10 BIG MATH IDEAS

Several years ago, Michael, one of my third graders, wrote this in his journal: "I never used to look forward to math. All we did was add and subtract. Now I like it more. We work together in class, and we still learn math but in a better way." In a sense, Michael described the challenge we face as math teachers—to help students become flexible thinkers who are comfortable with all the content areas of mathematics and able to apply their learning to problem-solving situations. I have to admit—my early teaching resembled the math class Michael described, but over time I have found more engaging and effective approaches. Here are the ten "big ideas" I now embrace for helping children learn, understand, and enjoy math class.

BY Marilyn Burns

1 SUCCESS COMES FROM UNDERSTANDING. Set the following expectation for your students: Do only what makes sense to you. Too often, students see math as a collection of steps and tricks that they must learn. And this misconception leads to common recurring errors—when subtracting, students will subtract the smaller from the larger rather than regrouping; or when dividing, they'll omit a zero and wind up with an answer that is ten times too small. In these instances, students arrive at answers that make no sense, and they rarely know why.

Help students understand that they should always try to make sense of what they do in math. Always encourage them to explain the purpose for what they're doing, the logic of their procedures, and the reasonableness of their solutions.

2 HAVE STUDENTS EXPLAIN THEIR REASONING. It's insufficient and shortsighted to rely on quick, right answers as indications of students' mathematical power. During math lessons, probe children's thinking when they respond. Ask: Why do you think that? Why does that make sense? Convince us. Prove it. Does anyone have a different way to think about the problem? Does anyone have another explanation?

When children are asked to explain their thinking, they are forced to organize their ideas. They have the opportunity to develop and extend their understanding. Teachers are accustomed to asking students to explain their thinking when their responses are incorrect. It's important, however, to ask children to explain their reasoning at all times.

3 MATH CLASS IS A TIME FOR TALK. Communication is essential for learning. Having students work quiet-

ly—and by themselves—limits their learning opportunities. Interaction helps children clarify their ideas, get feedback for their thinking, and hear other points of view. Students can learn from one another as well as from their teachers.

Make student talk a regular part of your lessons. Partner talk—sometimes called "turn and talk" or "think-pair-share"—encourages students to voice their ideas. Giving them a minute or so to talk with a neighbor also helps students get ready to contribute to a discussion. It's especially beneficial to students who are generally hesitant to share in front of the whole class.

4 MAKE WRITING A PART OF MATH LEARNING. Communication in math class should include writing as well as talking. In his book *Writing to Learn* (HarperInformation, 1993), William Zinsser states: "Writing is how we think our way into a subject and make it our own." When children write in math class, they have to revisit their thinking and reflect on their ideas. And student writing gives teachers a way to assess how their students are thinking and what they understand.

Writing in math class best extends from children's talking. When partner talk, small-group interaction, or a whole-class discussion precedes a writing assignment, students have a chance to formulate their ideas before they're expected to write. Vary writing assignments. At the end of a lesson, students can write in their math journals or logs about what they learned and what questions they have. Or ask them to write about a particular math idea—"what I know about multiplication so far," or "what happens to the sums and products when adding even and odd numbers." When solving a problem, encourage students to record how they reasoned. Writing prompts on the board can help students get started writing. For example: Today I learned ..., I am still not sure about ..., I think the answer is ..., I think this because....

5 PRESENT MATH ACTIVITIES IN CONTEXTS. Real-world contexts can give students access to otherwise abstract mathematical ideas. Contexts stimulate student interest and provides a purpose for learning. When connected to situations, mathematics comes alive.

"During math lessons, probe children's thinking. Ask: 'Why do you think that? Why does that make sense? Convince us. Prove it. Does anyone have a different way to think about the problem? Does anyone have another explanation?'"

Contexts can draw on real-world examples. For example, ask students to figure out what you might have bought and how much it cost if, after paying for it, you received $0.35 change. Or ask children to figure out how much money each of four children would get if they shared $5.00 equally. Or ask a group of children to estimate and then figure out how many raisins each of them would get if they shared a snack-size box.

Contexts can also be created from imaginary situations, and children's books are ideal starting points for classroom math lessons. After reading Eric Carle's *Rooster's Off to See the World* (Simon & Schuster, 1991), for example, ask children if they can figure out how many animals went traveling. Or ask children to follow the calculations in Judith Viorst's *Alexander, Who Used to Be Rich Last Sunday* (Simon & Schuster, 1978), and figure out how Alexander spent

his money. For a ready-to-use, literature-linked math lesson, see "A Step-by-Step Lesson with Marilyn Burns" on page 19.

6 SUPPORT LEARNING WITH MANIPULATIVES.
Manipulative materials help make abstract mathematical ideas concrete. They give children the chance to grab onto mathematics ideas, turn them around, and view them in different ways. Manipulative materials can serve in several ways— to introduce concepts, to pose problems, and to use as tools to figure out solutions. It's important that manipulatives are not relegated to the early grades but are also available to older students.

For teachers just getting started using manipulatives, classroom staples should include at least 400 color tiles (1" square tiles in four colors), three to six sets of pattern blocks (six dif-

ferent shapes which typically include green triangles, yellow hexagons, blue and tan parallelograms, orange squares, and red trapezoids), 500–1000 interlocking cubes (usually in 10 colors, about 3/4"), and a supply of measuring tools.

7 LET YOUR STUDENTS PUSH THE CURRICULUM.
Avoid having the curriculum push the children. Choose depth over breadth and avoid having your math program be a mile wide and an inch deep. As David Hawkins said in *The Having of Wonderful Ideas*, by Eleanor Duckworth (Teachers College Press, 1996), "You don't want to cover a subject; you want to uncover it." There are many pressures on teachers, and the school year passes very quickly. But students' understanding is key and doesn't always happen according to a set schedule. Stay with topics that interest children, explore them more deeply, and take the time for side investigations that can extend lessons in different directions.

8 THE BEST ACTIVITIES MEET THE NEEDS OF ALL STUDENTS.
Keep an eye out for instructional activities that are accessible to students with different levels of interest and experience. A wonderful quality of good children's books is that they delight adults as well. Of course, adults appreciate books for different reasons than children do, but enjoyment and learning can occur simultaneously at all levels. The same holds true for math. Look for activities that allow for students to seek their own level and that also lend themselves to extensions.

For example, challenge children to find the sum of three consecutive numbers, such as 4 + 5 + 6. Ask them to do at least five different problems and see if they can discover how the sum relates to the addends. (The sum is always the middle num-

ber tripled.) Allowing the children to select their own numbers to add is a way for students to choose problems that are appropriate for them. Even those students who don't discover the relationship will benefit from the addition practice. Invite more able students to write about why they think the sum is always three times the middle number, or to investigate the sums of four consecutive numbers.

9 CONFUSION IS PART OF THE PROCESS. Remember that confusion and partial understanding are natural to the learning process. Don't expect all children to learn everything at the same time, and don't expect all children to get the same message from every lesson. Although we want all students to be successful, it's hard to reach every student with every lesson. Learning should be viewed as a long-range goal, not as a lesson objective. It's important that children do not feel deficient, hopeless, or excluded from learning mathematics. The classroom culture should reinforce the belief that errors are opportunities for learning and should support children taking risks without fear of failure or embarrassment.

10 ENCOURAGE DIFFERENT WAYS OF THINKING. There's no one way to think about any mathematical problem. After children respond to a question (and, of course, have explained their thinking!), ask: Does anyone have a different idea? Keep asking until all children who volunteer have offered their ideas. By encouraging participation, you'll not only learn more about individual children's thinking, but you'll also send the message that there's more than one way to look at any problem or situation. That's when the potential for delight occurs. ■

A Step-by-Step Lesson with Marilyn Burns

While reading, I stopped to call attention to unusual numbers—or anything else I can connect to math.

We made a sticky note graph to represent how many letters are in each of our first names.

Students wrote their first and last names on sentence strips to see which one had more letters.

This bar graph showed how students' first names can be shorter, longer, or the same as their last.

More and more teachers are now taking advantage of math connections in favorite children's literature. Recently, I used *Chrysanthemum* by Kevin Henkes (HarperCollins, 1991) to help third graders practice addition, subtraction, and graphing skills. Here is my lesson plan, step-by-step:

●**COUNTING LETTERS:** After reading the book aloud, we discussed that the name Chrysanthemum has 13 letters—half as many letters as are in the alphabet! "My first name has seven letters," I told the children. "How many more letters does Chrysanthemum's name have than my name?" I encouraged the students to talk with a neighbor about this, and then had them say the answer together in a whisper voice.

●**USING COMPLETE SENTENCES:** After several children explained their answer, I invited students to figure out how many more letters there are in Chrysanthemum's name than in their own—reminding them that they would need to be able to tell the class their answers in a complete sentence. With coaxing, each child was able to report. Devin said, "Chrysanthemum has eight more letters than my name."

●**FINDING THE SHORTEST NAME:** Then I challenged the students to think about who had the shortest name in the class. Together we determined that Made, Isak, Anna, and Will had the shortest names with four letters each. "Stand up if you have five letters in your first name," I said. We counted five children. Then we counted five children with six letters, two with seven, one with eight, and finally

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Annapurna, who had the longest name with nine letters.

●**GRAPHING OUR FIRST NAMES:** The next step was to make a graph to show this information. The children returned to their desks. As they wrote their first names on sticky notes, I wrote "Letters in Our First Names" on the board and listed the numbers from 1 to 10. I posted my sticky note as a model. Then each student came up and posted his or her name next to the correct number.

●**READING THE GRAPH:** As a class, we discussed what we noticed about our graph. "The longest name has nine letters," Trent said. "There are the same number of names in the five row and the six row," Daniela said. "I'm the only one with eight letters," Danielle said.

●**SETTING A CHALLENGE:** "Your challenge is to figure out how many letters are in all of our first names together," I told the children. I invited them to share ideas of how they might do this using the information on the graph. Then I told them that they could work with the other students at their table, but that they each needed to write their own paper and explain their thinking.

●**CHECKING WITH MANIPULATIVES:** To verify the answer and link the activity to place value, I put a supply of interlocking cubes at each table and asked the children to each make a train of cubes as long as their first name.

●**WORKING TOGETHER:** I then asked the children at each table to combine their trains into tens and ones and be ready to report their results. I recorded these on the board and, together, we figured that there were 90 tens and 22 ones. "Talk at your table about what the total is," I said. After a moment, several students shared their ideas. Isak explained, "Ninety and one more ten is one hundred, and one more ten is one hundred ten, plus two is one hundred twelve."

●**EXTENDING THE LESSON:** "What about your last name?" I asked. "Are there more, fewer, or the same number of letters in your last name as in your first name?" I gave each student a 2" by 11" strip of two-centimeter squares. I modeled for the students how to write their first name in one row and their last name on the next row, and then trim the extra squares.

●**WRAPPING IT UP:** I prepared a chart with three columns and labeled them. I said, "You'll post your name strip to show if your first name is shorter than, the same length as, or longer than your last name." After posting my strip, the children posted theirs. I asked, "Do you think that there are more letters total in our last names or in our first names?" Most thought that there would be more letters in their last names combined. To end the lesson, I said, "Tomorrow we'll make a graph of our last names and figure it out." This repeat experience would provide the students with additional practice. On another day, I planned to give the students the first names from Chrysanthemum's class and have them individually make graphs of the names, figure out the total number of letters in their names, and compare this total with our class total. ■