Beyond Invert and Multiply: Making Sense of Fraction Computation

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Have you ever heard….

Yours is not to reason why,

Just invert and multiply!

In contrast to…..

"Children who are successful at making sense of mathematics are those who believe that mathematics makes sense."

-Lauren Resnick

CCSS Standards for Mathematical Practice

1. Make sense of problems and persevere in solving the
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Brendan, Grade 4

1. Without computing the exact answer, decide which of these expressions would produce the answer with the least value and the greatest value.
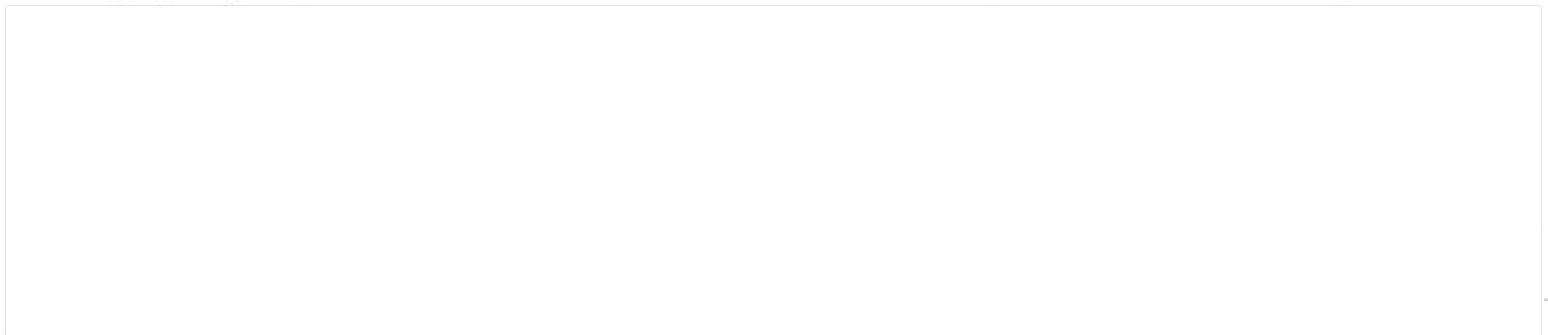
A. Addition: $\frac{3}{4} + \frac{5}{8}$

B. Subtraction: $\frac{3}{4} - \frac{5}{8}$

C. Multiplication: $\frac{3}{4} \times \frac{5}{8}$

D. Division: $\frac{3}{4} \div \frac{5}{8}$

Explain your thinking below:

Brendan, Grade 4

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Least Value _Division_

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Greatest Value _Multiplication_

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Greatest Value **Multiplication**

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Explain your thinking below:

Division because you have tol split it up.
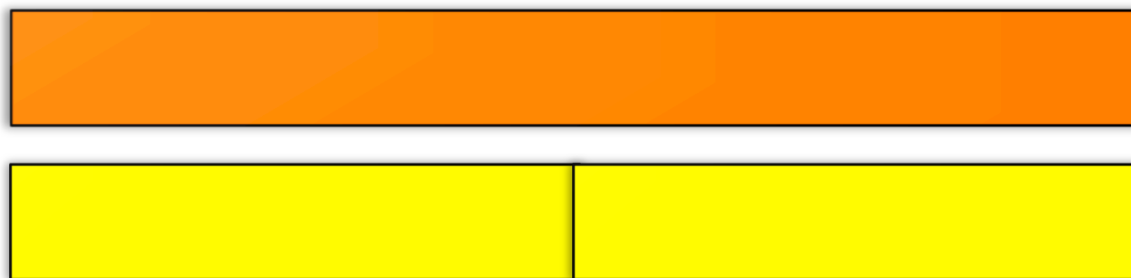Multiplication beause you have to add more than 2 of it.

Fractions as numbers…

"In mathematics, do whatever it takes to help you learn something, provided you do not lose sight of what you are supposed to learn. In the case of fractions, it means you may use any pictorial image you want to process your thoughts on fractions, but *at the end, you should be able to formulate logical arguments in terms of the original definition of a fraction as a point on the number line."*

-Wu, 2002, p. 13

Rod Relations

- Using your Cuisenaire rods, find as many fractional relationships as you can.

- For example:
  - 1 orange = 2 yellows, so 1 yellow = ½ orange

Developing Generalizations

Using Principles to Generalize

New Video: "¾ + ¾ : Belen's Strategy"

https://mathsolutions.wistia.com/medias/m3oc5e92qi

From *Beyond Pizzas and Pies: 10 Essential Strategies for Supporting Fraction Sense.* Math Solutions, 2015.

Making Sense of Fraction Addition

Whole Number Addition and Subtraction Strategies

- Decomposing/recomposing
- Associative property
- Commutative property
- Renaming

Use the Cuisenaire Rods to solve:

$\frac{1}{2}$ brown rod + $\frac{1}{2}$ brown rod

$\frac{1}{4}$ brown rod + $\frac{1}{4}$ brown rod

$\frac{1}{2}$ brown rod + $\frac{1}{4}$ brown rod

Addition with Cuisenaire Rods, V1 and V2

- Version 1:
  - All problems use brown rod as the whole
  - May need to rename one addend

- Version 2:
  - Problems use different rods as the whole
  - May need to rename both addends

Subtraction with Cuisenaire Rods, V1 and V2

- Version 1:
  - All problems use brown rod as the whole
  - May need to rename one addend

- Version 2:
  - Problems use different rods as the whole
  - May need to rename both addends

Get to the Whole!

Decomposing and recomposing fractions to "get to the whole" when adding and subtracting.

$$\frac{3}{4} + \frac{3}{4}$$

$\frac{3}{4} + \frac{3}{4}$: Will's Strategy

New Video: "¾ + ¾ : Will's Strategy"

https://mathsolutions.wistia.com/medias/ct9qxko5n3

From *Beyond Invent and Multiply: Making Sense of Fraction Computation.*
Math Solutions, 2015.

$\frac{3}{4} + \frac{3}{4}$: Belen's Strategy

New Video "¾ + ¾ : Belen's Strategy"

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Math Solutions, 2015.

$\frac{3}{5} + \frac{4}{5}$: Malaya's Strategy

New Video: "3/5 + 4/5 : Malaya's Strategy"

https://mathsolutions.wistia.com/medias/plerkbj369

From *Beyond Invent and Multiply: Making Sense of Fraction Computation.*
Math Solutions, 2015.

Student work

Student work

Student work

$$1) \frac{5}{9} + \frac{8}{9} = 1\frac{4}{9}$$

$$\frac{4}{9} + \frac{1}{9} + \frac{8}{9} = \frac{4}{9} + 1 = 1\frac{4}{9}$$

Making Sense of Fraction Multiplication

Tell Me All You Can

Before coming up with an exact answer,
consider what you know about the answer as a means
of getting a sense of the "neighborhood" of the answer.

Tell Me All You Can

- The answer will be less than _____ because _____.

- The answer will be greater than _____ because _____.

- The answer will be between _____ and _____ because _____.

What do you know about $6 \times 2\frac{1}{2}$?

New Video: "What Do We Know About 6 x 2½?"

https://mathsolutions.wistia.com/medias/8l3m3y621s

From *Beyond Invent and Multiply: Making Sense of Fraction Computation.*
Math Solutions, 2015.

What do you know about $4\frac{1}{2} \times 5$?

New Video: "4½ Is More Than 4 But Less Than 5"

https://mathsolutions.wistia.com/medias/qdsgv6hoox

From *Beyond Invent and Multiply: Making Sense of Fraction Computation.*
Math Solutions, 2015.

Connecting Multiplication to Addition

Using repeated addition to solve $6 \times \frac{1}{2}$

New Video: "Multiplication as Repeated Addition"

https://mathsolutions.wistia.com/medias/6b39lele6d

From *Beyond Invent and Multiply: Making Sense of Fraction Computation.*
Math Solutions, 2015.

Making Sense of Fraction Division

Two types of division situations:

Quotative (also called measurement division):
- Size of group is known; number of groups is unknown
- 6 ÷ 2: How many 2's are in 6?

Partitive:
- Number of groups is known; how many in each group is unknown
- 6 ÷ 2: Split 6 into 2 groups ➜ 6 is 2 groups of what?

Quotative Division

- $6 \div 2$: How many 2's are in 6?

- $\dfrac{3}{4} \div \dfrac{1}{2}$: How many $\dfrac{1}{2}$'s are in $\dfrac{3}{4}$?

- $1 \div \dfrac{1}{4}$: How many $\dfrac{1}{4}$'s are in 1?

How Long? How Far? Part 1

How many $\frac{1}{4}$ minutes are in 1 minute?

How many $\frac{1}{4}$'s are in 1?

New Video: "How Many ¼s Are in 1?"

https://mathsolutions.wistia.com/medias/mbae8pjeny

From *Beyond Invent and Multiply: Making Sense of Fraction Computation.*
Math Solutions, 2015.

Quotative Division

- Use your Cuisenaire rods and number line to show how many $\frac{1}{4}$'s are in 1.
- Use your Cuisenaire rods and number line to show how many $\frac{1}{4}$'s are in 2.
- Use your Cuisenaire rods and number line to show how many $\frac{1}{3}$'s are in 2.

Reasoning about $1 \div \frac{1}{6}$

$$1 \div \frac{1}{6} = 6$$

6 becaues the ~~red fod~~ 6 of the reel rod fit perfictly in 1.

Reasoning about $2 \div \frac{1}{6}$
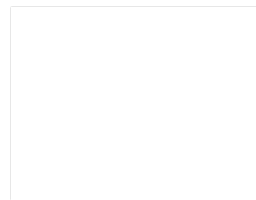
B.) $2 \div \frac{1}{6} =$

How many $\frac{1}{6}$ are in 2?

Reasoning about $10 \div \frac{1}{3}$

c.) $10 \div \frac{1}{3} = 30$

How many $\frac{1}{3}$s are in 10?
I Multiplied the Denominator
and the Problem got me 30.

Reasoning about $6 \div \frac{3}{4}$

d, $6 \div 3/4 = ?$

$$\frac{3}{4}, 1\frac{2}{4}, 2\frac{1}{4}, 3, 3\frac{3}{4}, 4\frac{2}{4}, 5\frac{1}{4}, 6$$

$$1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8$$

goes into
6   8 times

Ship counted to 6 wholes
and it took 8 times

Partitive Division

$6 \div 3$:   Split 6 into 3 groups ➜ 6 is 3 groups of what?

$\dfrac{3}{4} \div 3$:   Split $\dfrac{3}{4}$ into 3 groups ➜ $\dfrac{3}{4}$ is 3 groups of what?

How Long? How Far? Part 2

Beach Clean-Up (2 people)		
Distance	Expression	Each person cleans
8 miles	8 ÷2	4 miles
4 miles	4 ÷2	2 miles
2 miles	2÷2	1 mile
1 mile	1 ÷2	½ mile
½ mile	½ ÷2	?

How Long? How Far? Part 2

$$\frac{1}{2} \div 3$$

$$\frac{1}{6} \div 2$$

$$\frac{3}{4} \div 3$$

Do you always have to invert and multiply?

Your friend tells you she doesn't understand why your teacher makes you invert and multiply to divide fractions. She says you can just divide across the numerators and denominators to get your answer. She shows you the two examples below to prove her point:

$$\frac{4}{9} \div \frac{2}{3} = \frac{2}{3} \qquad \frac{15}{8} \div \frac{3}{4} = \frac{5}{2}$$

What do you think of her idea?
- Is she right?
- If so, why? If not, why not?

What about in this case?

$$\frac{3}{4} \div \frac{2}{3}$$

CCSS Number and Operations - Fractions

3.NF: Develop understanding of fractions as numbers.
4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
5.NF: Use equivalent fractions as a strategy to add and subtract fractions.
5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

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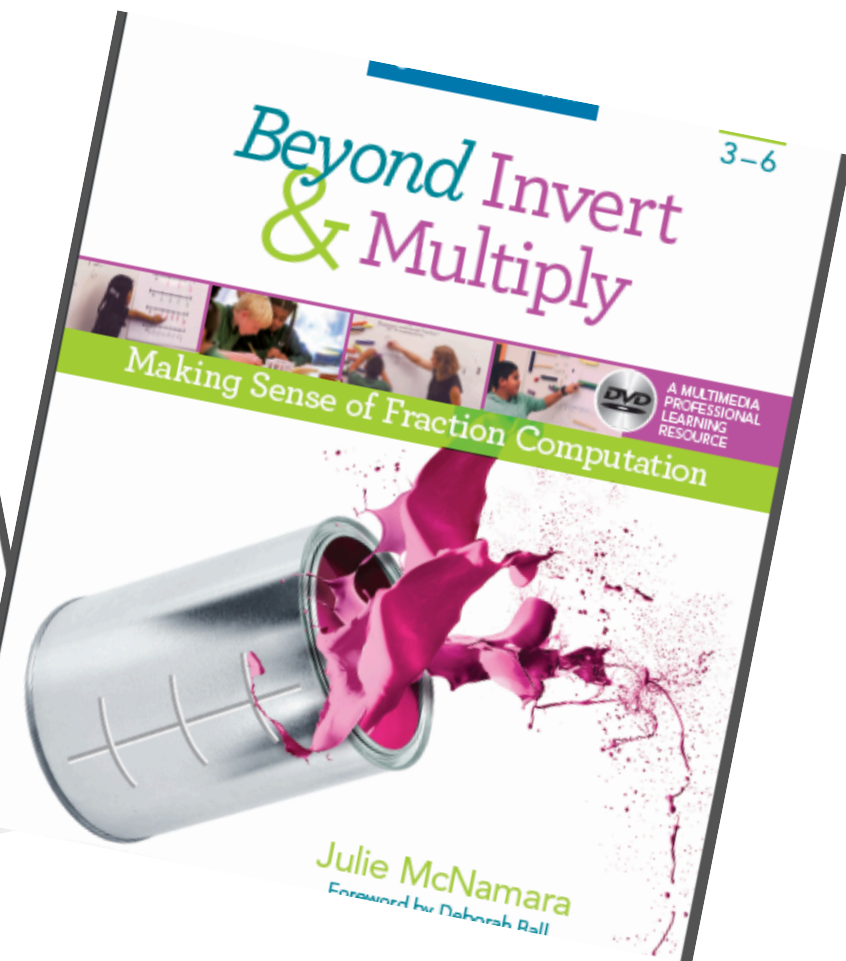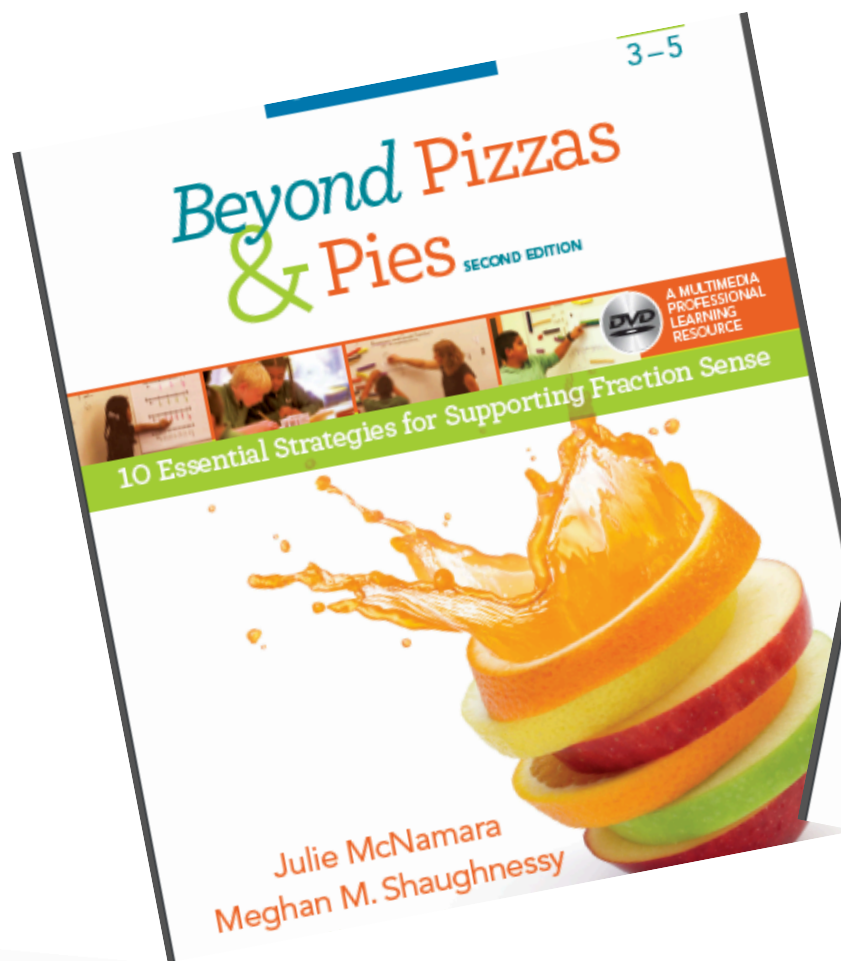
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Available Now!!

Thank you!!!

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