Math Solutions
FOUNDED BY MARILYN BURNS

Math Solutions Professional Learning

# AGENDA
## *Mathematical Practices Series™*

This series explores the central ideas embodied in the Standards for Mathematical Practice. We align what educators already know with what they need to learn about developing expertise in the "processes and proficiencies" outlined in the standards. Participants will leave each course with instructional skills and strategies they can use in their classrooms immediately.

Course/Day One: Making Sense of Math—Reasoning and Discourse
Course/Day Two: Mathematical Thinking—Representation and Procedural Fluency
Course/Day Three: Problem Solving—Developing Disposition, Competence, and Confidence

**FORMAT**
Designed to precede or follow an in-depth exploration of the content standards, this series is offered as a three-day institute or as individual courses over time.

**SERIES OUTCOMES**
- Strengthen participants' math content and pedagogical knowledge in order to understand various solution paths and students' reasoning
- Understand how students learn in order to make instructional decisions about tasks to complete and questions to pose
- Develop insight into individual learners' content mastery and math reasoning
- Cultivate new instructional strategies that promote thinking, reasoning, and sensemaking

**DAY ONE: MAKING SENSE OF MATH—REASONING AND DISCOURSE**

Among the highest priorities of the Common Core is for students to build a deep understanding of mathematics and use that understanding to reason about problems, make sense of new learning, and communicate their thinking to others.

This full-day course is designed to introduce participants to the Standards for Mathematical Practice, with particular emphasis on the role of reasoning and discourse in mathematics. During this course teachers will engage in reasoning and discourse, and discuss the implications for their students. In addition, they analyze the complexity of mathematical tasks and consider strategies for transforming grade-level tasks to increase their level of rigor.

**DEVELOPING TASKS THAT PROMOTE REASONING**
In order for students to develop habits of mind that rely on reasoning and making sense of mathematics, teachers must provide multiple practice opportunities with mathematical tasks and questions that require students to do more than memorize a procedure or answer.

**The National Council of Teachers of Mathematics (NCTM) recommends that teachers use tasks that:**

- Invite exploration of important mathematical concepts
- Allow students the opportunity to solidify and extend knowledge
- Encourage students to make connections and develop a coherent framework for mathematical ideas
- Call for problem formulation, problem solving, and mathematical reasoning
- Provide more than one solution path
- Promote the development of all students' disposition to do math

**DAY ONE: OUTCOMES**

- Discern how mathematical tasks and questions differ with respect to the level of thinking required to solve them
- Deepen participants' understanding that learning mathematics involves students' constructing ideas and systems
- Recognize the role of productive discourse in students' mathematical reasoning and sensemaking
- Understand the role of reasoning and discourse in Next Generation Assessments

**OPENING—WELCOME, LOGISTICS, AND EXPERIENCES**

Participants solve a problem that introduces them to the notion of using open tasks of high cognitive demand to engage students in the processes of thinking and reasoning to make sense of mathematics.

**LOGICAL REASONING AND CLASSROOM DISCOURSE**

Students should believe that mathematics makes sense. In this introductory experience, participants focus on the role of teachers and students in developing a classroom environment that supports thinking, reasoning, and sensemaking as key components of mathematics instruction and learning.

**HOW STUDENTS LEARN**

This session focuses on a view of learning in which people create/construct their own understanding of mathematical concepts/relationships through interactions between their minds and concrete expressions.

**ASPECTS OF LEARNING**

Through a series of mathematical investigations, participants examine progressions of content across their grade-level band domains to identify the mathematical ideas about which students need to reason and make sense.

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**COMPARING MATHEMATICAL TASKS**

Participants engage in and reflect on two different mathematical tasks. They compare and contrast the two tasks, identifying characteristics of tasks that require the learner to use thinking, reasoning, and problem solving skills. The two tasks focus on the mathematical concept of fractions.

**REFLECTION AND CLOSING**

Participants take time to reflect on the experiences of the day and ways that these experiences will affect their classroom instruction.

**DAY TWO: MATHEMATICAL THINKING—REPRESENTATION AND PROCEDURAL FLUENCY**

The Common Core calls for students to develop knowledge of computational procedures along with knowledge of when and how to use them appropriately. The goal is for students to become skillful in performing computational procedures flexibly, accurately, efficiently, and with understanding.

This full-day course provides teachers with a deeper understanding of procedural fluency beyond merely the ability to memorize procedures and apply them with little understanding. In addition, teachers will learn strategies to support students in representing ideas visually, symbolically, and verbally, as well as strategies for helping students make connections between these different representations.

**FLEXIBLE, ACCURATE, AND EFFICIENT**

For many students, procedures have been the mainstay of learning mathematics. "Yours is not to reason why, just invert and multiply" was a phrase used by teachers to help students remember the procedure for dividing fractions. The approach to learning computational procedures was based on a set of steps, or an algorithm, learned through repeated practice and memorization.

**DAY TWO: OUTCOMES**

- Expand understanding of procedural fluency to include carrying out procedures flexibly, accurately, and appropriately
- Broaden the definition of mathematical tools to include anything that students use to think about mathematics
- Connect multiple representations for the purpose of helping all students better understand underlying mathematical ideas
- Consider students' use of tools and representations for the purpose of assessing student understanding
- Understand the role of representation and procedural fluency in Next Generation Assessments

**OPENING—WELCOME, LOGISTICS, AND EXPERIENCES**

This introduction includes the course goals, an explanation of the structure and layout of the *Participant Guide*, an overview of the pillars and the practices addressed during the day, and pertinent logistical information.

**WHAT IS PROCEDURAL FLUENCY?**

Procedural fluency refers to knowledge of procedures; knowledge of when and how to use them appropriately; and skill in performing them flexibly, accurately, and efficiently. In this introductory experience, participants focus on aspects of procedural fluency beyond performing procedures such as estimation and sensemaking.

**USING TOOLS TO DEVELOP UNDERSTANDING**

Manipulative materials coupled with good questions can prompt students to think about mathematical ideas and reflect on their understanding of them. In this experience, teachers engage in a mathematical investigation and examine the questions used to focus students' work with manipulatives on important mathematical ideas.

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**CONNECTING MULTIPLE REPRESENTATIONS**
In this portion of the day, participants explore multiple representations of a mathematics problem as physical or mental constructs that describe aspects of the concept. Participants consider the various representations as forms of an idea that allow the learner to interpret, communicate, and discuss the idea with others.

**REFLECTION AND CLOSING**
Participants take time to reflect on the experiences of the day and ways that these experiences will affect their classroom instruction.

**DAY THREE: PROBLEM SOLVING—DEVELOPING DISPOSITION, COMPETENCE, AND CONFIDENCE**
The Common Core calls for students to "make sense of problems and persevere in solving them." Teachers' instructional practices directly affect students' confidence in their mathematical skills and their willingness to persevere to solve difficult problems.

This full-day course provides teachers with a deeper look at building perseverance in problem solving and applying mathematics to everyday situations. Participants will learn strategies for engaging students in appropriate levels of constructive struggle, thus allowing all students to approach mathematics with confidence and competence. Teachers learn how to maintain the integrity of high-level tasks by structuring lessons to allow students to make connections and develop new mathematical knowledge.

**SUPPORTING CONSTRUCTIVE STRUGGLE**
It is important for all students to experience some struggle in order to make sense of mathematics and develop new knowledge. Students will not persevere and be confident in their mathematical skills if we do not provide opportunities to make sense of the math and support them in the process.

Teachers maintain the integrity of high-level tasks by structuring lessons to allow students to make connections and develop new mathematical knowledge.

**DAY THREE: OUTCOMES**
- Broaden participants' understanding of how students learn and the features of a classroom environment that promotes confidence and perseverance in students
- Develop a working knowledge of constructive struggle as opportunities to involve students in problems that require critical thinking and connections across multiple mathematical concepts, skills, and ideas rather than those that entail superficial application of a rote procedure
- Examine three core features of the role of the teacher who teaches for understanding
- Consider how two cognitive processes, key in students' efforts to understand mathematics—reflection & communication—are also tools teachers use to assess student understanding.
- Understand the role of problem solving in Next Generation Assessments

**OPENING—WELCOME, LOGISTICS, AND EXPERIENCES**
This introduction includes the course goals, an overview of the practices addressed during the day, and pertinent logistical information.

**THE NATURE OF TASKS**
The session focuses on grade-level content to highlight the nature of tasks that promote confidence, competence, and perseverance in students. In this session, participants experience firsthand an example of a task that is rigorous yet accessible, at some level, to all students.

**THE PROBLEM-SOLVING LESSON**
The ability to identify and execute the critical phases of a problem-solving lesson, and to ask questions during each phase that compel students to think and reason, is vital to students' learning mathematics with understanding. This session uses the lesson "Hold and Fold" to illustrate these points.

**HOW LEARNING OCCURS**
This session focuses on a view of learning in which people create/construct their own understanding of mathematical concepts/relationships through interactions between their minds & concrete experiences.

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**HOW LEARNING OCCURS (CONTINUED)**

**TRANSFORMING TASKS**
Using two different methods, participants practice transforming grade-level-appropriate tasks from those requiring a low level of thinking and reasoning into those that requires a higher level of cognitive demand, as called for in the Common Core.

**PROBLEM SOLVING AND CONSTRUCTIVE STRUGGLE**
This session highlights the importance of constructive struggle in a classroom environment that supports students' mathematical practice of making sense of problems and persevering in solving them. Participants solve a problem, communicate orally about their solutions, and record and organize their thinking. In processing this experience, participants discuss important ideas about the role that constructive struggle plays in developing problem-solving skills in students.

**REFLECTION AND CLOSING**
This session reviews the connections between today's tasks and the Standards for Mathematical Practice from the Common Core, the five pillars for mathematics, and the learning outcomes for the day.

**MATH SOLUTIONS GUIDING PRINCIPLES**
Drawing upon academic work and our own classroom-grounded research and experience, Math Solutions has identified the following four instructional needs as absolutely essential to improving instruction and student outcomes:

- Robust Content Knowledge
- Understanding of How Students Learn
- Insight into Individual Learners through Formative Assessment
- Effective Instructional Strategies

These four instructional needs drive the design of all Math Solutions courses, consulting and coaching. We consider them our guiding principles and strive to ensure that all educators:

- Know the math they need to teach—know it deeply and flexibly enough to understand various solution paths and students' reasoning.
- Understand the conditions necessary for learning, what they need to provide, and what students must make sense of for themselves.

- Recognize each student's strengths and weaknesses, content knowledge, reasoning strategies, and misconceptions.
- Have the expertise to make math accessible for all students, to ask questions that reveal and build understanding, and help students make sense of and solve problems.