# Looking at How

*Mathematics teachers gain a wealth of information by delving into the thinking behind students' answers, not just when answers are wrong but also when they are correct.*

**Marilyn Burns**

First, a confession: Only during the last 10 to 15 years of my teaching career have I thought deeply about assessing students' understanding and learning progress. As a beginning teacher, I focused on learning to manage my classroom, plan lessons, and hold students' attention. Later, my focus shifted to improving my lessons and expanding my instructional repertoire. During those years, my attention was always firmly on my teaching. Assessment was not one of my concerns. Yes, I gave assignments and quizzes and examined the results, but I did so more to determine grades than to figure out what students were thinking.

Assessment plays a much different role in my teaching today. Although I'm no longer a full-time classroom teacher, I still spend time teaching students in elementary classrooms as I try out new instructional ideas. I now approach assessment in an intentional way and incorporate it into every lesson. No longer am I satisfied to simply record students' performance on assignments and quizzes; now, my goal is to find out, as I teach, what the students understand and how they think. I am still interested in honing my lessons, but along with planning the sequence of learning activities, I also prepare to question students about their thinking during class discussions, in individual conversations, and on written assignments. In addition, linking assessment with instruction has become a key issue in the professional development I provide to other teachers.

After teaching a lesson, we need to determine whether the lesson was accessible to all students while still challenging to the more capable; what the students learned and still need to know; how we can improve the lesson to make it more effective; and, if necessary, what other lesson we might offer as a better alternative. This continual evaluation of instructional choices is at the heart of improving our teaching practice.

## Uncovering the Way Students Think

In my early teaching years, I was a devotee of *discovery learning,* sometimes called *inquiry learning.* This instructional approach involves designing learning activities that help



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# Students Reason

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students discover concepts and make sense of facts and principles for themselves, rather than relying on textbooks or teacher explanations. I implemented this approach by asking the class a carefully prepared sequence of questions, in the style of Socrates. If a student's response was correct, I continued to the next question. If a student's response was incorrect, other students would typically raise their hands to disagree, and I'd let a class discussion unfold until someone proposed the correct response. Then I'd continue with the next question. If no students objected to an incorrect response, I'd ask a slightly different question to lead students to the right answer.

Years later, I thought about why the discovery method of instruction seemed flawed. The problem was that when a student gave a correct response, I assumed that both the student who had answered correctly and his or her classmates understood the mathematics behind the problem. I never probed students' level of understanding behind their responses; I just happily continued on my teaching trajectory.

**Students need to be able to look at mathematical situations from different perspectives.**

As a result, I never really knew what students were thinking or whether their correct answers masked incorrect ideas. I only knew that they had given the answer I sought.

I no longer teach this way. Although I still believe in the value and importance of using questions to present ideas for students to consider, I've broadened my use of both oral and written questions so that I now attempt to probe as well as stimulate students' thinking.

For example, when teaching fractions to a class of 4th graders, I wrote five fractions on the board—$^{1}/_{4}$, $^{11}/_{16}$, $^{3}/_{8}$, $^{1}/_{16}$, and $^{3}/_{4}$—and asked the students to write the fractions in order from smallest to largest. I then added another step by asking them to record their reasons for how they ordered the fractions. After giving the students time to solve the problem, I initiated a whole-class discussion. Robert reported first. He said with confidence, "The smallest fraction is $^{1}/_{16}$." In my early days of teaching, I was accustomed to questioning students when their answers were incorrect, but not when they were correct. Now, however, I asked Robert to explain how he knew that $^{1}/_{16}$ was the smallest fraction. Robert read from his paper, again with confidence, "Because $^{1}/_{16}$ is the lowest number in fractions." The students had previously cut and labeled strips of construction paper to make fraction kits, and $^{1}/_{16}$ happened to be the smallest piece in their kits. The fraction kit—a standard tool in my instructional repertoire that I've always found effective for developing students' understanding of fractions—had led Robert to an incorrect generalization.

By questioning Robert's correct response, I was able not only to clear up his misunderstanding but also to improve on the fraction kit lesson to avoid this problem in the future. When I teach this lesson now, I always ask students to consider how we would name pieces that are smaller than $^1/_{16}$; I talk with them about how we could continue to cut smaller and smaller pieces and find fraction names for even the teeniest sliver.

Incorporating students' reasoning into both written assignments and classroom discussions was a crucial step toward making assessment an integral and ongoing aspect of my classroom instruction. Now it's a staple of my math teaching.


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## Incorporating students' reasoning into classroom discussions makes assessment an integral aspect of classroom instruction.

### Assessment Through Students' Written Work

One of the main strategies I use to assess students' learning is incorporating writing in math assignments. There are many ways to present writing assignments that yield as much information as possible about what students are thinking (see Burns, 2004).

*Ask for more than one strategy.* Solving math problems often requires making false starts and searching for new approaches. Students need to develop multiple strategies so that they become flexible in their mathematical thinking and are able to look at mathematical situations from different perspectives. Even when students are performing routine computations, asking them to offer more than one way to arrive at an answer provides insight into their thinking.

For example, I worked with one 2nd grade class that had been focusing on basic addition. When they encountered more difficult problems—those involving numbers above 5, such as 9 + 6 and 7 + 8—the students' fallback strategy was always counting. Over time, I helped them develop other strategies for addition. One day, to assess their progress, I asked them to add 6 + 7 and to explain how they could figure out the answer in more than one way. Their work was revealing. Daniel described five methods, including the following method, that showed the progress he had made with the important skill of *decomposing* numbers—taking numbers apart and combining them in different ways:

> You take 1 from the 6 and 2 from the 7 and then you add 5 + 5. Then you add on the 1 and the 2 and you get 13.

Ryan, in contrast, was able to offer only two methods, even when I pushed him for more. He wrote,

> (1) You start with the 6 and count on 7 more. (2) You start with the 7 and count on 6.

Although Ryan's work showed that he understood that addition was commutative, it also showed that his addition strategies were limited to counting.

*Let students set parameters.* A good technique for assessing students' understanding as well as differentiating instruction is to make an assignment adjustable in some way, so that it is accessible and appropriate for a wider range of students. For example, I worked with one 3rd grade class in writing word problems. For several days, the students discussed examples as a group and completed individual assignments. Sometimes I gave a multiplication problem—3 x 4, for example—and asked students to find the answer and also write a word problem around the problem. At other times I gave them a word problem—such as, "How many wheels are there altogether on seven tricycles?"—and asked them to write the related multiplication problem and find the answer. Finally, I gave students the assignment of choosing any multiplication problem, writing a word problem for it, and finding the answer in at least two ways.

Having the students choose their own problems allowed them to decide on the parameters that were comfortable for them. Carrie chose 5 x 2 and wrote a problem about how many mittens five children had. Thomas chose 102 x 4 and wrote a problem about the number of wheels on 102 cars. Each student's choice gave me information about their numerical comfort as well as their skill with multiplication.

*Assess the same concept or skill in different ways.* I've often found that a student's beginning understanding, although fragile, can provide a useful building block or connection to more robust learning. Sometimes a familiar context can help a student think about a numerically challenging problem. Using flexible assessment approaches enables us to build on students' strengths and interests and help them move on from there.

**By building and using a wide repertoire of assessment strategies, we can get to know more about our students than we ever thought possible.**

In a 4th grade class, I watched Josh, who was fascinated by trucks, overcome his confusion about dividing 96 by 8 when I asked him to figure out how many toy 8-wheeler trucks he could make if he had 96 toy wheels. Although his numerical skills were weak, he was able to make progress by drawing trucks and examining the pattern of how many wheels he needed for two trucks, then three trucks, and so on.

*Take occasional class inventories.* Compiling an inventory for a set of papers can provide a sense of the class's progress and thus inform decisions about how to differentiate instruction. For example, after asking a class of 27 5th graders to circle the larger fraction— $^2/_3$ or $^3/_4$ —and explain their reasoning, I reviewed their papers and listed the strategies they used. Their strategies included drawing pictures (either circles or rectangles); changing to fractions with common denominators ($^8/_{12}$ and $^9/_{12}$); seeing which fraction was closer to 1 ($^2/_3$ is $^1/_3$ away, but $^3/_4$ is only $^1/_4$ away); and relating the fractions to money ($^2/_3$ of \$1.00 is about 66 cents, whereas $^3/_4$ of \$1.00 is 75 cents). Four of the students were unable to compare the two fractions correctly. I now had direction for future lessons that would provide interventions for the struggling students and give all the students opportunities to learn different strategies from one another.

### Assessment Through Classroom Discussion

Incorporating assessment into classroom discussion serves two goals: It provides insights into students' thinking, and it ensures that no student is invisible in the class, but that all are participating and working to understand and learn. Here are some strategies to get the most out of class discussions.

*Ask students to explain their answers, whether or not the answers are correct.* When I follow up on both correct and incorrect answers by asking students to explain their reasoning, their responses often surprise me. Some students arrive at correct answers in unexpected ways. For example, when comparing $^4/_5$ and $^3/_4$, Brandon changed the fractions so that they had common numerators—$^{12}/_{15}$ and $^{12}/_{16}$. He knew that 16ths were smaller than 15ths, so $^{12}/_{15}$, or $^4/_5$, had to be larger! Students may also surprise us by using incorrect reasoning to arrive at the correct answer. Lindsay, a 3rd grader, used 7 x 3 = 21 to conclude that 8 x 4 = 32. "Each number is just one bigger," she said, and went on to explain that 1 more than 7 is 8, 1 more

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than 3 is 4, and 1 more than each of the digits in 21 makes the number 32. Although Lindsay's method worked for this problem, it doesn't work for all problems!

*Ask students to share their solution strategies with the group.* After a student responds to a question that I pose and explains his or her reasoning, I ask the group, "Who has a different way to solve the problem?" or "Who has another way to think about this?" I make sure to provide sufficient wait time to encourage students to share ideas. In addition to providing insights into students' thinking and understanding, this method also reinforces the idea that there are different ways to think about problems and lets the students know that I value their individual approaches.

*Call on students who don't volunteer.* For many years, I called only on students who had the confidence to offer their ideas. For students who were less confident, I relied on their written work. I didn't want to intrude on shy students and put them under additional stress. I've since changed this practice, partly because of the insights I gained from the excellent professional resource *Classroom Discussions* (Chapin, O'Connor, & Anderson, 2003). I now tell students that it's important for me to learn about how each of them thinks and, for that reason, I need to hear from all of them. I reassure them, however, that if I call on them and they don't know the answer, they should just let me know. I tell them, "It's important for me to know when a student isn't able to explain so I can think about what kind of support to give." I'm always careful to check in with the student later to determine what kind of intervention I need to provide.

**Solving math problems often requires making false starts and searching for new approaches.**

*Use small-group work.* This technique is especially useful for drawing out students who are reticent about talking in front of the whole class. After posing a problem, I'll often say, "Turn and talk with your partner" or "Talk with your group about this." Then I eavesdrop, paying especially close attention to the students who don't typically talk in class discussions.

*Ask students to restate others' ideas.* This is another strategy I learned from *Classroom Discussions*. After a student offers an idea or answer, I call on someone else with the prompt, "Explain what Claudia said in your own words." If the student can't do this, I prompt him or her to ask Claudia to explain again. If the student still isn't able to restate Claudia's idea, I ask another student to try, reminding the first student to listen carefully and see whether this alternate explanation helps. After a student shares, I ask Claudia, "Does that describe your idea?" Depending on my professional judgment about the student and the situation, I may also return to the first student and ask him or her to try again.

### Improving Mathematics Teaching

According to the National Council of Teachers of Mathematics (2000),

> To ensure deep, high-quality learning for all students, assessment and instruction must be integrated so that assessment becomes a routine part of the

ongoing classroom activity rather than an interruption. Such assessment also provides the information teachers need to make appropriate instructional decisions.

Making assessment an integral part of daily mathematics instruction is a challenge. It requires planning specific ways to use assignments and discussions to discover what students do and do not understand. It also requires teachers to be prepared to deal with students' responses. Merely spotting when students are incorrect is relatively easy compared with understanding the reasons behind their errors. The latter demands careful attention and a deep knowledge of the mathematics concepts and principles that students are learning.

But the benefits are worth the effort. By building and using a wide repertoire of assessment strategies, we can get to know more about our students than we ever thought possible. The insights we gain by making assessment a regular part of instruction enable us to meet the needs of the students who are eager for more challenges and to provide intervention for those who are struggling. EL

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