Whole-Class Instruction

The following suggestions for whole-class instruction introduce students to important and basic ideas relating to place value. They are organized into four categories: counting large quantities, using money, a logic game, and looking for patterns.

A Problem-Solving Lesson: How Many Fingers?

Engage the students in a variety of problems in which they estimate and count large numbers of objects. Relating the problems to the students and their school environment helps them apply mathematics in contexts that are real to them. The following whole-class sample lesson engages students in figuring out how many fingers the students in the class have altogether.

Materials

interlocking cubes, at least as many as fingers in the room

Introducing

1. **Present or review concepts.** On the day you do this investigation, determine the number of people in the class—students who are present, you, and any other visitors. Write this number on the board.

2. **Pose a part of the problem or a similar but smaller problem.** Ask seven or eight students to come to the front of the room and hold up their hands. Ask the class for different ways to count how many fingers they have in all. Typically, students will suggest counting by twos, fives, tens, and, perhaps, ones. As each way is suggested, have the class count aloud with you. Also, suggest any of these ways to count that students didn't think of.

   Then ask, *How many students need to come up to the front of the room so there are 100 fingers altogether?* For some students, this will be obvious; others may be uncertain or have no idea at all. Whatever number students suggest, have that many come to the front. Have the students count along with you as you count their fingers. If no one suggests ten students, be sure to do so. For each suggestion, go through the process of counting by twos, fives, and tens, and even ones, if the students suggest it, to see how many fingers there are.

   Ask the students to estimate the total number of fingers in the room.

3. **Present the investigation.** Then present the problem for students to solve in pairs or small groups: *How many fingers are there altogether on all of our hands?*

4. **Discuss the task to make sure students understand what they are to do.** Remind the students of the number of people that you wrote on the board earlier.

For Younger Students
When smaller numbers are more appropriate, switch the investigation to *How Many Thumbs?* Have each student take two interlocking cubes to represent their two thumbs and then work together to snap them into trains with ten cubes in each, and then count them. Then break apart trains so that there are five of each and recount.
—MSB

Teaching Tip
Sometimes I've begun by asking only three students to come to the front of the room, which makes this introductory experience a little easier but a number large enough so that I can point out that counting by ones takes much longer and that it's always good to look for ways to solve math problems more efficiently.
—MSB

Teaching Tip
To guide students to think about a reasonable estimate (as well as assess students' understanding and reasoning) ask, Are there enough people in the class to have two hundred fingers altogether? How do you know? More than two hundred? More than three hundred?
—MSB

Exploring

Circulate and offer assistance as needed. Ask pairs or groups that finish more quickly than others to figure out how many thumbs are in the room altogether.

Summarizing

Ask student groups to report their solutions. Be sure to ask students to explain how they figured out their answers. To verify the correct answer, have each person in the room make a train of ten interlocking cubes, one for each finger. Then count the trains by tens. Also, record the final answer on the board in several different ways; for example, *two hundred and seventy, 27 tens, 100 + 100 + 70, 270.*

A Problem-Solving Lesson: How Many Pockets?

The following whole-class sample investigation builds students' number sense through engaging them in estimating how many pockets they have.

Teaching Tip

Students too often think that answers always have to be exact to be right. I try and reinforce for them that estimates are "about" numbers and there typically isn't only one estimate that is reasonable.

—MSB

Materials

interlocking cubes such as Multilink, Snap, or Unifix cubes, at least 200

Introducing

1. **Present or review concepts.** Talk with the class about what an estimate is. Explain that estimating gives the opportunity to think about numbers, better understand what they mean, and make guesses about what answers might be. Also, tell the students that they'll be solving a problem that could have a large number as the answer. Ask them for ideas about what a large number might be.
2. **Pose a part of the problem or a similar but smaller problem.** Invite a student to come to the front of the room. Be sure to choose a student who is wearing clothing that has pockets. Ask the student to put one cube into each pocket. Then have the student remove the cubes, snap them together, and count them.

Teaching Tip

Using the "think, pair, share" routine is appropriate when asking students to make an estimate.

—MSB

3. **Present the investigation.** Ask, *About how many pockets do you think there are altogether in our class?* Have students think first and then talk in pairs. Finally, have them report their estimates and explain why they think their estimates make sense.

   Tell them that they'll now each find out how many pockets they have. Explain that they'll put one cube in each of their pockets, as their classmate demonstrated. Then they'll remove the cubes from their pockets, snap them together, count them, and compare how many they have with their neighbors. Make cubes available in several locations so students have easy access to them.
4. **Discuss the task to make sure students understand what they are to do.** Ask several students to state the directions for what they are to do.

For more information about the "think, pair, share" routine, see "Starting Point 14: The Importance of Classroom Discussions" in Part 1 on page 67.

Exploring

Circulate and offer assistance if needed.

Summarizing

After students have had the opportunity to compare, bring them to attention for a classroom discussion. Begin by having students report the number of pockets they have. Ask, *Who has zero pockets? One? Two? Three?* Continue by asking, *Who has the most number of pockets?*

If you'd like, create a class graph by listing the numbers *0, 1, 2,* and so on, and having students each mark an *X* or a check next to the number of pockets they have. Then pose another problem: *How many pockets do you have altogether?* Ask students to combine their cubes, work with their neighbors to make trains with ten cubes in each, and bring each train of ten to the front of the room. Have the class count aloud with you by tens. Then have students bring up their extra cubes and use them to make additional trains of ten. Again, have them count aloud by tens. Count the extras. Record the number of tens and extras on the board as well as the final number (e.g., *6 tens and 8 ones = 68*).

Extension

How Many Pockets? Revisited. Tell the students they'll repeat the investigation for the next few days to see if the number of pockets changes. Continue the investigation for as many days as the students are interested.

Teaching Tip

After students put a cube in each pocket, I've found that it's good to collect the extra cubes so that they don't get mixed up with the cubes from students' pockets. (This tip comes from my experience where one boy got so involved snapping cubes together that he ignored the pocket connection and made a train that was seventeen cubes long.)

—MSB

FYI

It may seem that grouping by tens is the most efficient way to count the cubes, but left to their own choices, students may prefer to group objects in other ways, by twos or fives, for example. While grouping by tens relates to our place-value system, it takes time and many experiences for this to be compelling or even make sense to all the students.

—MSB

Additional Counting Investigations

How Many Fingers? and *How Many Pockets?* are examples of lessons that can be repeated during the year. Spacing out experiences like these provides the opportunity to assess changes that occur over time in how students think about large numbers. It's important to remember that teaching students the usefulness and logic of grouping objects into tens to make sense of large quantities is not a lesson objective, but a long-range goal that's best attached to repeated investigations. Students need many experiences over time to learn to connect the idea of counting by tens (which many can do by rote) to the structure of our place-value system.

While students are developing understanding of place value, it's important they feel that their own methods for counting large numbers of objects are also valid. The pedagogical challenge is to make the connection between grouping by tens and our number system. The goal is for students to have the chance to consider the connection, understand it, see its usefulness, and eventually use their experience to construct an understanding of place value for themselves. Additional counting investigations follow.

ADDITIONAL PROBLEMS FOR COUNTING LARGE QUANTITIES

- How many feet are there in class altogether? Thumbs?
- How many buttons are there on everyone's clothing?
- How many cubes would be in a box after each student puts in a handful?
- How many letters are there altogether in everyone's first names? Last names?
- How many books are in the class library?

When doing other explorations, such as figuring the number of buttons or books, vary the materials. For example, use beans to represent buttons and provide small cups for students to group them into tens. Use tally marks to record the number of feet in the room and, after counting by fives, draw circles around groups to count them by tens.

Classroom discussions are important because students have the opportunity to hear each other's ideas and you can get a glimpse into students' understanding. In classroom discussions, raise questions such as the following:

- *If you group the cubes (or other objects) by twos or fives or tens, will you get the same number when you count how many there are altogether? Always? Explain your thinking.*
- *Some people say it's easy to count by tens. Why do you think they say that?*
- *Which takes fewer numbers—to count to one hundred by fives or by tens? Why? How else can you count to one hundred?*

FYI

Though many young students can count by tens, when I've asked them to continue beyond one hundred, many continue incorrectly, next typically saying two hundred, three hundred, and so on, or 101, 102, and so on. I've found it useful to make time to listen to each student count by tens, hear how they extend the pattern past one hundred, and then provide experiences counting objects to develop their understanding.

—MSB