



FRACTION COMPUTATION

*Building on Previous
Understandings*

Julie McNamara
CMC South



*Yours is not to reason why
Just invert and multiply*



Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Brendan, Grade 4

1. Without computing the exact answer, decide which of these expressions would produce the answer with the least value and the greatest value.

A. Addition: $\frac{3}{4} + \frac{5}{8}$

Least Value Division

B. Subtraction: $\frac{3}{4} - \frac{5}{8}$

C. Multiplication: $\frac{3}{4} \times \frac{5}{8}$

Greatest Value multiplication

D. Division: $\frac{3}{4} \div \frac{5}{8}$

Explain your thinking below:

Division because you have to split it up.
Multiplication because you have to add more than 2 of it.

Grade 4: Number and Operations - Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

- *Implications for addition/subtraction*
- *Implications for multiplication*



Grade 5: Number and Operations - Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- *Implications for addition/subtraction*
- *Implications for multiplication/division*



Grade 3: Number and Operations - Fractions

Develop understanding of fractions as numbers.

- *Implications for addition/subtraction*
- *Implications for multiplication/division*

Fractions as numbers...

“In mathematics, do whatever it takes to help you learn something, provided you do not lose sight of what you are supposed to learn. In the case of fractions, it means you may use any pictorial image you want to process your thoughts on fractions, but at the end, you should be able to formulate logical arguments in terms of the original definition of a fraction as a point on the number line.”

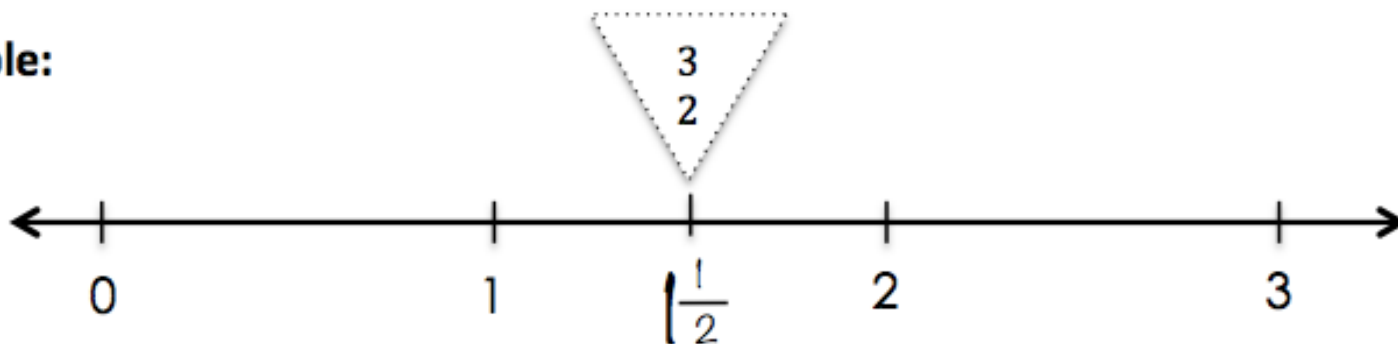
Wu, 2002, p. 13



Fractions Greater than One

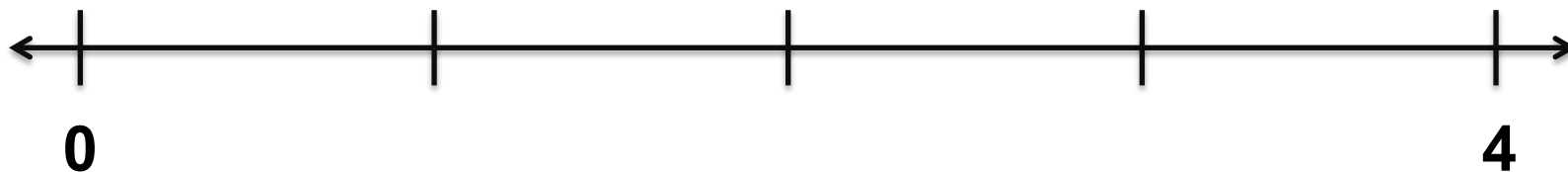
- Placing mixed numbers and fraction equivalents on the number line

Example:





Placing $\frac{1}{2}$ on the number line





Grade 3: Number and Operations - Fractions

Develop understanding of fractions as numbers.



4.NF.3b

Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.



Whole number addition strategies

- Decomposing/recomposing
- Associative property
- Commutative property



Get to the Whole!

- Decomposing and recomposing fractions to “get to the whole” when adding and subtracting.

$$\frac{3}{4} + \frac{3}{4}$$



Get to the Whole!

$$\frac{3}{8} + \frac{7}{8}$$

$$\frac{4}{6} + \frac{3}{6}$$

$$\frac{5}{9} + \frac{8}{9}$$



Student work

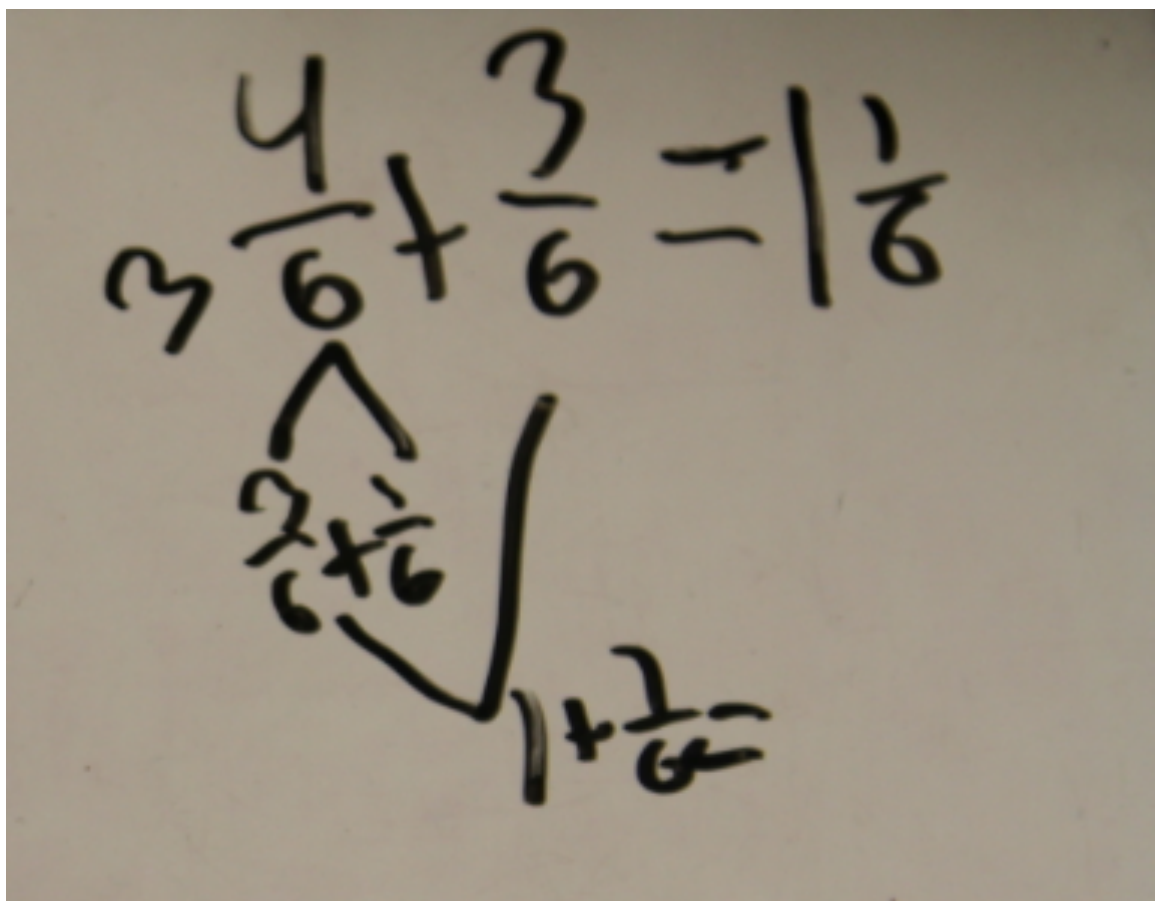
Handwritten student work showing the addition of two fractions:

$$\frac{3}{8} + \frac{7}{8}$$

$$\frac{10}{8} + \frac{1}{8}$$

$$1 + \frac{2}{8} = 1\frac{2}{8}$$

Student work



Handwritten student work showing the addition of two mixed numbers:

$$3\frac{4}{6} + \frac{3}{6} = 1\frac{1}{2}$$

The student has drawn a bracket under the fractions $\frac{4}{6}$ and $\frac{3}{6}$ with the label $2 + \frac{1}{2}$ written below it. Below this, the student has written $1 + \frac{1}{2}$.

Student work

$$1) \frac{5}{9} + \frac{1}{9} = 1 \frac{4}{9}$$

$$\frac{4}{9} + \frac{1}{9} + \frac{1}{9} = \frac{4}{9} + 1 = 1 \frac{4}{9}$$



Multiplication Patterns

Students consider patterns of products in problems with factors that decrease in value.



Multiplication Patterns

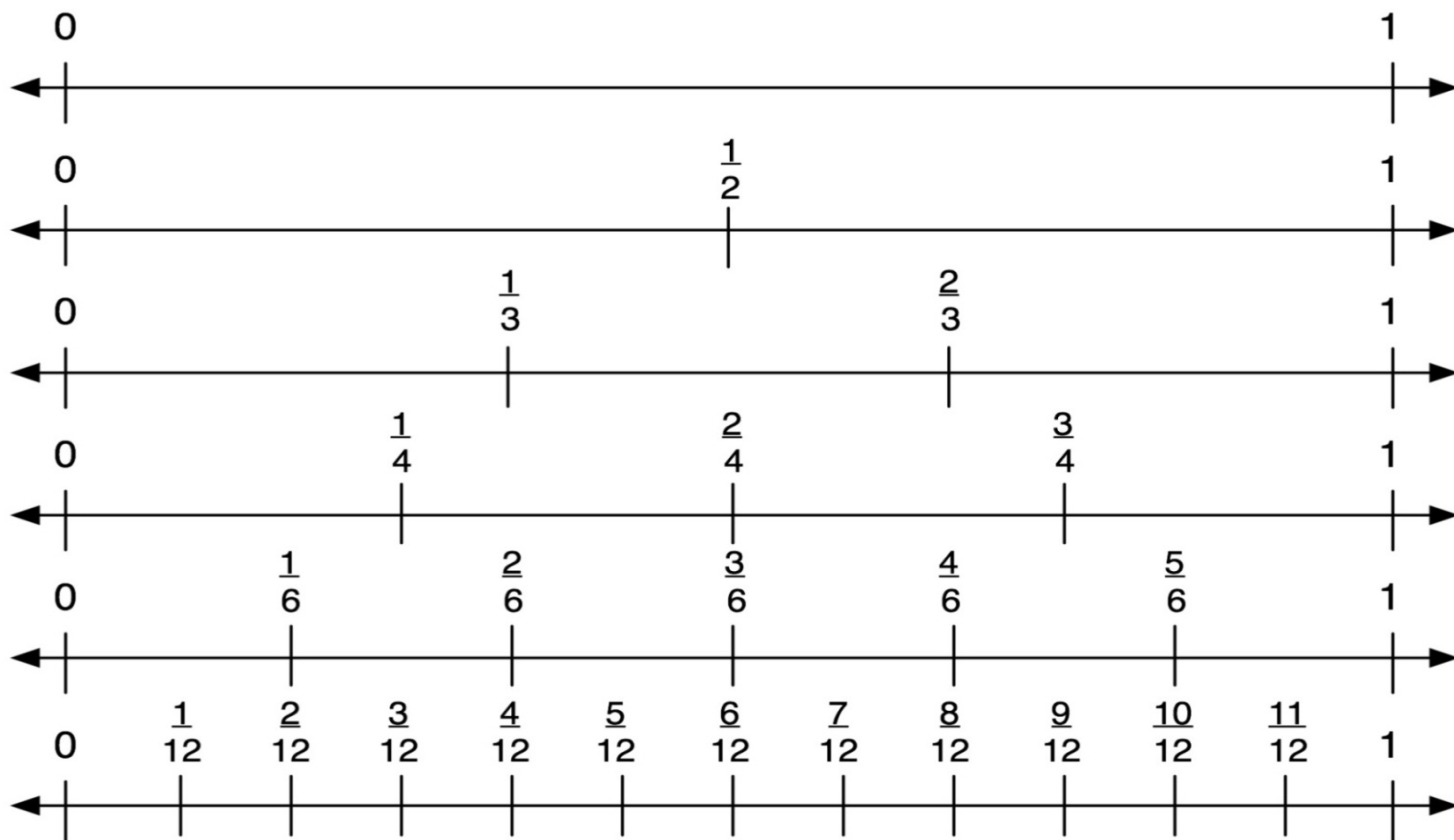
$$6 \times 8 = 48$$

$$6 \times 4 = 24$$

$$6 \times 2 = 12$$

$$6 \times 1 = 6$$

$$6 \times \underline{\quad} = \underline{\quad}$$





Tell Me All You Can

Before coming up with an exact answer, consider what you know about the answer as a means of getting a sense of the “neighborhood” of the answer.

Tell Me All You Can

- The answer will be less than _____
because _____.
- The answer will be greater than
_____ because _____.
- The answer will be between _____
and _____ because _____.



Tell Me All You Can

$$6 \times 2\frac{1}{2}$$



Tell Me All You Can

$$4\frac{1}{2} \times 5$$



Grade 5: Number and Operations - Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- *Implications for multiplication/division*



Is $60 \div 2$ the same as $60 \div \frac{1}{2}$?

What strategies will students have to answer this question after engaging in *How Long? How Far?*



How Long? How Far?

Dividing a whole number by a fraction

Dividing a fraction by a whole number



Reasoning about $1 \div \frac{1}{4}$

- How many $\frac{1}{4}$ s are in 1?

Reasoning about $1 \div \frac{1}{6}$

Q) $1 \div \frac{1}{6} = 6$
 6 becauses ~~the red rod~~
 6 of the red rod fit
 perfectly in 1.



Reasoning about $2 \div \frac{1}{6}$

B.) $2 \div \frac{1}{6} =$

How many $\frac{1}{6}$ are in 2?



Reasoning about $10 \div \frac{1}{3}$

C.) $10 \div \frac{1}{3} = 30$

How many $\frac{1}{3}$ s are in 10?
I Multiplied the Denominator
and the Problem got me 30.

Reasoning about $6 \div \frac{3}{4}$

$6 \div \frac{3}{4} = ?$
 goes into
 6 8 times

$\frac{3}{4}, 1\frac{1}{4}, 2\frac{1}{4}, 3, 3\frac{3}{4}, 4\frac{1}{4}, 5\frac{1}{4}, 6$
 1 2 3 4 5 6 7 8
 Ship counted to 6 wholes
 and it took 8 times



Is $60 \div 2$ the same as $60 \div \frac{1}{2}$?



How Long? How Far? Part 2

Beach Clean-Up (2 people)	
Distance	Each person cleans
8 miles	4 miles
4 miles	2 miles
2 miles	1 mile
1 mile	$\frac{1}{2}$ mile
$\frac{1}{2}$ mile	?



How Long? How Far? Part 2

$$\frac{1}{2} \div 3$$

$$\frac{1}{6} \div 2$$

$$\frac{3}{4} \div 3$$

Number and Operations - Fractions

3.NF: Develop understanding of fractions as numbers.

4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

5.NF: Use equivalent fractions as a strategy to add and subtract fractions.

5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Fractions as numbers...

“In mathematics, do whatever it takes to help you learn something, provided you do not lose sight of what you are supposed to learn. In the case of fractions, it means you may use any pictorial image you want to process your thoughts on fractions, but at the end, you should be able to formulate logical arguments in terms of the original definition of a fraction as a point on the number line.”

Wu, 2002, p. 13



Thank You

mathsolutions.com

800.868.9092

info@mathsolutions.com