Beyond Invert and Multiply: Making Sense of Fraction Computation

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CALIFORNIA STATE UNIVERSITY
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Have you ever heard….

Yours is not to reason why,

Just invert and multiply!

In contrast to…..

"Children who are successful at making sense of mathematics are those who believe that mathematics makes sense."

-Lauren Resnick

CCSS Standards for Mathematical Practice

1. Make sense of problems and persevere in solving the
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Brendan, Grade 4

1. Without computing the exact answer, decide which of these expressions would produce the answer with the least value and the greatest value.

A. Addition: $\frac{3}{4} + \frac{5}{8}$

B. Subtraction: $\frac{3}{4} - \frac{5}{8}$

C. Multiplication: $\frac{3}{4} \times \frac{5}{8}$

D. Division: $\frac{3}{4} \div \frac{5}{8}$

Explain your thinking below:

Fractions as numbers…

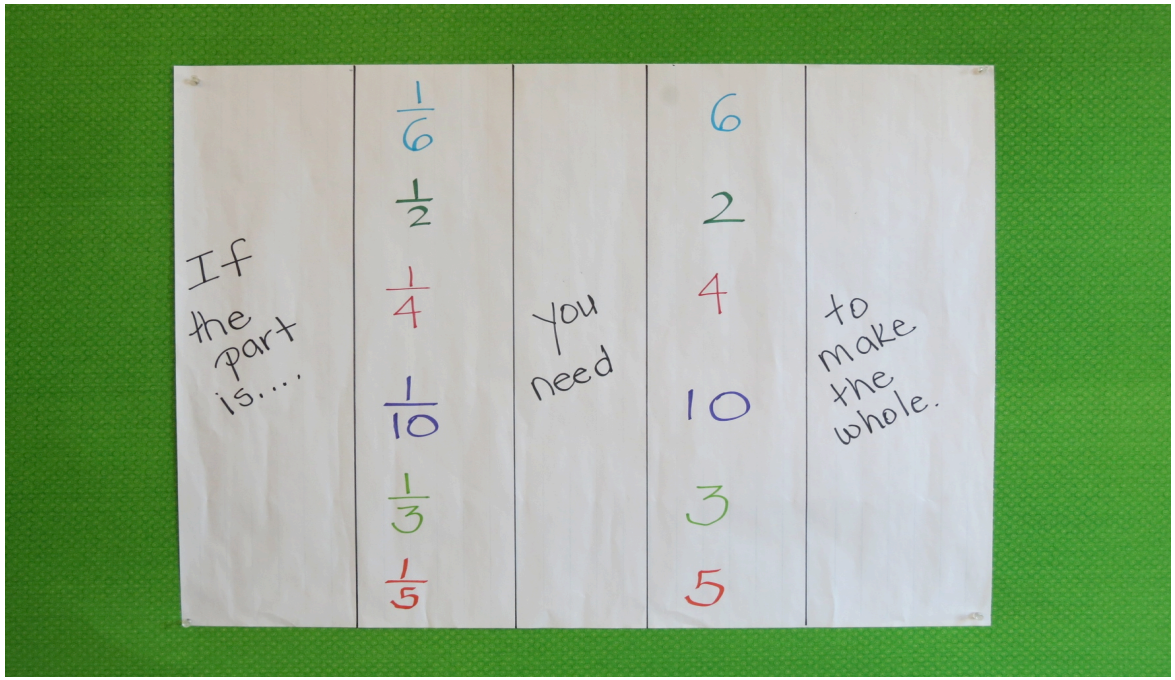
"*In mathematics, do whatever it takes to help you learn something, provided you do not lose sight of what you are supposed to learn*. In the case of fractions, it means you may use any pictorial image you want to process your thoughts on fractions, but *at the end, you should be able to formulate logical arguments in terms of the original definition of a fraction as a point on the number line.*"

-Wu, 2002, p. 13

Rod Relations

- Using your Cuisenaire rods, find as many fractional relationships as you can.
- For example:
  - 1 orange = 2 yellows, so 1 yellow = $\frac{1}{2}$ orange

Developing Generalizations



Using Principles to Generalize

Video 3d from *Beyond Pizzas and Pies: 10 Essential Strategies for Supporting Fraction Sense.* Math Solutions, 2015.

MAKING SENSE OF FRACTION ADDITION

Whole Number Addition and Subtraction Strategies

- Decomposing/recomposing
- Associative property
- Commutative property
- Renaming

Use the Cuisenaire Rods to solve:

$$\frac{1}{2} \text{ brown rod} + \frac{1}{2} \text{ brown rod}$$

$$\frac{1}{4} \text{ brown rod} + \frac{1}{4} \text{ brown rod}$$

$$\frac{1}{2} \text{ brown rod} + \frac{1}{4} \text{ brown rod}$$

Addition with Cuisenaire Rods, V1 and V2

- Version 1:
  - All problems use brown rod as the whole
  - May need to rename one addend
- Version 2:
  - Problems use different rods as the whole
  - May need to rename both addends

Subtraction with Cuisenaire Rods, V1 and V2

- Version 1:
  - All problems use brown rod as the whole
  - May need to rename one addend
- Version 2:
  - Problems use different rods as the whole
  - May need to rename both addends

Get to the Whole!

Decomposing and recomposing fractions to “get to the whole” when adding and subtracting.

$\frac{3}{4} + \frac{3}{4}$: Will's Strategy

Video 3c from *Beyond Invent and Multiply: Making Sense of Fraction Computation.* Math Solutions, 2015.

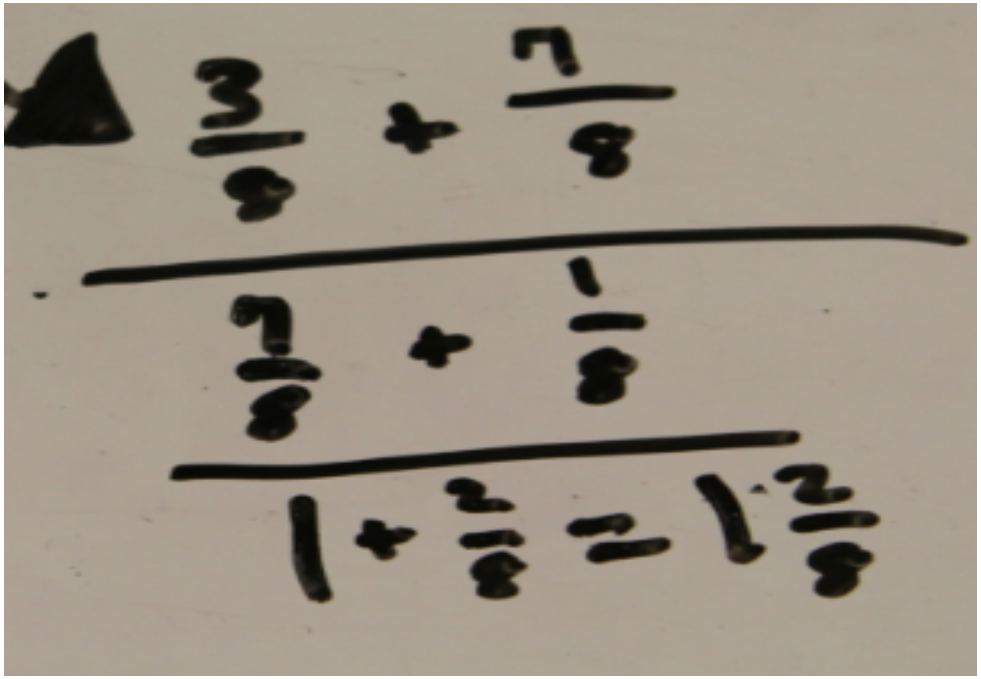
$\frac{3}{4} + \frac{3}{4}$: Belen's Strategy

Video 3d from *Beyond Invent and Multiply: Making Sense of Fraction Computation.* Math Solutions, 2015.

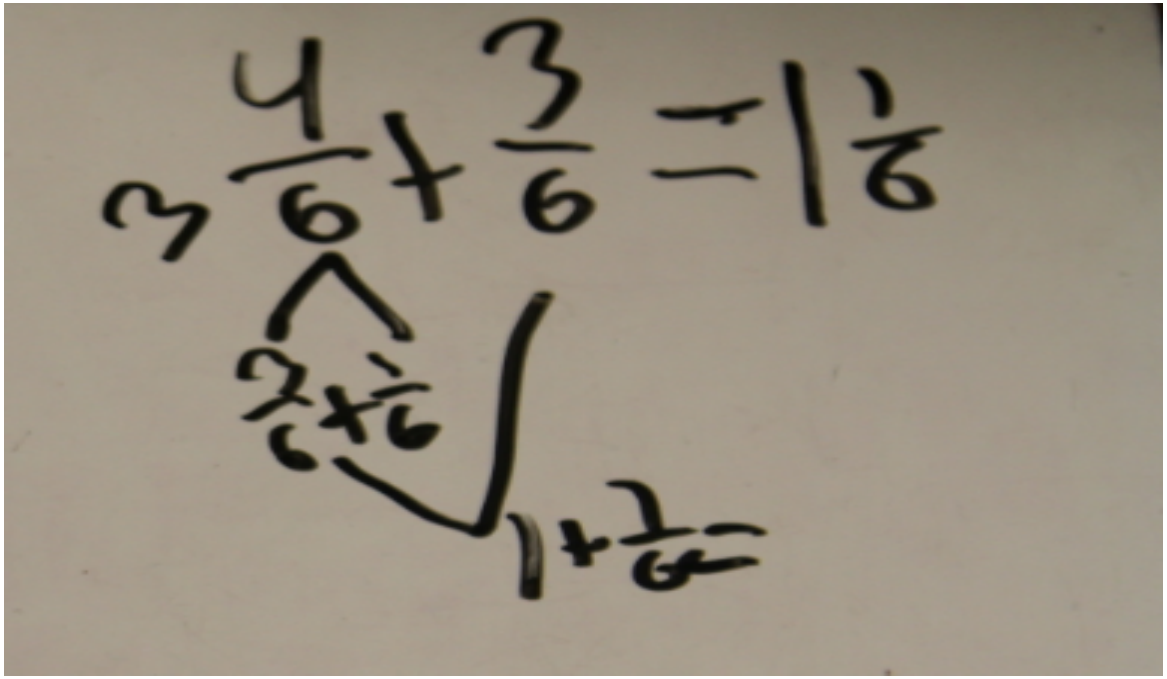
$\frac{3}{5} + \frac{4}{5}$: Malaya's Strategy

Video 3e from *Beyond Invent and Multiply: Making Sense of Fraction Computation.* Math Solutions, 2015.

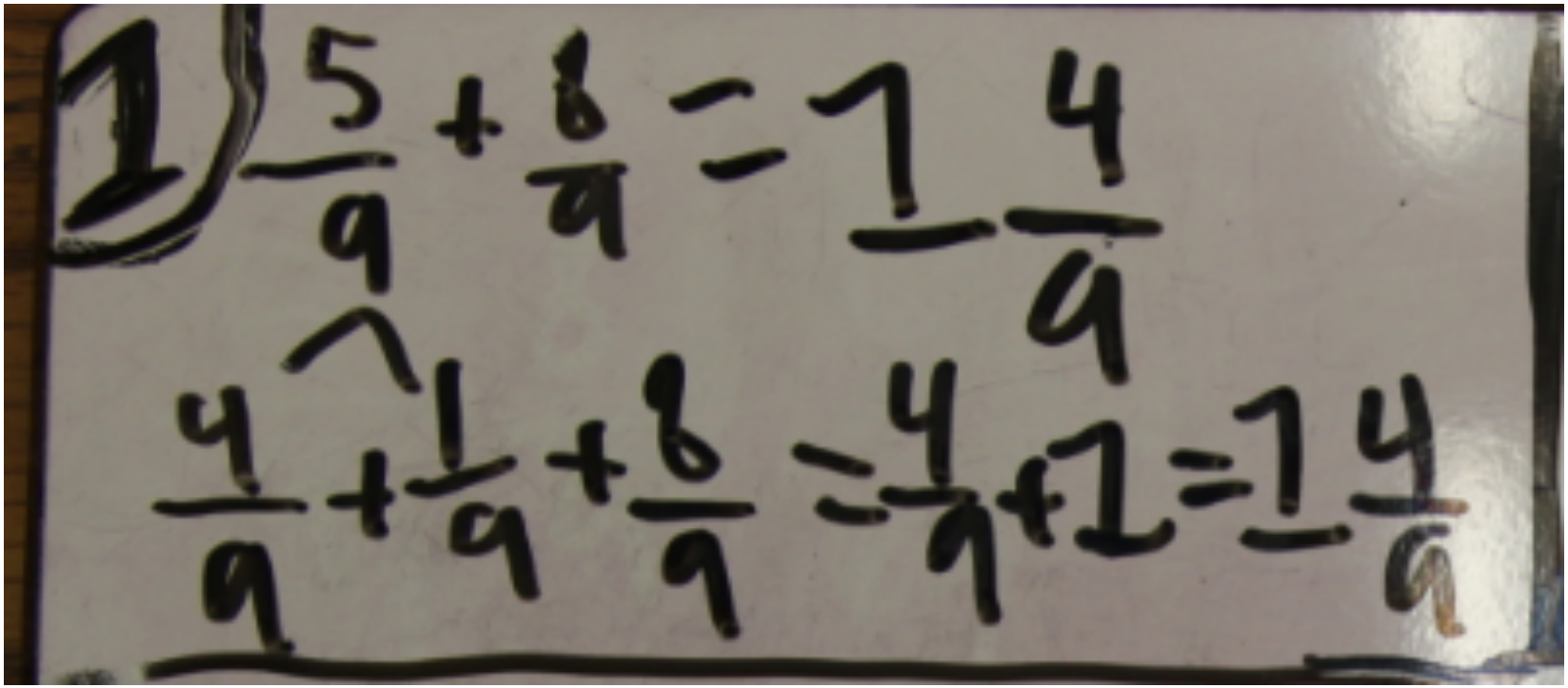
Student work



Student work



Student work



MAKING SENSE OF FRACTION MULTIPLICATION

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Tell Me All You Can

Before coming up with an exact answer, consider what you know about the answer as a means of getting a sense of the “neighborhood” of the answer.

Tell Me All You Can

- The answer will be less than _____ because _____.
- The answer will be greater than _____ because _____.
- The answer will be between _____ and _____ because _____.

What do we know about $6 \times 2\frac{1}{2}$?

Video 6f from *Beyond Invent and Multiply: Making Sense of Fraction Computation.* Math Solutions, 2015.

What do we know about $4\frac{1}{2} \times 5$?

Video 6i from *Beyond Invent and Multiply: Making Sense of Fraction Computation.* Math Solutions, 2015.

CONNECTING MULTIPLICATION TO ADDITION

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Using repeated addition to solve $6 \times \frac{1}{2}$

Video 6e from *Beyond Invent and Multiply: Making Sense of Fraction Computation.* Math Solutions, 2015.

MAKING SENSE OF FRACTION DIVISION

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Two types of division situations:

Quotative (also called measurement division):

- Size of group is known; number of groups is unknown
- 6 ÷ 2: How many 2 ’s are in 6?

Partitive:

- Number of groups is known; how many in each group is unknown
- 6 ÷ 2: Split 6 into 2 groups ➔ 6 is 2 groups of what?

Quotative Division

- $6 \div 2$: How many 2 ’s are in 6?

- $\frac{3}{4} \div \frac{1}{2}$: How many $\frac{1}{2}$’s are in $\frac{3}{4}$?
- $1 \div \frac{1}{4}$: How many $\frac{1}{4}$’s are in 1?

How Long? How Far? Part 1

How many $\frac{1}{4}$ minutes are in 1 minute?

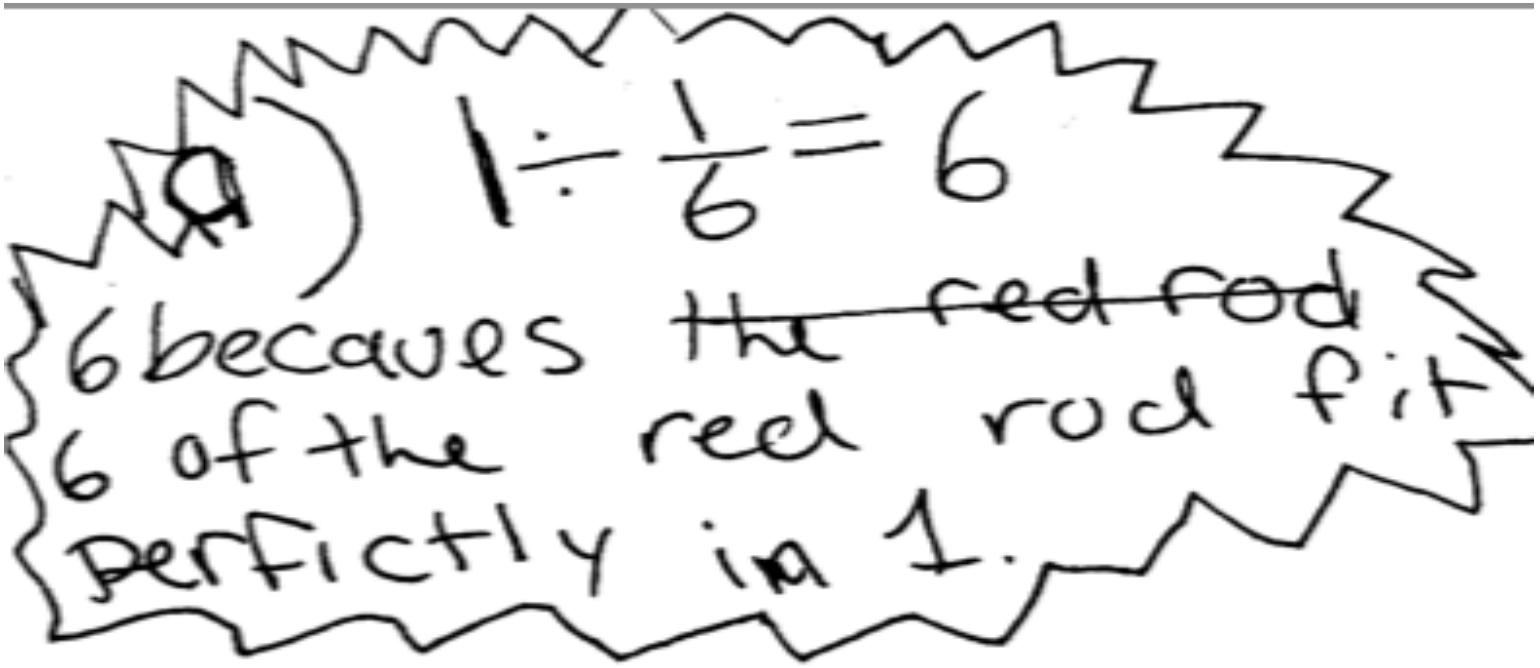
How many $\frac{1}{4}$'s are in 1?

Video 7c from *Beyond Invent and Multiply: Making Sense of Fraction Computation.* Math Solutions, 2015.

Quotative Division

- Use your Cuisenaire rods and number line to show how many $\frac{1}{4}'$s are in 1.
- Use your Cuisenaire rods and number line to show how many $\frac{1}{4}$,s are in 2.
- Use your Cuisenaire rods and number line to show how many $\frac{1'}{3}$s are in 2.

Reasoning about $1 \div \frac{1}{6}$

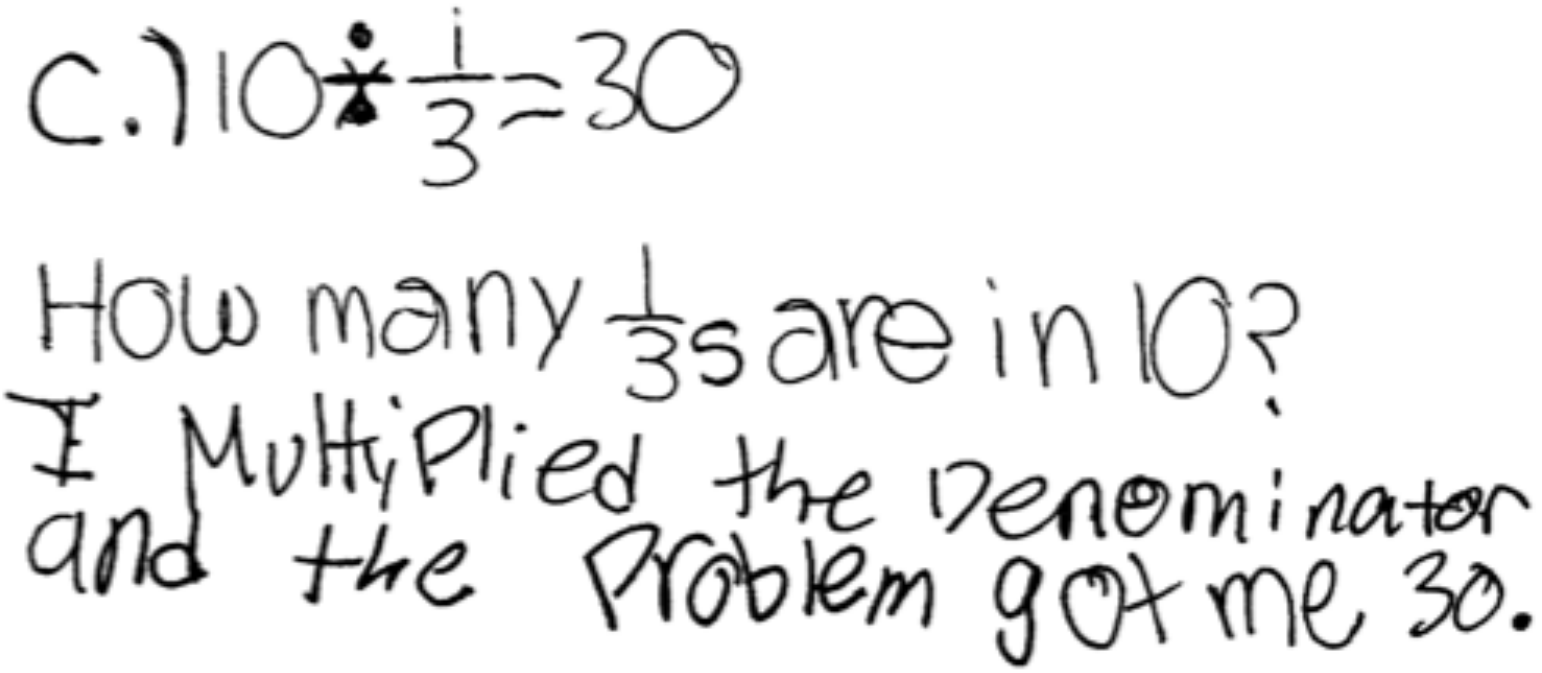


Reasoning about $2 \div \frac{1}{6}$

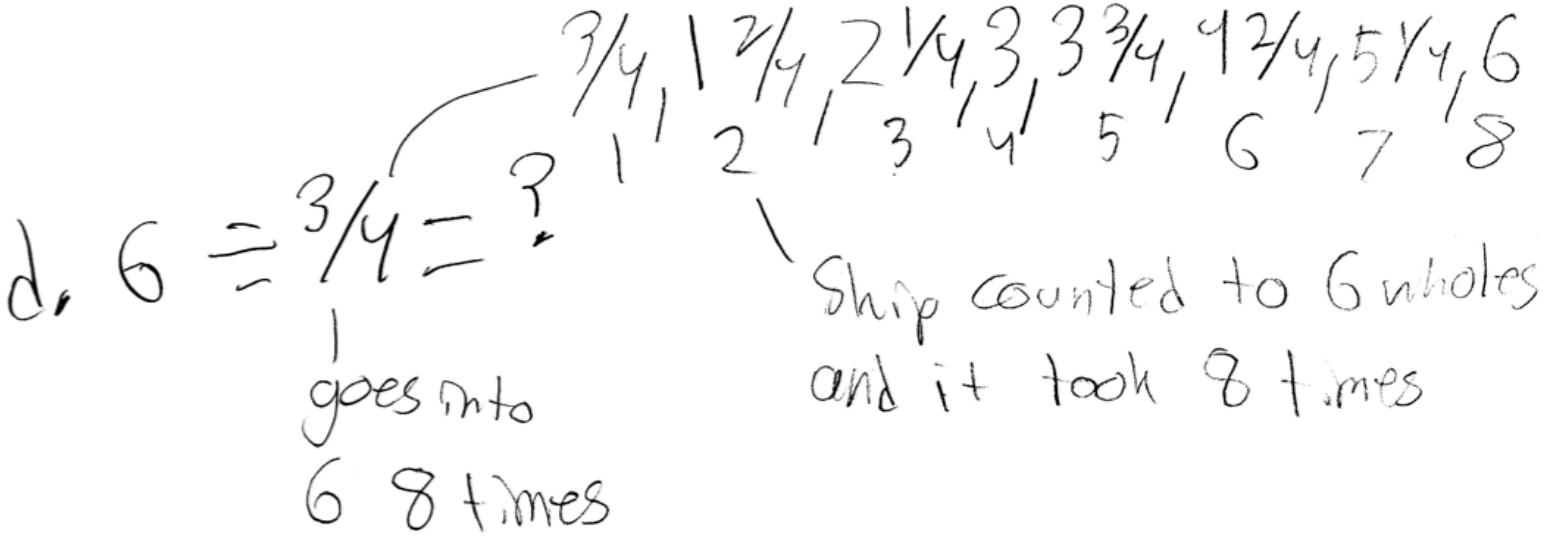
B.) $2 \div 1/6 =$

How many 1/6 are in 2?

Reasoning about $10 \div \frac{1}{3}$



Reasoning about $6 \div \frac{3}{4}$



Partitive Division

$6 \div 3$: Split 6 into 3 groups ➔ 6 is 3 groups of what?

$\frac{3}{4} \div 3$: Split $\frac{3}{4}$ into 3 groups ➔ $\frac{3}{4}$ is 3 groups of what?

How Long? How Far? Part 2

Beach Clean-Up (2 people)		
Distance	Expression	Each person cleans
8 miles	8 ÷2	4 miles
4 miles	4 ÷2	2 miles
2 miles	2÷2	1 mile
1 mile	1 ÷2	½ mile
½ mile	½ ÷2	?

CCSS Number and Operations - Fractions

3.NF: Develop understanding of fractions as numbers.
4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
5.NF: Use equivalent fractions as a strategy to add and subtract fractions.
5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Fractions as numbers…

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Thank you!!!

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