Maximizing Learning or Missing Opportunities?

Helping Teachers Foster Algebra Readiness

Presenters: Amy Mayfield and Lu Ann Weynand
Solve Mentally

• $1000 - 998 = ?$

• $99 + 17 = ?$

• $12.6 \times 10 = ?$

How might a student with strong number sense solve this problem?

How might a student with weak number sense solve this problem?
A child who does not see patterns often does not expect things to make sense and sees all events as discrete, separate, and unrelated.”

A student who expects things to ‘make sense’ looks for patterns and generalizations and from these develops understanding.
# Pathway to Algebra

<table>
<thead>
<tr>
<th></th>
<th>K</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<tbody>
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<td>Represent and solve problems involving addition and subtraction</td>
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<td>Understand the place value system</td>
<td>Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers</td>
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<td>Understand and apply the relationship between addition and subtraction</td>
<td>Solve problems involving the four operations, and identify and explain patterns</td>
<td>Use place value understanding and properties to perform multi-digit arithmetic</td>
<td>Use equivalent fractions as a strategy to add and subtract fractions</td>
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**Math Solutions**

**Founded by Marilyn Burns**
Understanding Arithmetic: Three Pillars

• Understanding numbers
• Developing computational fluency
• Examining the behavior of the operations
Arithmetic ↔ Algebraic Thinking

...using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (multiplication/division).
## Setting the Stage for Algebra Readiness

### Decompose Small Numbers

<table>
<thead>
<tr>
<th>Number</th>
<th>Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1 + 5</td>
</tr>
<tr>
<td>6</td>
<td>2 + 4</td>
</tr>
<tr>
<td>6</td>
<td>3 + 3</td>
</tr>
</tbody>
</table>

### Decompose to Find Sums

<table>
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<tr>
<th>Expression</th>
<th>Calculation</th>
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</thead>
<tbody>
<tr>
<td>18 + 6</td>
<td>18 + (2 + 4) = (18 + 2) + 4 = 20 + 4 = 24</td>
</tr>
</tbody>
</table>
Linking to Larger Numbers

28 + 14 =  

20 + 10 = 30
8 + 4 = 12
20 + 12 = 32
Linking to Fractions

\[3 \cdot \frac{3}{4} + \frac{1}{2} = \]

\[3 \cdot \frac{3}{4} + \left(\frac{1}{4} + \frac{1}{4}\right) = \]

\[(3 \cdot \frac{3}{4} + \frac{1}{4}) + \frac{1}{4} = \]

\[4\frac{1}{4}\]
Working Towards Algebra Readiness

? 16 × 3

10 6

30 18

30 + 18 = 48

? 16 × 13

10 6

100 30

18

100 + 60 + 30 + 18 = 208
Linking to Fractions

\[ 4 \times 3 \frac{1}{3} = \]
\[ (4 \times 3) + (4 \times \frac{1}{3}) = \]
\[ 12 + \frac{4}{3} = \]
\[ 13\frac{1}{3} \]
Building Critical Algebra Foundations

\[(x + 4) \times (x + 2) = x^2 + 8\]

\[
\begin{array}{ccc}
 x & & 4 \\
 x & x^2 & 4x \\
 2 & 2x & 8 \\
\end{array}
\]

\[x^2 + 6x + 8\]
Understanding Arithmetic: Three Pillars

- Understanding numbers
- Developing computational fluency
- Examining the behavior of the operations
Interpreting the Equal Sign

"1 plus 7 makes 8"

1 + 7 = 8

\[ \frac{2}{3} + \frac{3}{5} = \frac{6}{10} + \frac{6}{10} \]

It means to put together.

Then,

\[ \frac{2}{3} + \frac{3}{5} = \frac{6}{10} + \frac{6}{10} \]

That a equals sign. It means it's a sum.
Interpreting the Equal Sign

\[8 + 4 = \square + 5\]
Interpreting the Equal Sign

7 = 3 + 4

8 = 8

5 + 8 = 8 + 5

6 - □ = 7 - 4
True or False – How do you know?

7 = 3 + 4

8 = 5 + 13

6 − 1 = 7

27 = 7 + 10 + 10

10 − 3 = 11 - 4
1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.
Relational thinking occurs when one observes and uses number relationships between the two sides of the equal sign instead of actually computing amounts.
6 + □ = 5 + 9

Since 5 + 9 is 14, I need to figure out 6 plus what equals 14. It is 8, so the box is 8.

Six is one more than the 5 on the other side. That means the box should be one less than 9, so it must be 8.
Encouraging Relational Thinking

37 + 54 = 38 + 53

48 + 63 − 62 = 49

625 + 450 = 700 + 400

126 − 37 = □ - 40
Exploring True and False Sentences

1. Give each other think time on each number sentence before talking.

2. Share with each other your reasoning. How did you decide if it was true or false?

3. Which examples were interesting to you?
Exploring Open Sentences


2. Share with each other your reasoning.

3. Will that strategy work for other numbers?
Look for and make use of structure.
True or False?

6 + 9 = 9 + 6
4 - 3 = 3 - 4
90 - 0 = 0 - 90
7 + 50 = 50 + 7
6 + □ = 10 + 6

10 + □ = □ + 10
Commutative property of addition states that changing the order of the addends does not change the sum.
Supporting Teachers

\[ 7 + 3 = \square + 9 \]

\[ 6 + 2 = 1 + 7 \]
Inverse Operations

2. How can knowing $8 \times 4 = 32$ help you to understand $32 \div 4$?

\[
\begin{align*}
32 \div 4 &= 8 \\
32 \div 8 &= 4 \\
8 \div 32 &= 4 \\
4 \div 32 &= 8
\end{align*}
\]
Understanding Arithmetic: Three Pillars

• Understanding numbers
• Developing computational fluency
• Examining the behavior of the operations
“A focus on the operations emphasizes noticing, describing, representing, and explaining consistencies across many problems. Generalizing in this way about the properties and behaviors of the operations is not about solving particular problems but about regularities that are foundational to arithmetic and algebra.”

Connecting Arithmetic to Algebra by Susan Jo Russell, Deborah Schifter, and Virginia Bastable (Portsmouth, NH: Heinemann, 2011).
Impacting Teacher’s Practice

• What two or three things to support algebra readiness do you want to see in your classrooms?

• What ideas do you have for making those happen?
How to Support Algebra Readiness

• Provide teachers with professional development that builds:
  – Robust content knowledge
  – Understanding of how students learn
  – Effective instructional strategies
  – Strategies for gathering information about what students do and do not understand

• Ensure teachers and students have quality materials and programs
Thank you!

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## Progress to Algebra in Grades K-8

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<th>7</th>
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<tr>
<td>Know number names and the count sequence</td>
<td>Represent and solve problems involving addition and subtraction</td>
<td>Represent &amp; solve problems involving multiplication and division</td>
<td>Use the four operations with whole numbers to solve problems</td>
<td>Apply and extend previous understandings of multiplication and division to divide fractions by fractions</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>Count to tell the number of objects</td>
<td>Understand and apply properties of operations and the relationship between addition and subtraction</td>
<td>Understand properties of multiplication and the relationship between multiplication and division</td>
<td>Generalize place value understanding for multi-digit whole numbers</td>
<td>Work with radical and integer exponents</td>
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<td></td>
</tr>
<tr>
<td>Compare numbers</td>
<td>Add and subtract within 20</td>
<td>Multiply &amp; divide within 100</td>
<td>Use place value understanding of numbers to solve problems using strategies based on place value, the properties of operations, and/or the relationship between addition and subtraction</td>
<td>Understand the connections between proportional relationships, lines, and linear equations</td>
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<td>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from</td>
<td>Work with addition and subtraction equations</td>
<td>Solve problems involving the four operations, and identify &amp; explain patterns in arithmetic</td>
<td>Extend understanding of fraction equivalence and ordering</td>
<td>Analyze proportional relationships and use them to solve problems</td>
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<tr>
<td></td>
<td>Extend the counting sequence</td>
<td>Develop understanding of fractions as numbers</td>
<td>Build fractions from unit fractions by applying and extending previous understandings of operations</td>
<td>Analyze and extend previous understandings of algebraic expressions</td>
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<tr>
<td></td>
<td>Measure and estimate lengths in standard units</td>
<td>Solve problems involving measurement and estimation of intervals of time, liquid volumes, &amp; masses of objects</td>
<td>Understand concepts of area and relate area to multiplication and addition</td>
<td>Reason about and solve one-variable equations and inequalities</td>
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</tr>
<tr>
<td></td>
<td>Use place value understanding and properties of operations to add and subtract</td>
<td>Relate addition and subtraction to length</td>
<td>Geometric measurement: understand concepts of area and relate area to multiplication and addition</td>
<td>Represent and analyze quantitative relationships between dependent and independent variables</td>
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<td>Measure lengths indirectly and by iterating length units</td>
<td>Geometric measurement: understand concepts of area and relate area to multiplication and addition</td>
<td>Understand decimal notation for fractions, and compare decimal fractions</td>
<td>Represent and analyze quantitative relationships between dependent and independent variables</td>
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<td>Graph points in the coordinate plane to solve real-world and mathematical problems*</td>
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*Indicates a cluster that is well thought of as part of a student’s progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the two asterisked exceptions, the clusters listed here are a subset of those designated as Major in both of the assessment consortia’s draft documents.
**True or False?**

Sort these into two groups – those that are true and those that are false. Try to use relational reasoning rather than performing the calculations on both sides of the equal sign.

<table>
<thead>
<tr>
<th>True</th>
<th>False</th>
</tr>
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<tbody>
<tr>
<td>345 + 71 = 70 + 344</td>
<td>$\frac{6}{7} + \frac{3}{7} = 1 + \frac{3}{7} - \frac{1}{7}$</td>
</tr>
<tr>
<td>37 + 56 = 39 + 54</td>
<td>33 − 27 = 34 − 26</td>
</tr>
<tr>
<td>9 × 7 = 10 × 7 − 7</td>
<td>4 × 6 = 6 + 6 + 6 + 6</td>
</tr>
<tr>
<td>0.3 × 7 = 0.7 × 3</td>
<td>7 × 8 = (2 × 8) + (5 × 8)</td>
</tr>
<tr>
<td>$6 \times \frac{1}{3} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$</td>
<td>$7 \times \frac{4}{5} = \frac{4}{5} \times 7$</td>
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## Open Sentences

Determine the number that goes in the box without performing calculations.

<table>
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</table>
| \[
\frac{\square}{3} = \frac{7}{7}
\]                                | \[
18 + 22 = 32 + \square
\]                                |
| \[
2.4 \div \square = 4.8 \div 6
\]                                 | \[
\frac{7}{8} = \square + \frac{4}{8}
\]                                 |
| \[
10 + 8 + \square = 12 + 9
\]                                 | \[
126 - 37 = \square - 40
\]                                 |
| \[
\square \times \frac{3}{4} = 12 \times 3 \times \frac{1}{4}
\]                 | \[
8 \times 7 = \square \times 7 + 7
\]                                 |
| \[
(4 \times 3) + (4 \times 0.25) = 4 \times \square
\]                      | \[
12.8 - 1.9 = \square - 2
\]                      |