Effective Classroom Teaching for Developing Numerical Understanding and Skills

Marilyn Burns

NCTM Annual Conference
Thursday, April 16, 2015
## 15% & 20% Tip Table

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Tip N Split
Tip Calculator
Calculate Tips & Split the Bill in 5 seconds!
WEAR TIP CALCULATOR

Exclusively on Android Wear
FIRST THINGS FIRST.
SELECT YOUR TIPPING PHILOSOPHY BELOW.

GENEROUS
AMAZING = 25%  GREAT = 20%
OKAY = 15%  SUCKED = 10%

FAIR
AMAZING = 20%  GREAT = 15%
OKAY = 10%  SUCKED = 5%

STINGY
AMAZING = 15%  GREAT = 10%
OKAY = 7%  SUCKED = 5%
IF YOUR BILL WAS $42.42 AND

YOUR SERVER WAS
AMAZING!

YOUR SERVER WAS
GREAT

YOUR SERVER WAS
OKAY

YOUR SERVER
SUCKED

THEN YOU SHOULD TIP:
$8.48

Swipe left for total.
Gratuity Not Included:

(Tipping Guide)

15%: ____________________

18%: ____________________

20%: ____________________

Total Charge 25.40

Room Number 1354

Print Name BURNS

Signature

Manly
3 R’s

- Reading
- ‘Riting
- ‘Rithmetic
ARITHMETIC

COMPUTATION

PROBLEM SOLVING

NUMBER SENSE
3 R’s

- Reading
- ‘Riting
- Reasoning
Yours is not to reason why, just invert and multiply.
Do only what makes sense to you. And persist until it does.
Mathematical Practices

• Numerical Reasoning
• Teachers’ Math Knowledge
• Classroom Instruction
10 − 5
Justin: Grade 1

\[ \frac{0}{0} \div \frac{5}{5} \]
Ellen: Grade 3

100 – 3
Ellen: Grade 3

100 – 3

100 – 98
School Bus Problem

There are 295 students in the school.
School buses hold 25 students.
How many school buses are needed to fit all of the students?
295 students, 25 per bus
Marisa: School Bus Problem

View at https://mathreasoninginventory.com/
Click on Resources Tab, then Video Library
Marisa

\[
\begin{array}{c}
\frac{1}{4} + \frac{1}{17} = \frac{295}{221} \\
\frac{25}{1} + \frac{25}{1} = \frac{25}{1} \\
\frac{25}{3} + \frac{25}{20} = \frac{50}{60} + \frac{25}{60} = \frac{75}{60} = \frac{5}{4} = 1\frac{1}{4}
\end{array}
\]

\[
\frac{320}{320}
\]
99 + 17
Ana: 99 + 17

View at https://mathreasoninginventory.com/
Click on Resources Tab, then Video Library
Zakari: 99 + 17

View at https://mathreasoninginventory.com/
Click on Resources Tab, then Video Library
99 + 17: Go to MRI site

Jada

Manuel

Dina

Amir
\[
\frac{1}{2}\left(\frac{2}{3}\right) + \frac{2}{3}\left(\frac{2}{2}\right) = \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1 \frac{1}{6}
\]
Amelia

\[ \frac{1}{2} + \frac{2}{3} \]

\[ \frac{1}{2} = \frac{1.5}{3} + \frac{2}{3} = \frac{3.5}{3} = \frac{1.5}{3} \]
“That the quality of mathematics teaching depends on teachers’ knowledge of the subject should not be a surprise.”

*What Mathematical Knowledge Is Needed for Teaching Mathematics?*
Deborah Loewenberg Ball
University of Michigan
Alexa

\[
\begin{array}{c}
49910 \\
5000 \\
-328 \\
\hline
4672
\end{array}
\]
Jesús

\[
\begin{array}{c}
5000 \\
-328 \\
\hline
4672
\end{array}
\]

\[
\begin{array}{c}
4700 \\
4680
\end{array}
\]
Are students using reasoning strategies that are appropriate for the numbers at hand?
Can students be successful in math class without having to reason?
503 − 398
Is the answer greater or less than 100?

503 \(-\) 398
500 \(-\) 400
Camille
Fran

\[ 398 + 10 - 5 + 100 = 400 \]
\[ 403 - 5 = 400 \]
\[ 300 + 100 = 500 \]

\[ 398 + 5 + 5 = 408 \]
\[ 408 - 5 = 403 \]
\[ 500 - 500 = 500 \]
Zoe
$12 \div 0.3$
$12 \div 0.3$

Which is the exact answer?

0.04  0.4  4  40
\[
\text{Jackson} = 12 \div 3 = 4
\]
Answer has to be higher because 0.03 is smaller.
It has to be 40.
lars

Did it on mental chalkboard

0.3\overline{12}
Maverick

$12 \div 0.3$ is the same as

$0.3 \times \_ = 12$
12 ÷ 0.3

Which is the exact answer?

0.04 0.4 4 40

Jackson
\[
\frac{12}{3} = 4
\]
Answer has to be higher, because 0.3 is smaller.
It has to be 40

Lars
Did it on mental chalkboard

Maverick
\[
12 \div 0.3 \text{ is the same as}
0.3 \times \_ = 12
\]
\[
0.3 \overline{12}
\]
Quick Write

Explain why 40 is the answer to $12 \div 0.3$. 
Well, $12 \div 3 = 4$ then you multiply .03 by all the possible answers until you get 40. Also the answer couldn't be less than 5.

$\frac{.03}{40}$

$\frac{00}{12}$
$12 + 0.3$ is the same thing as $0.3 \times \_ = 12$

so $\times 0.3$ and there is one decimal point so $120$ would turn to $\boxed{12}$ so the

$\underline{12} \times \boxed{40}$ blank space = $\boxed{40}$
The way I solved the problem was I knew $12 \div 3 = \frac{12}{3}$ and I knew that because the numerator was bigger than the denominator, it had to be a whole number. I also thought that $\frac{12}{3} = 4$, so $\frac{12}{3}$ must be bigger than $4$, by $10$ times. $10$ times $4$ is $40$ so I knew that must be the answer.
If done using a mental checkerboard you will always get the same answer using paper, unless human error.
Question 9

For this division problem, look at the choices and decide which is the exact answer.

12 ÷ 0.3

0.04 0.4 4 40

Answer
- Correct (40)
- Incorrect
- Self-corrected (40)
- Did Not Answer

Explanation
- How did you figure out the answer?
- Divided 12 by 3 and then adjusted decimal point
- Multiplied and then adjusted decimal point
- Analyzed choices and chose one that seemed most reasonable
- Moved decimal point one place to the right in both numbers and divided
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

Notes

record student response
Amir

View at https://mathreasoninginventory.com/
Click on Resources Tab, then Video Library
Dina

View at https://mathreasoninginventory.com/
Click on Resources Tab, then Video Library
The Two-Dice Sum Game

You need: counters, 11 per player
1–6 dice, 2 per pair or group of students

This is a game for two or more players. Each player draws a number line from 2 to 12, with spaces between the numbers large enough so counters fit on the numbers.

Place your eleven counters on your number line in any arrangement. (You may put more than one counter on some numbers and none on others.)

Take turns rolling the dice. On each roll, every player removes one counter that’s on the number that matches the sum of numbers that come up on the two dice. (If players have more than one counter on a sum, they may remove only one of them. If players don’t have a counter on a sum, they don’t do anything.) The winner is the first player to remove all eleven counters.

Decide on the best winning arrangement of counters on the number line. Explain your thinking.
Math Menu

Pattern Block Explorations (I)

The Two-Dice Sum Game (P)

Comparing Names (I)
Is It Fair? (Version 1)

Player A wins 6 times, Player B wins 6 times.

Possible Outcomes:

- 2, 4, 6, 8, 10, 12
- 3, 5, 7, 9, 11, 12

It is fair.
36 Equally Likely Ways to Get 11 Possible Sums

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Two Dice Sum Game

what your board should look like:

Dear Second Grader(s),

A good strategy for this game is to put 1 cube on each number. This is a good strategy because then you have at least one cube for every number. Although, this strategy doesn’t always work because once you take off that one cube you can never take off another cube.

Good Luck!

-Sarah Fraschetti
Dear second graders,

when you play ten or I recommend making a “mountain around the seven,” this will give you a high probability of winning

sincerely, Jackson

[Diagram of a number line with points marked at 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]
Dear second grade Students from Ms.Narvases class,

My advice for winning the two dice Sum game is not to pile them all up on one number but to scatter them out. The odds are higher on 3, 8, and 6. There are the most possible combinations for these, there are 6 possible combos for 7, and 5 possible combos for 6 and 8. There are the most possible combos for 7, and it goes down from there.

Possible → 2 3 4 5 6 7 8 9 10 11 12
Combos

Sincerely,

Charlie Dyer

from Ms.Adams class.
Dear Second Graders,

Your strategy for the game should be: learn from your past games. Know generally which numbers are rolled the most, usually 5-9, and place your cubes accordingly. 8 works for me.

Please reply so I know how this strategy works out for you. Sincerely,

Anonymous
Our teaching goal is not to *cover* the Standards . . .
Our teaching goal is not to *cover* the Standards . . .

but to *uncover* them.
ABOUT TEACHING Mathematics
Marilyn Burns