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Math Solutions Online Newsletter
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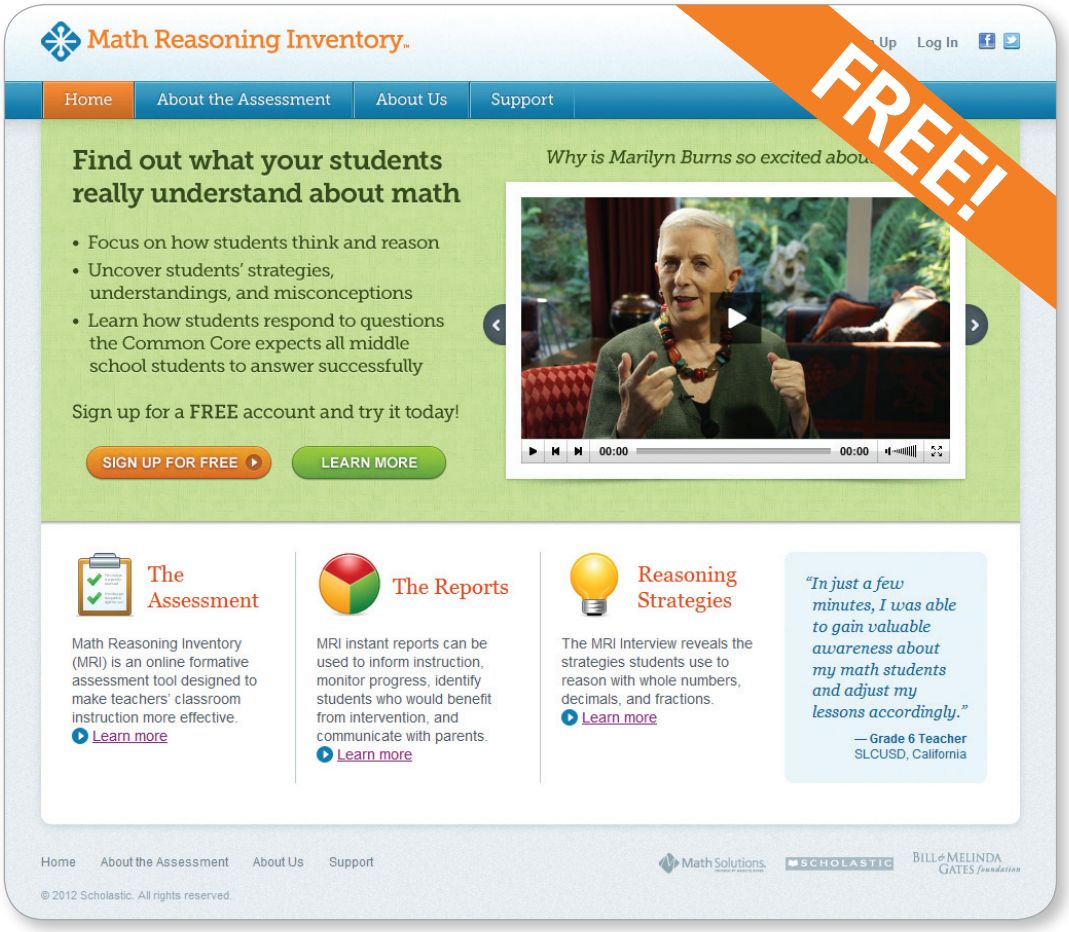
What Do Your Students Really Understand About Math?

Introducing

Math Reasoning Inventory™



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Math Reasoning Inventory™

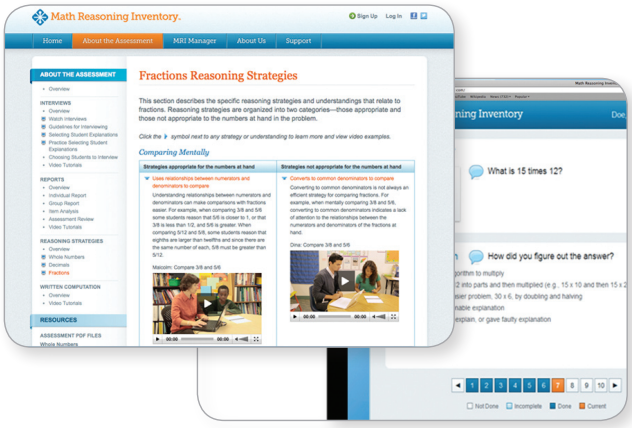
www.mathreasoninginventory.com

What Do Your Students Really Understand About Math? Introducing Math Reasoning Inventory (MRI)

Marilyn Burns, Founder, Math Solutions

I began teaching math in 1962, which makes this my 50th year as a math educator! Over the past two years, I feel as if I've made a huge leap in my own learning about teaching mathematics. During this time, with support from the Bill & Melinda Gates Foundation, I've worked with a team of colleagues to develop the Math Reasoning Inventory (MRI). I'm excited to introduce this online assessment to you. It's available now, free of charge to all teachers.

MRI is a formative assessment tool that focuses on numerical reasoning. The core of MRI is a face-to-face interview: you ask your students questions, you probe their thinking and listen to how they reason, and you learn what they understand.



Over the years, I've integrated a variety of formative assessments into my teaching. However, from this work to develop MRI, I've come to believe that none of the assessments I used have been as informative, reliable, or compelling as the MRI interviews. From the many conversations I've had with students, I've learned that answers by themselves, without access to how students reason, can be misleading. Correct answers can hide confusion and misconceptions, and incorrect answers can mask what students really understand.

What Is MRI?

First, MRI has been designed to help make your classroom instruction more effective. Formative assessments are not meant to replace high stakes summative assessments, like our state tests. They fill a very different role. They help shift the focus from "covering" the curriculum, to a focus on "uncovering" the curriculum for the students.

Second, MRI is a free to all teachers – all you need is a computer with a decent internet connection. The questions and how to record students' answers and explanations are provided, and the computer generates instant reports.

Third, MRI asks questions that we expect…and hope…all middle school students to answer successfully. As part of the goal of the Gates Foundation for all K–12 students to be college or career ready, the questions in the MRI assessment tool address content from the Common Core State Standards for Mathematics that is taught prior to sixth grade.

MRI focuses on the basics of number and operations. We know that this is only part of the full range of the math curriculum, but it's the part that is the cornerstone of elementary mathematics, that's essential for students' continued success with math and also for their success in their daily lives. It saddens me that Tip Tables and Tip Apps are so regularly used by people who feel the need for crutches for numerical problems they should be able to solve in their heads.

There are three assessments in MRI – Whole Numbers, Decimals, and Fractions. Each assessment has two sections – the Interview and the Written Computation sections. You give the interview (10–12 questions) face to face. Students complete the Written Computation section (4 problems) independently.

The Value of Probing Students' Thinking

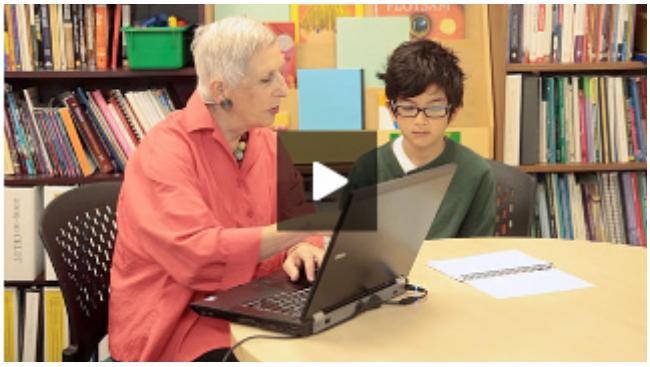
Asking questions and calling on students to give answers has always been a regular part of classroom teaching. In my experience, especially as a beginning teacher, when students answered questions correctly, I usually accepted their responses with a nod or comment of approval, rarely prodding them to explain their reasoning. When students were incorrect, however, I was more likely to probe further by asking, "Are you sure about that?" or "Why do you think that's right?" Follow-up prompts like these then became signals to the students that their response was not correct or acceptable.

What we learned from our work developing MRI is that asking students to explain their reasoning, even when they solve problems correctly (or, perhaps even especially when they were right), provides insights into their thinking and understanding in a way that answers alone can not provide. Also, the interview experience itself models for students what's important about learning mathematics, that while answers are important, reflecting on their thinking and communicating how they reason are integral to their math learning.

MRI reveals students' reasoning strategies

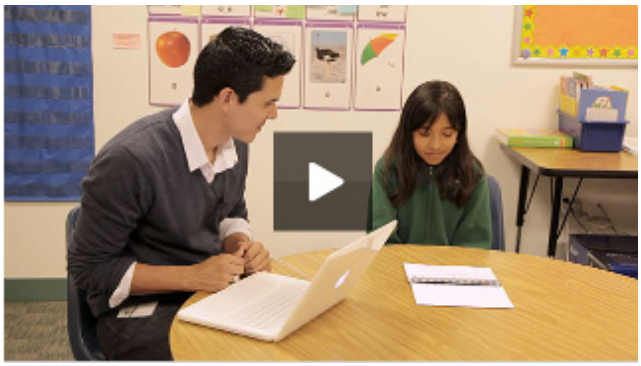
I've learned that it's important during an MRI Interview to listen carefully to students as they explain their reasoning. The goal is to understand how they think, not to listen for a particular way of reasoning. Also, since we're interested in how students reason, most of the interview questions in MRI are questions for students to answer in their heads, without using paper and pencil.

The first question in the MRI Fractions assessment asks students to compare two fractions—Which is greater, 3/8 or 5/6? I've asked this question to many students as we developed MRI, and I've heard a variety of different explanations. Some students use 1/2 as a benchmark and reason that since 5/6 is greater than 1/2 and 3/8 is less, 5/6 has to be greater than 3/8. Other students explain that since eighths are smaller than sixths, and there are fewer eighths, 3/8 has to be less. Some convert to fractions with a common denominator, choosing 24 or 48 (which is laborious to do mentally).



Marilyn Burns and Amir: Compare 3/8 and 5/6

When I presented the question last October to Amir, a sixth grader, his reasoning was strong and appropriate, but I was surprised by the way he constructed his argument. Amir related the fractions to percents. He knew that 3/8 was equivalent to 37 1/2%. Then, to figure the percent equivalent for 5/6, he used 4/6 as a benchmark, which he knew was equivalent to 66 2/3%. So, using these percents, he determined that 5/6 had to be more than 3/8.

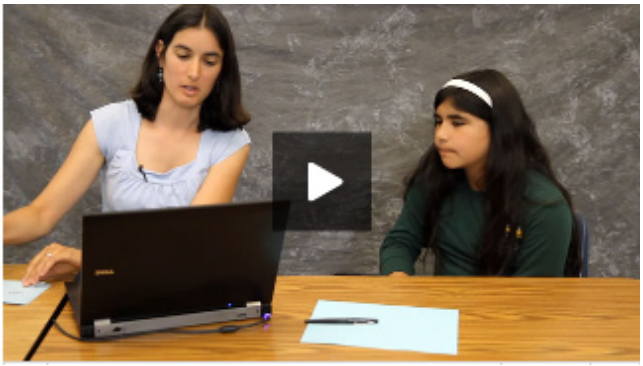


Jesus Martinez Monica: 15 x 12

One of the problems in the Whole Numbers assessment is 15 x 12. When Jesús Martínez presented this problem to Monica, one of Amir's classmates, she used an appropriate reasoning strategy to figure the correct answer of 180 – multiplying 15 x 10, 15 x 2, and then adding 150 + 30. Monica's explanation was clear and demonstrated her comfort with breaking numbers apart, an indicator of understanding the Distributive Property.

MRI also identifies misconceptions and lack of understanding

The last of the ten questions in the MRI Interview for Whole Numbers is a word problem about school buses: There are 295 students in the school. School buses hold 25 students. How many school buses are needed to fit all of the students?



Mallika Scott and Marisa: School Bus Problem

Mallika Scott presented this problem to Marisa in June when Marisa was completing fifth grade. After thinking for a moment, Marisa decided to add 295 + 25, using paper and pencil to get

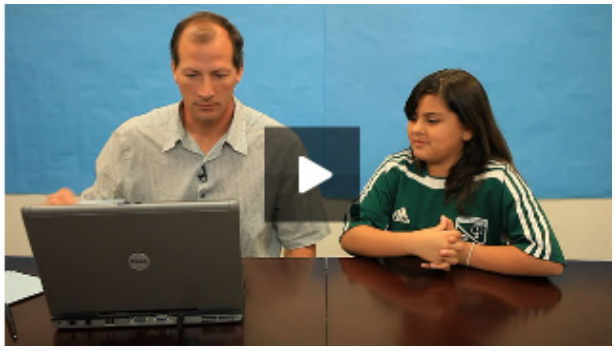
the answer of 305 buses. (We give students the option to use paper and pencil for this problem, and also for two word problems in the Fractions interview). Marisa came up with an answer for the number of school buses that is greater than the number of students going on the trip, which she didn't notice! She explained, "Since I heard all, I figured, oh that's plussing, so I plused 295 and 25, and I got 320." Marisa was not able to model the situation mathematically and connect it to the appropriate operation. She had been taught to use word cues to decide what to do to solve word problems, and she applied that learning inappropriately here.

MRI reveals when students use procedures inappropriately

A few years ago, when we were focused on assessing younger students, I had an experience with a third grader, Ellen, that then influenced our work on MRI. When assessing what Ellen understood about subtraction, I asked her, "How much is 100 minus 3?" Ellen thought for a moment and answered correctly, "It's 97." Then I asked, "How much is 100 minus 98?" She thought and then said, "I can't count back that far. Can I use pencil and paper?" I agreed and Ellen set up the subtraction problem, writing 98 under 100, then crossing out to borrow and figure the answer.

What understanding did Ellen lack? She didn't understand the inverse relationship between addition and subtraction, a key number property that is essential for students, not only with problems like these, but also when they learn to operate with positive and negative integers, and when they learn to solve algebraic equations. It's a foundational necessity for algebra.

But Ellen was a third grader, and since MRI was focused on assessing incoming sixth graders, we changed the problem to 1000 – 998. We initially included 1000 minus 3 as well, but we eliminated that question since all students we tested answered it easily. When developing MRI, we tested many, many questions. We eliminated questions that all students easily answered correctly, and we eliminated questions that hardly any students answered correctly. We looked for questions that gave us a good statistical spread and that produced a variety of reasoning strategies from students. So this question, 1000 – 998, is now the first question in the Whole Number interview.



Leo Kostelnik and Ana: 1000-98

Leo Kostelnik presented the problem to Ana. The only strategy that Ana had for solving the problem was the same as third grader Ellen. Without access to paper and pencil, Leo watched as Ana used her finger, tracing the numbers on the desk, working to figure out the problem as Ellen had done on paper. When Leo asked her how she figured out the answer, Ana explained the crossing-out-and-borrowing steps she used to subtract. Marisa and Ana were in the same class, and our interviews with them were as much a surprise to their teacher as to us.

MRI and the Common Core

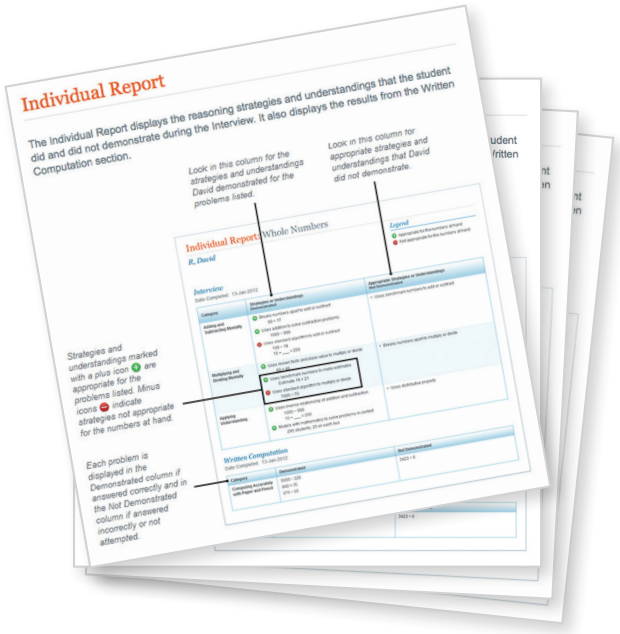
The Common Core State Standards for Mathematics includes **Standards for Mathematical Practice** and **Standards for Mathematical Content**.

The **Standards for Mathematical Practice** describe the expertise that we're looking to develop in students. They define the practices that rest on "processes and proficiencies with longstanding importance in mathematics education". The **MRI Interview** focus on reasoning directly addresses this section of the Common Core. In a way, reasoning is the core within the core of the Standards for Mathematical Practice.

The **Standards for Mathematical Content** include "a balanced combination of procedures and understanding." The questions in MRI draw from three Domains – Numbers & Operations in Base Ten, Numbers & Operations – Fractions, and Operations & Algebraic Thinking. The Common Core cautions that "students who lack understanding of a topic may rely on procedures too heavily." This was the case with Ana, for sure. This caution challenges us as teachers not to judge students' numerical proficiency solely or primarily on their ability to perform procedures. We've all known students who borrow, carry, invert and multiply, and more, yet are unaware when their answers are unreasonable. These students typically have difficulty making estimates, they typically lack numerical intuition, and they often don't see the sense in mathematics.

MRI Reports

MRI provides four different kinds of reports, all instantly available after giving the assessment. An **Individual Report** displays the strategies and understandings students demonstrated that were both appropriate and not appropriate to the numbers at hand, and the specific problems they solved correctly using those strategies. It also displays the appropriate strategies that we wish the student had demonstrated, but that he or she didn't.



A **Group Report**, for a whole class or part of a class displays shows which strategies and understandings were demonstrated consistently, often, sometimes, and rarely. We feel this is especially useful information when thinking about instruction. One teacher told us that this report helped inform the choices she made for her daily "Do Now" warmups.

From MRI, you can get an instant **Item Analysis**, a report that gives the data about the percentage of students who answered correctly, incorrectly, or who did not answer. The Item Analysis also reports on the strategies that students used for those problems, valuable data for informing instruction.

An **Assessment Review** shows the status of any individual student's assessment. It indicates the questions completed, whether the student's answer for these was correct or not, and what explanations the students gave.

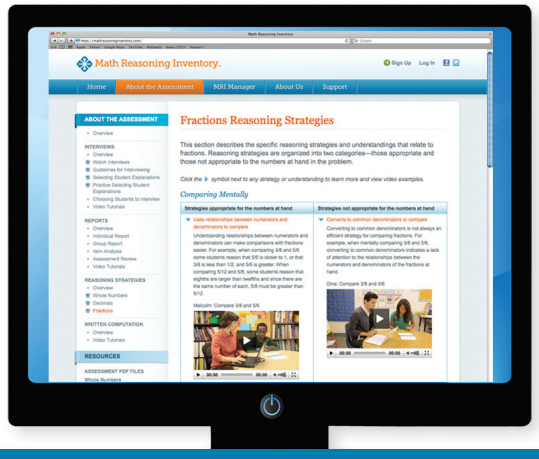
Marilyn Burns and The MRI Author Team:
Ruth Cossey, Leo Kostelnik, Melody Randel,
Mallika Scott, Lynne Zolli

A Final Note

As we interviewed students, discussed our results, and revised and tinkered with questions, we saw patterns in students' responses that helped us identify the reasoning strategies that we think are essential for numerical proficiency. This was one of the most exciting results for me from working on the MRI project. I've long been a believer in the importance of developing students' number sense. Now I feel as if I have much more concrete information about what that really means. We've made those strategies that we really want students to have in their toolkits for reasoning numerically available to all MRI users.

I invite you to use MRI with your students and enjoy the benefits I've experienced that I believe are essential for teaching math effectively. Visit the **MRI website (www.mathreasoninginventory.com)**. There you'll find information about preparing for interviews, tips for giving interviews, information about the reasoning strategies students need to be numerically proficient, guidelines for analyzing the MRI reports, and more. You can view more than 100 clips from the MRI Video Library and practice recording sample interviews. And you can register, free of charge, and begin using MRI with your students.

Please keep in touch about your experience with MRI. We're interested in your feedback.



"In just a few minutes, I was able to gain valuable awareness about my math students and adjust my lessons accordingly."

– Diana Jones, Grade 6 Teacher
San Luis Obispo, California

Visit the MRI Website:
www.mathreasoninginventory.com

From Marilyn Burns

"During my 50 years as an educator, I've integrated a variety of formative assessments into my teaching to help students develop numerical understandings and skills. After developing MRI, I've come to believe that none of those tools have been as informative, reliable, or compelling as face-to-face interviews. I've also learned that answers by themselves, without access to how students reason, can mask confusion and misconceptions. During one-on-one MRI interviews, where I ask students to explain how they reason, probe their thinking, and listen intently to what they say, I gain insights into students' reasoning strategies and understandings that are essential for my teaching."

To learn more about MRI, visit **www.mathreasoninginventory.com**.
Find out how you can use MRI with your students, listen to Marilyn Burns, view an extensive collection of videos of actual interviews, and gain access to MRI assessments and reports.