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The Power of Proportional Reasoning

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Ratios and Proportional Relationships

"The study of ratios and proportional relationships extends students' work in measurement and in multiplication and division in the elementary grades.

Ratios and proportional relationships are foundational for further study in mathematics and science and useful in everyday life."

CCSS Writing Team. 6-8, Ratios and Proportional Reasoning. Page 2. Progressions for the Common Core State Standards in Mathematics. Retrieved from http://commoncoretools.me/category/progressions/

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Standards for Mathematical Practices

1) Make sense of problems and persevere in solving them.
2) Reason abstractly and quantitatively.
3) Construct viable arguments and critique the reasoning of others.
4) Model with mathematics.
5) Use appropriate tools strategically.
6) Attend to precision
7) Look for and make use of structure.
8) Look for and express regularity in repeated reasoning.

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Why Focus on Proportionality?

Teaching proportionality-infused lessons helps students make connections among concepts.

By including proportional reasoning in lessons, we can begin to think of proportionality as the "big idea" that helps make unifying connections across domains.

Whitman, Carmen. *It's All Connected,* page xii

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Task

- **Consider the following:**

  Sam the snake is 4 feet long. When he is fully grown, he will be 8 feet long. Sally the snake is 5 feet long. When she is fully grown, she will be 9 feet long.

  Which snake is closer to being fully grown?

  (Lamon 1994)

- **Write a statement explaining how you know.**

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Student Response -- Tanya

"Sally the snake is closer to being fully grown because she is 5 feet long and will only get to 9 feet. This means she is over half of her total growth. Although Sam will only get to be 8 feet, he is exactly halfway to his total growth, and this means Sally is further along in growth than Sam."

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Student Response – Lily

"They are both the same distance to being fully grown. Although Sam is only 4 feet long and will be 8 feet long, Sally is 5 feet long and will be 9 feet. No matter how you look at it, they are both 4 feet away from being fully grown."

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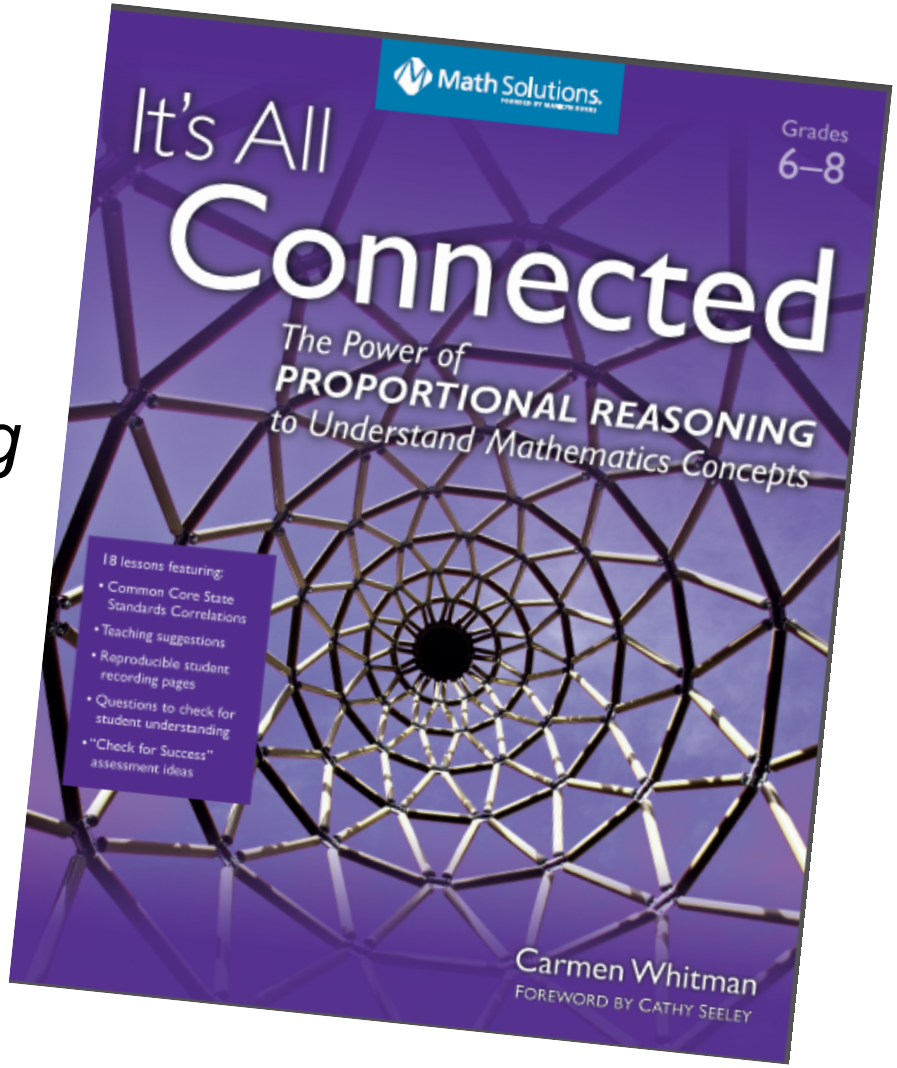
It's All Connected:

The Power of

Proportional Reasoning

by

Carmen Whitman



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Standards for Mathematical Content (CCSS)
Ratios and Proportional Reasoning

6. RP Understand ratio concepts and use ratio reasoning to solve problems.

7. RP Analyze proportional relationships and use them to solve real-world and mathematical problems.

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Equivalence: Many Names for Fractions

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Considering One-Half

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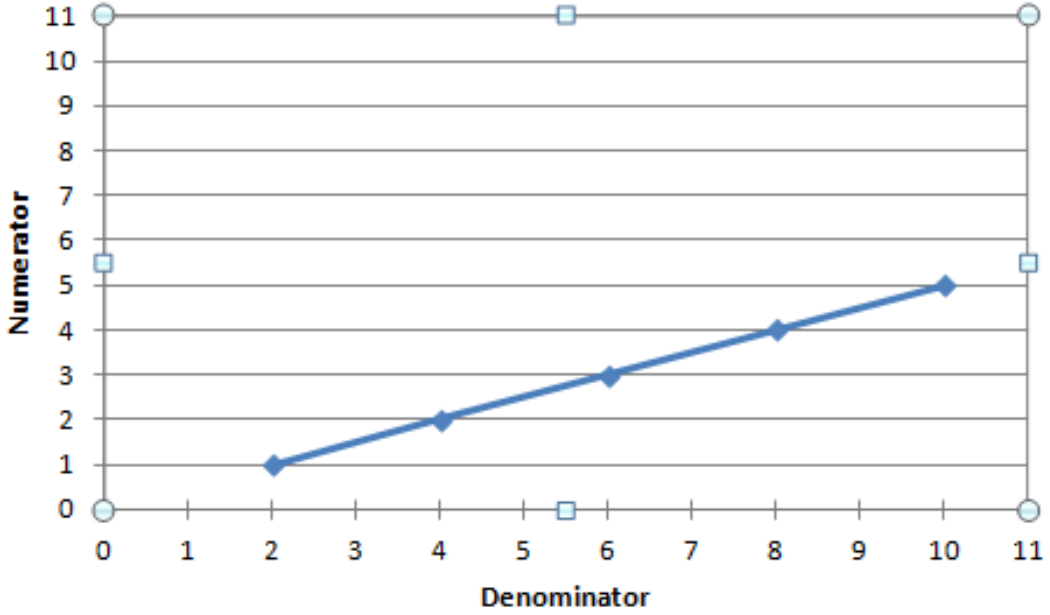
Considering One-Half

D		N
2	2/2 or 2 X ½	1
4	4/2 or 4 X ½	2
6	6/2 or 6 X ½	3
8	8/2 or 8 X ½	4
10	10/2 or 10 x ½	5



$$Y = \tfrac{1}{2}x$$

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Individual Task

- Choose one of the fractions below.

$\frac{2}{3}$ $\frac{6}{8}$ $\frac{4}{5}$ $\frac{4}{16}$

- Write five other fractions equivalent to the fraction you choose.
- Record the fractions in a t-chart.
- Graph the data using the paper on your table.
- Describe the graph in words and with an equation.

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Processing

- Describe the graph.
- What is the equation of your graph?
- Examine the ratios of numerator to denominator. What do you notice?

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Standards for Mathematical Practice

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

For example, "The ratio of wings to beaks in a bird house at the zoo was 2:1 because for every 2 wings there was 1 beak."

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Thinking about Proportionality

- Proportional relationships are functions.
- The graph of a proportional relationship results in a straight line through the origin.
- In situations involving proportional relationships, one quantity is a constant multiple of the other. The constant multiplier is called the constant of proportionality.

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Centimeters to Inches

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In your small groups, discuss….

- What is a ratio?
- What is a rate?
- How are they alike? How are they different?

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Look for 10 different-sized items to measure. List each item on the table. Measure each item to the nearest tenth of a centimeter (millimeter); then measure each item to the nearest $\frac{1}{8}$ of an inch.

Name of Item	Length (cm)	Length (inches)		
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10				

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Representing the relationship in words and graphically

- Discuss the patterns that you notice in the measurements on your recording sheet.
- Work together to write a statement that describes the relationship.
- Graph the data using the grid paper on your tables.

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Identifying Ratios and Unit Rates

Name of Item	Length (cm)	Length (inches)	Ratio Cm: Inches	Unit Rate
1.				
2.				
3.				
4.				
5.				
6.				
7.				
8.				
9.				
10				

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Processing the Mathematics

- Why does the information in the rows in column 4 look different while the information in the rows in column 5 look the same (or very similar)?
- How does the table help to highlight the proportional relationship between inches and centimeters?

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Looking at the graphs…..

- What patterns do you notice in the graph?
- If you drew a line connecting the points on the graph what do you know about this line?
- How do the patterns in the graphs relate to the patterns in the table of data?

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Representing the relationships symbolically….

- Write an equation that describes the relationship.
  - For your variables, use C for centimeters and I for inches.

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Final Processing

- Where do you see evidence of the relationship between centimeters and inches in the table, graph, and equation?
- What is the constant of proportionality? What does it represent?

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Standards for Mathematical Practice

6.RP.3d Use ratio reasoning to convert measurement units, manipulate and transform units appropriately when multiplying or dividing quantities

7.RP.2a Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane

7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

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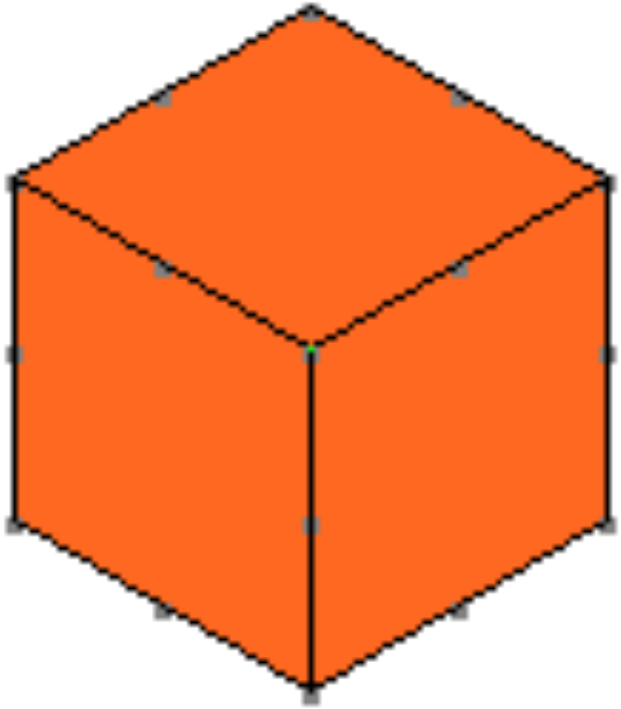
Candy Boxes



- The dimensions of this box are 1 X 1 X 1.
- If the owner wants her signature gold ribbon along the perimeter of each box lid, how much ribbon is needed for this box?
- What is the volume of the box?

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Group Task

The owner likes the shape of the box and the ribbon along the perimeter of the lid, but she wants bigger boxes that are similar.

Make three more boxes according to the directions:

A-- Double the original dimensions

B -- Triple the original dimensions

C -- Quadruple the original dimensions

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Group Task

- Create and fill in a table like the one here:

Box	Length	Width	Height	Perimeter of Lid	Volume

- What patterns do you notice in the table?
- Create a graph showing (use a different color for each):
  - The side length and perimeter of box lids
  - The side length and volume of boxes

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Processing the Math

- Examine the ratios of side lengths to perimeters. What do you notice?
- Examine the ratios of side lengths to volumes. What do you notice?
- Compare and contrast the graphs.
- What kind of relationship does each graph show?

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"The classical school presentation of proportionality in terms of *solving proportions* does not lead easily to a useful and flexible understanding of the idea."

The ideas of functions as a way of expressing the mutual variation of two quantities need to more thoroughly and consistently replace the classical presentation."

Stanley, Dick, Ph.D., McGowan, Diane, and Hull, Susan Hudson, Ph.D. "Proportionality." 2003. Retrieved from http://www.utdanacenter.org/mathtoolkit/downloads/support/proportionality.pdf

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Final Processing

Discuss with your table partners:

- How is a table helpful in considering proportional relationships?
- In what ways is the graph of a set of data helpful in considering proportional relationships?
- How do these two representations support students' efforts to write the equation for proportional relationships?

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“These standards are not intended to be new names for old ways of doing business. They are a call to take the next step.”

Common Core State Standards, page 5

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