was sixteen pounds."

I then asked, "Does the graph indicate a trend between the estimates and the actual weights?" The students agreed that there seemed to be a general trend.

Hector said, "The scatter plot mostly shows small estimates for the light backpacks and big estimates for the heavier backpacks."

I agreed, explaining that two variables are considered to be *correlated* when there is a strong general trend that connects them. Here, there was a correlation between the estimates and the actual weights because as the actual weights increased, the estimates increased. Both the numerical data and the scatter plot showed this trend.

Next each student constructed a personal scatter plot to display his or her own estimates versus the actual weights of the backpacks. I reminded the students to circle the points that represented the weights and estimates for Daniel's, Alice's, and Jack's backpacks in red.

When all had completed their scatter plots, I had students respond to several questions. First I asked, "What does your scatter plot show about your estimates for the weights of the backpacks?"

Sergio volunteered, "Mine shows that I was a good estimator." He went on to explain that most of his y values (estimates) were close to his x values (the actual weights).

"How does that affect the way the scatter plot looks?" I asked. He replied, "When you look at my scatter plot, the points are not all over the page. They are sort of a group of points that go from the bottom left to the upper right of the graph."

In answer to the question "Where are the points for low estimates?" Leah replied, "Low on the graph."

Similarly, when asked how to identify high estimates, Vito answered, "Higher on the graph than other points close to them."

Then I asked, "What would the graph look like if every single estimate matched the actual weight?"

Reid said, "Each point would be a corner of a square." He went to the class scatter plot, pointed to 14 on the x-axis and 14 on the y-axis, traced up and across with his fingers and said, "See, the point would be at the corner of a square that is fourteen on each side."

"And where would all the points, or corners of all those squares, appear on your scatter plot?" I asked.

Reid frowned a bit, then smiled. "They would make a line," he said.

Knowing that some students did not follow Reid's explanation, I had each student who had made a perfect estimate for a backpack place a blue dot on the graph. I repeated that each blue point had a *y*-coordinate (estimate) that matched the *x*-coordinate (actual weight). Many students were surprised to see that the points all fell on a line that went diagonally up to the right from the origin (Figure 2).

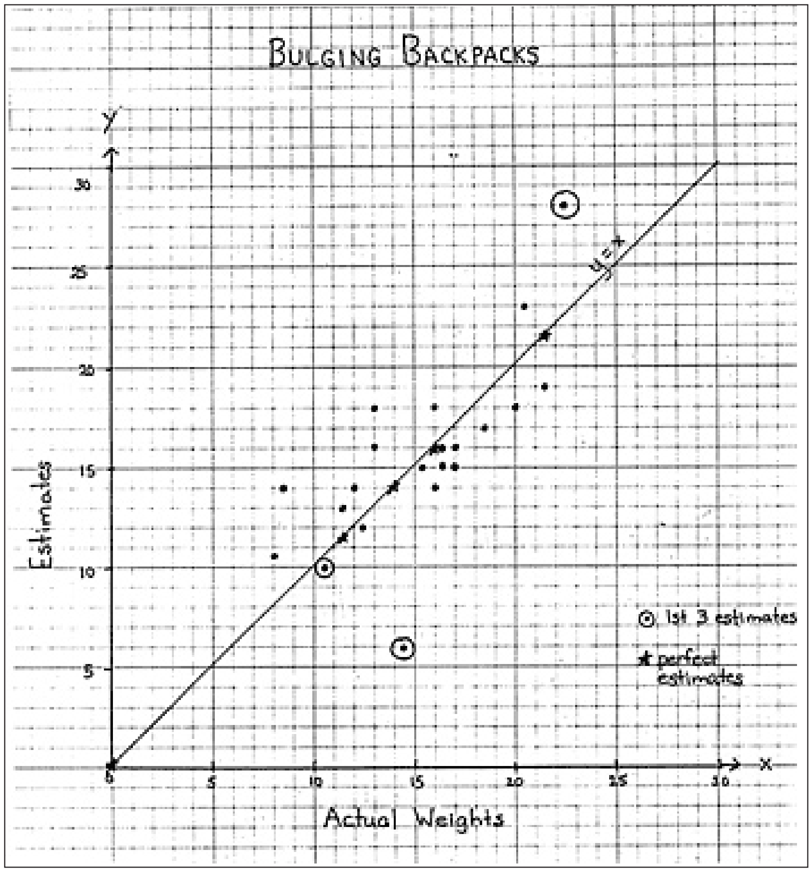

Figure 2. Class scatter plot #2.

I asked what they noticed about the new set of points. Nashalie stated, "There is a diagonal line of dots when every estimate matches the actual weight."

I continued, helping the students see that the line of points with exact estimates could be represented as *estimate = actual*, or $y = x$.

I continued the lesson by having the students think about how points would be placed if each estimate were two pounds more than the actual weight $(y = x + 2)$ and we plotted points in green to show the line. I repeated this for estimates that were three pounds less than the actual weights of the backpacks $(y = x - 3)$. We then discussed and graphed what the graph would look like if every estimate were seven pounds $(y = 7)$.

For the last part of the lesson, I asked each student to go back to his or her personal scatter plot and add points for perfect estimates (the $y = x$ line). When everyone had done so, I asked, "What do you notice?"

Callie offered, "If you were a good estimator, the points of your original graph should be close to the new line. And, except for the first three points, mine are!" I asked Callie to pass her scatter plot around the room for her classmates to view (Figure 3).

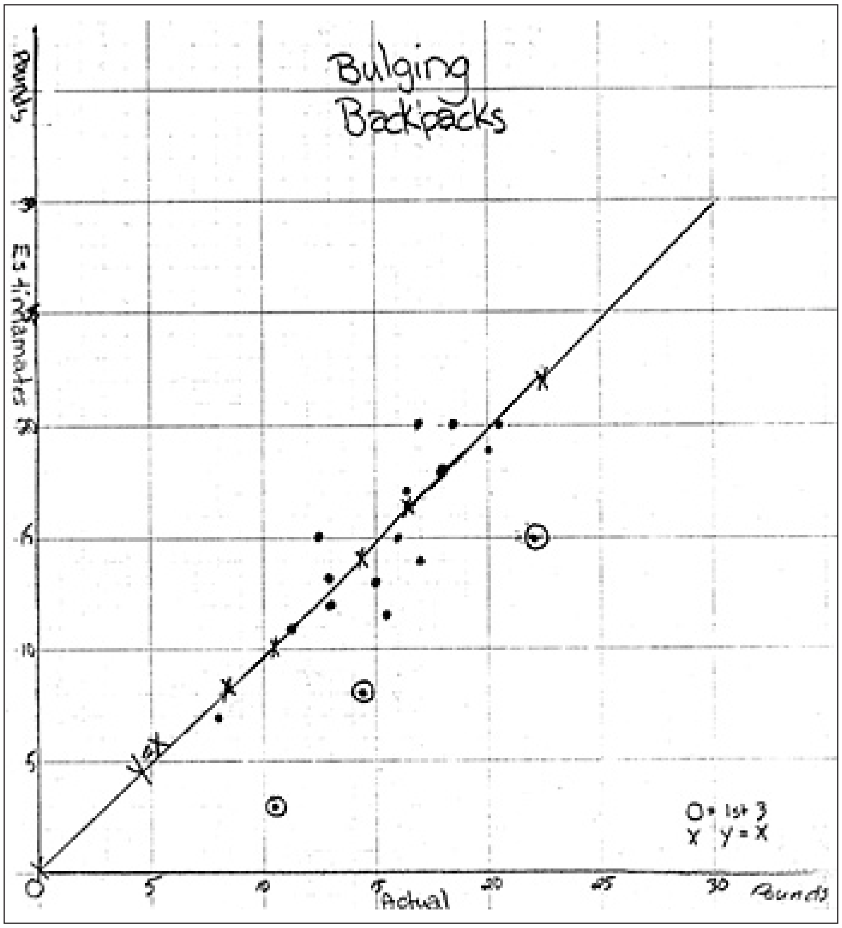

Figure 3. Class scatter plot #3.

Ryan commented, "Almost all my points are below the line" (Figure 4 on following page).

When I asked Alice what this told her about Ryan's estimates, she replied, "I think it means Ryan mostly estimated too little for the weights of the backpacks." Ryan agreed, and I asked him to pass his scatter plot around.

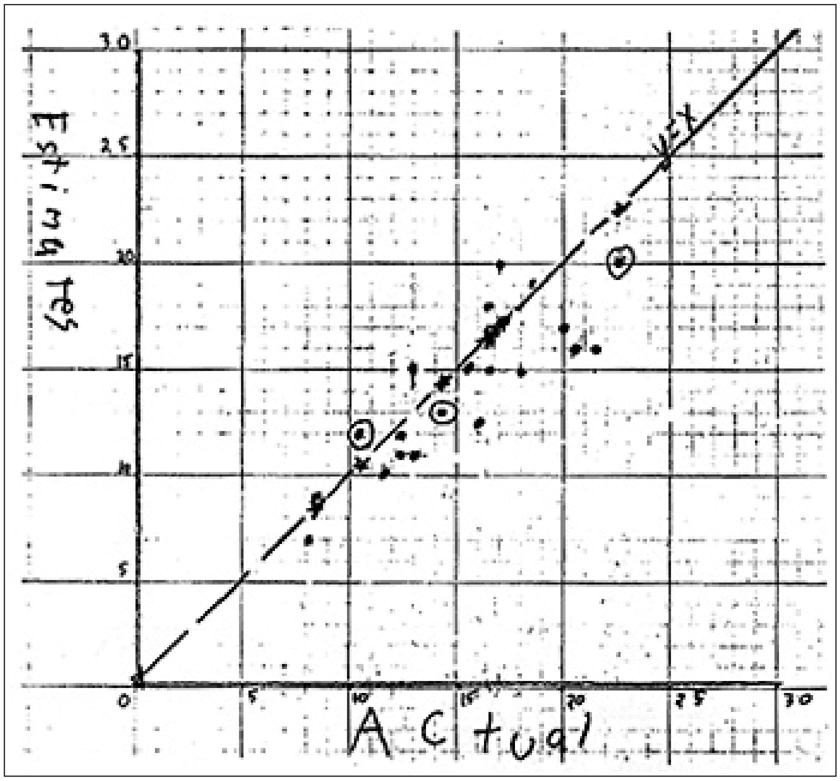

Figure 4. Class scatter plot #4.

Jack stated, "My points are mostly above the line, so that means I estimate too high." Again, Jack's observation was affirmed as his scatter plot was passed among his classmates.

I asked the students to summarize what an informed observer might conclude from the appearance of the various scatter plots. Marianna volunteered, "If the points are close to the 'matching line,' you are a good estimator. If most of the points are below the line, you estimate too low. And if the points are mostly above the line, you estimate too high."

"What does the scatter plot tell you about the correlation of the actual weights and the estimates?" I asked.

Luis said, "Your estimates are correlated with the actual weights if the points on the scatter plot are clumped around the y equals x line we graphed."

To end the lesson, I wrote the following three questions on the board and asked the students to respond to each of them in writing:

1. *What should a person know by looking at your original scatter plot?*
2. *What should a person know by looking at the points and line you added later to your scatter plot?*
3. *What are the main things you learned from this activity?*