Snapshots of Student

To find out what students really understand about that math lesson, try one-on-one interviews—and be ready for some surprises.

Marilyn Burns

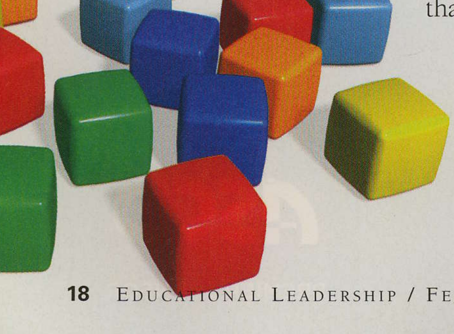
"How much is 100 minus 3?" I asked Alicia, a 3rd grader. I gave an additional direction, asking her to try solving the problem in her head. "But you can use paper and pencil if you need to," I assured her.

After a moment, Alicia responded confidently and correctly. "Ninety-seven," she said.

I then gave Alicia another problem. "How much is 100 minus 98?"

Alicia frowned and said, "I'll have to use paper and pencil. I can't count back that far." She wrote the problem on her paper and subtracted, regrouping as she had learned and getting the correct answer of 2.

Alicia's response indicated that she had not learned about the relationship between addition and subtraction. She did not think, "I know that 98 plus 2 equals 100, so 100 minus 98 has to be 2." On numerous assignments, Alicia had demonstrated proficiency with paper-and-pencil skills for both addition and subtraction. Her written work, however, hid this important gap in her understanding. I now had information about the instructional help that I could provide Alicia.

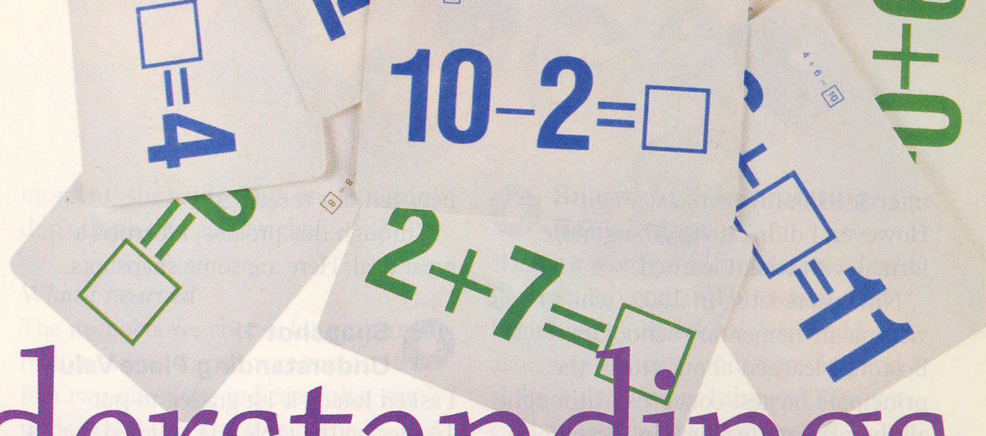


I showed Gerald, a 6th grader, two fractions—6/10 and 7/10. "Which fraction is greater?" I asked. As with Alicia, I asked him to try and decide in his head but assured him that it was OK to use paper and pencil.

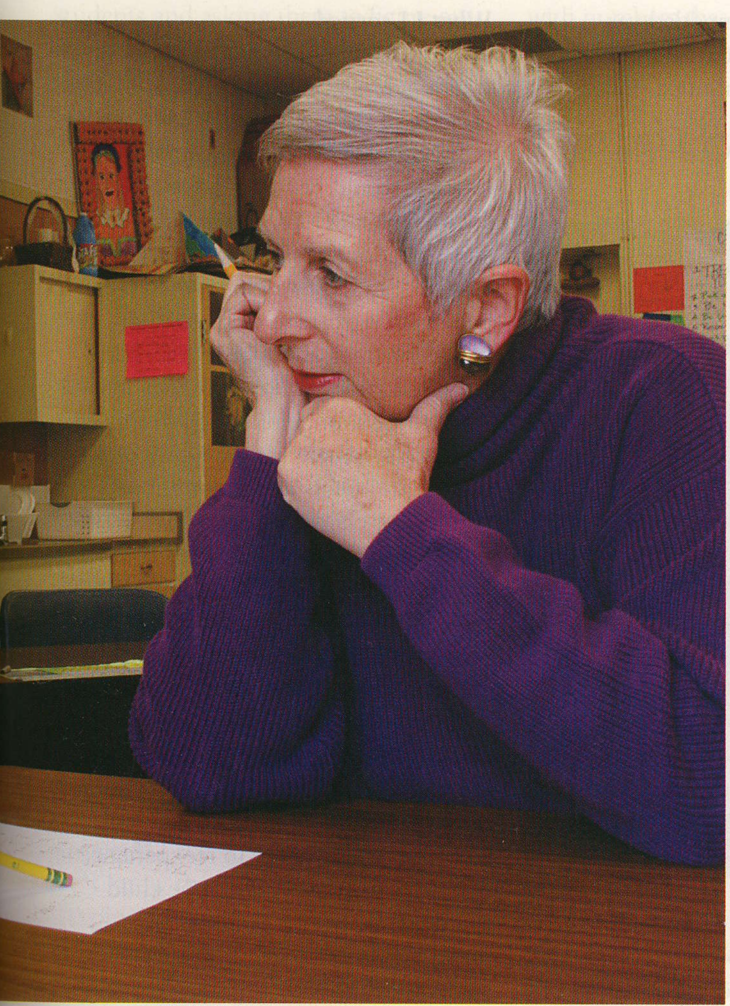
Gerald thought for a moment and then answered incorrectly, "Six-tenths."

"How do you know that?" I probed.

"The smaller the number, the bigger the fraction," he replied. Gerald incorrectly applied what he had learned about comparing fractions with different denominators—for



Misunderstandings



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applying a different rule he had been taught about fractions. I now had information about the instructional help that I could provide him.

Surprised by Student Thinking

We teachers often rely on students' written assignments to assess their skills and understanding. However, as with Alicia and Gerald, I've learned that one-on-one interviews reveal valuable information that is not available when I rely solely on students' written work. This information is essential for guiding appropriate instructional decisions.

Some history on how I came to this conclusion. In 1993, I was working on a series of videotapes *Mathematics: Assessing Understanding* for ETA/Cuisenaire. The videos included individual student interviews. In preparation, I practiced by teaching many lessons and conducting dozens of one-on-one interviews with students at different grade levels. During the interviews, I probed mathematical strengths and weaknesses so I could construct a mathematical profile of the student. The practice experiences were always revealing and sometimes astonishing, uncovering students' misconceptions and gaps in their understanding that I hadn't recognized before.

I was even more stunned during the actual videotaping. The lesson called for the students to time the teacher, Carol Brooks, for one minute while she drew stars on the board. Carol then talked with the students about how they might figure out how many stars she had drawn. The discussion led them to circle groups of 10 stars and count how many 10s and extras there were. During the lesson, the responses of two of the 2nd graders—Cena and Jonathan—indicated a firm foundation of understanding place value. However, when I interviewed each of them the next day to probe their understanding one-on-one, I was shocked. The interviews revealed the fragile conceptual base of their understanding in ways that their teacher had no way of knowing from the context of the classroom lesson. (The videotaped interviews are available at www.mathsolutions.com/placevalue/Cena and www.mathsolutions.com/placevalue/Jonathan.) As a result of this experience, I began to incorporate more and more individual

example, 1/8 and 1/5. One piece of a whole that is cut into five equal pieces is obviously greater than one piece of the same whole cut into eight equal pieces.

Gerald would most likely have given the incorrect answer on a written assignment if asked to circle the greater fraction. However, without talking to Gerald and having the opportunity to probe his thinking, I would not have known that his misconception was a result of inappropriately interviews in my own teaching. However, I didn't do anything more formal with what I learned.

Nine years later, in 2002, while I was visiting an elementary school in Boston, I learned about one of the principal's biggest concerns: Although all the primary teachers in the school conducted one-on-one assessments in reading with each of their students, no such counterpart existed for mathematics. The principal worried that teachers didn't have the same degree of understanding about their students' mathematical ability as they did about their students' reading ability and therefore might be unable to make appropriate instructional shifts or identify appropriate interventions.

The principal's concern rekindled my interest in student interviews in math. I formed a study group with several colleagues to create and test individual assessments for kindergarten through grade 6 that focused on the basics of number and operations. We met weekly to work on the interviews and separately tested the assessments with students in different schools between our meetings.

Through this process, I learned a great deal. Here are some snapshots.

Snapshot 1: Understanding Place Value

I asked Jonah, a 1st grader, to put 14 tiles on the table. He counted out 14 tiles correctly. I asked him to write 14 on his paper, and he correctly wrote the numeral.

"Are there more than 10 tiles?" I asked. Jonah quickly nodded yes.

"If you give me 10 of the 14 tiles, will there be extras?" I continued. Jonah again quickly nodded yes.

"How many extras would there be?" I continued. Jonah was quiet for a moment. Then he looked intently at the tiles, nodding his head as he silently tried to track the tiles with his eyes to count them. But he couldn't manage.

"I'm not sure," he finally said and then added, "Maybe 2 or 3."

Understanding our place value system is an essential foundation for all computations with whole numbers. Teachers talk with students about 10s and 1s, and students learn early on which number is in the 10s place and which is in the 1s place in a two-digit number. However, merely being able to identify the place of the numbers is not a reliable indicator that a student understands the structure of our base-10 number system. Students need to understand the role of 10s and learn to see 10 objects both as one group (the 1 in the 10s place) and as 10 individual objects.

What I Learned

The difficulty in developing understanding of place value is compounded because of the language we use for numbers in English, especially for the teens. It would be helpful if we read 11 as "one ten and one" instead of "eleven," and 12 as "one ten and two" instead of "twelve," and so on. This is the verbal pattern that exists in Chinese. Our number names of *eleven*, *twelve*, and *thirteen* do not help to reveal the role of 10 in those numbers. Although this is unfortunate, these are the words we have for numbers and the number names that children have to learn. That said, we can support students' conceptual learning by providing them with many opportunities to count quantities of objects by grouping them into 10s and recording how many there are.

Cena and Jonathan had one such experience when they figured out how many stars Carol had drawn on the board. After students complete that activity, they are ready to reexperience that activity in pairs, with one child timing while the other draws stars, dollar signs, letters, or any other symbols. They need experience counting other collections of objects in the classroom—cubes, beans, pencils, paper clips, and so on. For each of these experiences, the teacher needs to help students see the pattern that exists when they record how many there are—that the digit on the left represents the number of groups of 10s and the other digit represents the number of extras.

Snapshot 2: Solving Missing Addend Problems

I took two tiles from a container and showed them to Rosa, a 2nd grader. "How many more do I need so I have 10 tiles?" I asked her. Rather than showing her the problem in written form (2 + ___ = 10) I presented the problem verbally and with the concrete material of the tiles. This question presented Rosa with a missing addend problem: Instead of giving numbers to students and asking them to figure out the sum, students already know the sum and one of the parts and have to figure out the missing part.

"You need 8 tiles," Rosa answered quickly with confidence.

"How did you figure out the answer?" I probed.

Rosa replied, "It's easy. I know that 8 plus 2 makes 10."

Next, I showed Rosa a picture of a jar and explained, "This jar can hold 100 marbles when it's totally filled." I showed her another picture of the same size and shape jar, this one with *30 marbles* written underneath. I hadn't drawn actual marbles, but had roughly scribbled to show a jar that was about one-third filled. "Can you figure out in your head how many more marbles I need to put into the jar so there are 100?"

"You need 70 more marbles," Rosa answered, again quickly with confidence.

"How did you figure out the answer?" I asked.

"I know that 7 plus 3 makes 10, so 70 plus 30 makes 100," she replied. I've found that using the known fact of 7 + 3 to figure out the answer to this problem is a typical response.

I gave Rosa another problem. This time, I showed her 5 tiles. "Figure out in your head how many more tiles I need so I have 30 tiles in all," I said.

Rosa was quiet. After a moment, she counted softly by 5s to 30, putting up a finger each time. She looked at her six fingers and thought. After another moment, she said, "This one is hard. I don't think that six is right."

What I Learned

The numbers we choose for problems matter. Rosa's correct responses to the first two questions indicated that she understood the structure of the problem. The known facts of 8 + 2 and 7 + 3 gave her an anchor that helped her reason out both problems. Children usually have a good deal of experience with numbers that add to 10, with their fingers as a backup support.

The interviews revealed the fragile conceptual base of the students' understanding.

With 5 and 30, however, Rosa had no useful anchor, except for counting by 5s, which didn't help her solve the problem. Rosa understood the structure of the problem—that she was to find the missing addend—but she lacked facility with the particular numbers. Students need experience developing strategies for mentally computing with numbers that are not as "friendly" as single-digit numbers or multiples of 10.

The marbles-in-the jar problem can help students figure out, in their heads, how many more are needed to make 100. The teacher can start with multiples of 10, then move to numbers that end in 5, and then move to all numbers. Having students share their strategies is valuable. Also useful are 10-by-10 grids; students can color in the number they have and see how many more 1s and 10s they need to fill the grid. The goal is to help students develop their number sense so they increase the range of their numerical comfort.

Snapshot 3: Interpreting Remainders

"Here's a word problem to solve," I told Randy, a 5th grader. This was at the beginning of the school year, and I was assessing students' understanding and skills with division. I showed Randy a card on which I had written "30 students, 4 students in a car." The card was to help Randy keep track of the information in the problem. I continued, "Thirty students are going on a field trip. Four students fit in a car. How many cars are needed to fit all the students?"

"Can I use paper and pencil?" Randy asked. I nodded and watched Randy solve the problem as a long division problem. He wrote the answer as "7 R 2."

"How many cars are needed to fit all the students?" I asked.

"It's 7 remainder 2," he said.

What I Learned

Even though Randy was able to compute correctly, the computation alone was not a sufficient indication of his proficiency with division. His answer of "7 remainder 2" made sense numerically but not in the context of the problem. Students often lack experience solving problems that call for relating numbers to real-world situations.

Dividing up things in their lives is a common experience for students, and it's valuable to build on this experience and situate a good deal of their division work in word problems. Randy, like all students, needs many experiences solving division word problems with a focus on making sense of the answer.

Sometimes pictorial representations of problems can help. In my experience, when asking students like Randy who don't make sense of their answers to draw a picture that shows the 30 students getting into cars in groups of 4, they often self-correct and give the answer of 8, which does make sense. But if students do the bulk of their division work on naked numbers—

numbers without connections to contexts that require students to interpret the answer—they may not learn the importance of reasoning to decide what those answers actually mean.

Snapshot 4: Estimating the Sum of Fractions

During a fraction assessment with Heidi, a 6th grader, I wrote a problem on her paper: 1/3 + 2/5. I said to her, "Without using paper and pencil, decide if the answer is greater or less than 1."

Heidi considered the problem for a moment and then gave the correct answer, "Less than 1."

I followed up by asking, "How did you figure that out?" I was listening for Heidi to explain that because both fractions were less than one-half, their sum had to be less than 1. However, that's not what Heidi said. Instead, she made a classic error.

"1 plus 2 is 3," she answered, pointing to the numerators of each fraction. Then, as she pointed to the denominators, she added, "3 plus 5 is 8, and 3/8 is less than 1."

What I Learned

Sometimes correct answers hide misconceptions or gaps in learning. Too often, we probe students' thinking only when they answer incorrectly. Students quickly catch on that if we ask them a follow-up question, this indicates that they've made an error.

What is missing is the opportunity to give all students the experience of communicating how they reason, an important aspect of their math learning. Also, students benefit from hearing one another's ideas. Having students explain their thinking and asking the others to listen and react not only benefits all the students but also makes the important point that often there are different ways to solve a problem.

If we don't ask students to explain their reasoning, we can't be sure how they are reasoning or what they truly understand. Heidi's initial response masked her faulty reasoning, which indicated that she was relying on a procedure instead of trying to reason numerically.

Putting It into Practice

I can't imagine starting to teach math at the beginning of the year without having these conversations. Not only do they give me a mathematical profile of each student, but they are also a wonderful way to connect with the students. When starting any one-on-one assessment, I'm careful to tell the student that I'm asking the questions so I can be a more effective teacher—and not to give them a grade.

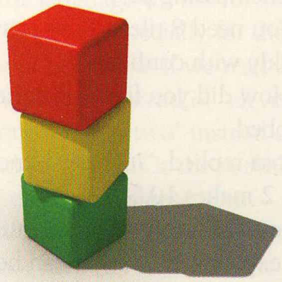
Information from student interviews guides my instruction.

I spend about 10–15 minutes interviewing each student. Finding the time to do this requires that the rest of the class be productively engaged. To accomplish this, I involve the students in a variety of math games that give them practice with skills and strategic thinking. Then I can pull individual students aside. For this to work, however, the students need to understand what I plan to do; I always carefully explain what I expect students to do independently while I converse with individuals.

During the year, I return to questions I asked in these early conversations, sometimes changing the numbers, to do spot checks. In this way, the technique is useful for both formative and interim assessing.

What I learn from interviewing students spills over to my classroom teaching. First, I use the information to inform instructional decisions. I'm deliberate about taking the time, especially before teaching a new topic, to find out what students do and do not understand. And I'm more careful now about not relying solely on students' written work to gauge what they know.

I now regularly probe students' thinking during classroom lessons, even when their answers are correct. I ask them such questions as, Why do you think that? How did you figure that out? How would you explain your answer to someone who disagreed? I have them comment on their classmates' answers as well, asking them to explain what a



peer said in their own words or asking students whether they have a different way to explain the answer. If a student is stuck, it's sometimes useful to have him or her turn and discuss the problem with a partner and then return to a whole-class discussion. This gives more students a chance to practice explaining their thinking.

We know that the students in our classes have a range of mathematical skills, understanding, intuition, interests, approaches to learning, and needs. The more information we have about them, the better prepared we are to make effective instructional decisions. The challenge is to balance the teaching of math with the teaching of students. Talking to students one-on-one can help teachers find that balance. EL

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