EL *Educational Leadership*

October 2014 | Volume **72** | Number **2**
Instruction That Sticks Pages 64-68

Uncovering the Math Curriculum

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Too often, mathematics instruction gives students the erroneous notion that learning math is all about learning procedures, rather than making sense of ideas.

Teachers often express to me their worry about the need to "cover the curriculum." To respond, I draw on one of my favorite quotes: "You don't want to cover a subject; you want to uncover it." This quote is from *The Having of Wonderful Ideas and Other Essays on Teaching and Learning* by Eleanor Duckworth (Teachers College Press, 1986), a book that's been on my shelf for more than 25 years and one that I return to time and again for inspiration and guidance.

One of the most important steps in my own growth as a math teacher has been to understand the difference between covering and uncovering the curriculum. I've given a good deal of thought to how to incorporate this difference into my teaching practice.

Uncovering Pi

As a young middle school teacher, I primarily taught each topic the way I had been taught. For example, when my students and I reached the part of the curriculum in which we were required to cover the properties of circles, I presented the formulas for circumference ($c = \Pi d$ or $2\Pi r$) and area ($A = \Pi r^2$), introduced Π as the symbol for pi, explained that we could use either 3.14 or 3 1/7 for the value of Π, and then had students apply the formulas to solve problems. In other words, I covered the subject, but I didn't uncover it. I taught the formulas for area and circumference and how to apply them, but I didn't help students understand why those formulas made sense.

Since those early years of teaching, I've come to realize that our challenge as teachers is not to find better ways to explain to our students what we want them to learn, but rather to find better ways to ask our students to make sense of what they're learning for themselves. What does a lesson look like when we make the shift from telling to asking? As an example, here's what I think is a better way to help students understand pi.

Because pi is a constant relationship that exists in the physical world, my goal as a teacher is to engage students in a firsthand investigation that can help reveal that relationship to them. To do this, I assemble a variety of circular objects—plates of different sizes, a variety of cups and glasses, coasters, jar lids—whatever I can gather. I ask students to measure the circumference and diameter of these circles. Sometimes I ask each student to measure one circle, and I collect their data on the board for a class discussion. Other times, I ask each student to measure a few circles and then have the students compile and investigate their data in small groups. I ask them, "What do you notice?" And after they've had time to think, I ask, "Now, what do you wonder?"

Asking students what they *notice* focuses them on looking for patterns, structure, and regularity, all important for making sense of mathematical ideas and procedures. Asking students what they *wonder* focuses them on extending what they've noticed to make conjectures. This kind of thinking is fundamental to doing mathematics.[1]
As the teacher, my role is to guide a discussion that helps students see that, for each circle they measure, the circumference is always about three times the diameter. It's important to point out to students that measurement is

never exact, and even the best of measurements are approximations. That said, if we measure carefully with the best measuring tools we can find, for any circle, the result of dividing the circumference measurement by the diameter measurement will always be close to 3.14 or 3 1/7. That relationship—the ratio of a circle's circumference to its diameter—is what we call pi.

I've found interesting ways to assess my middle school students' understanding about circles. Although I want to know whether they've learned the relevant formulas, I also want to know whether they can apply that knowledge. One way to assess this is to ask them to solve the problem of measuring the diameter of a tree trunk. To help them envision the task, I model with a standard tape measure and a cylindrical container. The circumference of a container of bread crumbs, for example, measures about 12 1/4 inches, or 31 centimeters. (This is a good opportunity to reinforce for students that measurement is never exact.) After I ask students to predict the diameter of the container and explain their reasoning for their predictions, I measure to show that the diameter is about 4 inches, or a little more than 10 centimeters.

Then I point out that because we can't easily measure across a tree trunk to find its diameter, their task is to design a tape measure that does this. When you wrap this special tape measure around the circumference of the tree, the tape shows a number that's the tree's diameter. Instead of the marks on the tape measure indicating units that are inches or centimeters, they indicate "diameter units." In this problem, the goal is for students to do the math, not to do the page.

Procedures Versus Understanding

All instruction must foster students' ability to think, reason, and solve problems. Being able to compute answers without also understanding the underlying mathematics is an insufficient and shallow goal for students' mathematics learning. It builds the erroneous notion for students that learning math is all about learning procedures, rather than making sense of ideas. The expertise we should seek to develop in our students is much broader and embraces understanding.

The Common Core State Standards for Mathematics recommend a "balanced combination of procedures and understanding" and caution that "students who lack understanding of a topic may rely on procedures too heavily." The standards describe the consequences when students lack understanding:

> Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.[2]

Embracing the Common Core mathematical practices requires that we help students uncover knowledge by conducting firsthand investigations, working with physical materials when appropriate, and having opportunities to interact with others. However, we also need to recognize that some mathematical knowledge is based on agreed-upon social conventions, not logic. Students acquire this social knowledge by relying on outside sources—a book, the teacher, another student, TV, the Internet, and so on.

Examples of social knowledge are the term *pi* and the symbol Π that we use to name the ratio of the circumference to the diameter of circles. No amount of thinking and reasoning alone will reveal this knowledge to students. This is content that we as teachers need to cover. In such a case, teaching by telling is appropriate and necessary. But the actual ratio of the circumference to the diameter is a mathematical constant that exists in the physical world for all circles. Students can uncover this for themselves through firsthand learning experiences. And they should.

Exploring the "Why"

Teaching for understanding demands going beyond basic facts and procedures to ask, Why do we do this? Why does this make sense? The mathematics instruction we provide to students should emphasize meaning, relationships, and connections to help them uncover the curriculum. We should be mindful of what our students understand, not merely what they can do.

To help students understand why things work the way they do, teachers need to think deeply about the underpinnings of numerical concepts. Here are some questions that relate to making sense of mathematics that teachers can think about themselves and explore with their students.

1. Why is it OK to add a zero when multiplying whole numbers by 10 but not when multiplying decimals by 10?

Discussing this question helps students uncover several important mathematical ideas. One is that in our place-value system, which enables us to represent any number with only 10 digits, the same digits can have different values depending on their position in numbers. The difference between 36 and 63, for example, although obvious to adults, is not always easy for young students to understand.

Discussions can help students understand that when we multiply 25 by 10, we can add a zero and get 250—the digit 2 shifts from the tens place to the hundreds place and the digit 5 shifts from the ones place to the tens place, so the value changes. But we cannot just add a zero at the end when we multiply 2.5 by 10 because in both 2.5 and 2.50, the 2 is in the ones place and the 5 is in the tenths place, so their values are the same. Another important idea emerges by talking about the fact that .5 and .50 are the same and are both equal to 1/2.

2. Why is the sum of two odd numbers always even?

Before students discuss this question, it's important to give them time to verify that adding two odd numbers always results in an even sum. I typically have students do this in pairs and discuss with their partners why this might be so. This helps them prepare to offer ideas in a class discussion.

I've seen students come up with several explanations for why the sum of an odd number plus an odd number is even. For instance, one student explained that when you take an odd number of things and put them in pairs, there will always be one extra without a partner. But when you put *two* odd numbers of things in pairs, each of them will have one extra without a partner. These two extras can always be paired, so there won't be an extra anymore.

This question combines investigating how the parity of numbers (that is, whether they are odd or even) connects to the operation of addition. Questions like this help students develop understanding of the properties of numbers and of operations while building their number sense. For follow-up questions, you might ask, Why is the product of two odd numbers always odd? Why is the sum of an odd number and an even number always odd, but their product is always even? What happens when we do a subtraction problem with two odd numbers, two even numbers, or one of each?

3. Why is zero an even number?

Even numbers describe integers that are divisible by 2; for example 26 ÷ 2 = 13, so 26 is even. (A number is *divisible* by another when the result is a whole number without a remainder.) You can also use multiplication to explain this instead of division. An integer is even if you can write it as "2 times something;" for example, 26 = 2 × 13, so 26 is even. Or you can use addition: Even numbers can be represented as a number plus itself (13 + 13 = 26, so 26 is even.) Zero passes all three tests: It's divisible by 2 (0 ÷ 2 = 0); you can write it as 2 times something (2 × 0 = 0); and it can be represented as a number plus itself (0 + 0 = 0).

4. Why does canceling zeros produce an equivalent fraction in the fraction 10/20, but not in the fraction 101/201?

I presented this question to a class of 4th graders. First we discussed several examples of equivalent fractions that demonstrated we could cancel zeros in their numerators and denominators and have equivalent fractions:

10/20 = 1/2

20/30 = 2/3

20/40 = 2/4

Then I presented another fraction and asked whether it was OK to cancel the zeros to produce an equivalent fraction:

Does 101/201 = 11/21?

Some students initially thought the answer was yes; others thought it was no. We had a spirited discussion. Trey argued for yes, because "It works for 102/204 and 12/24—both would be 1/2." Russell supported Trey with another example, 100/200 and 10/20, saying, "It doesn't matter which zeros you cancel."

Elissa argued that those examples were different because you could reduce both of them to 1/2, "but you can't reduce 101/201 or 11/21 to anything." Tina argued that the fractions should be the same because, "If you add 1 to each denominator, you get 101/202 and 11/22, and these are both equal to 1/2."

Sophia used a calculator to divide, and reported that it didn't work: 101 ÷ 201 was 0.5024875, and 11 ÷ 21 was 0.5238095. She came up to the board and recorded these numbers.

Then Nick came to the board and wrote the sequence of equivalent fractions he had written starting with 11/21 to show that 101/201 wasn't in the sequence:

11/21, 22/42, 33/63, 44/84, 55/105, 66/126, 77/147, 88/168, 99/189, 110/210

Emmy gave a place-value argument for why you can't cross out the middle zeros. She said, "If you cross out the zeros, you suddenly are making hundreds into tens, and math doesn't work like that."

Actually, there were three sides, with Leslie offering a minority opinion that the discussion was moot since both fractions were very, very close to 1/2, so you should just say that they're just about the same.

Too often, students learn rules without the depth of understanding that tells them when and when not to apply them. Here students have the opportunity to investigate what happens when "cancelling" to compare fractions, first with fractions for which cancelling maintains the proportional relationship between the numerators and denominators and then with fractions for which it doesn't. The question allows a variety of entry points for students to analyze what makes sense mathematically.

Those Who Understand, Teach

Learning how to best uncover the curriculum for students has been a long process for me. I've had to learn when to *ask* and when to *tell*. Even more important, I've had to learn *what* to ask and *what* to tell, which calls for thoroughly understanding the mathematical content I'm teaching.

Glenda Lappan, a past president of the National Council of Teachers of Mathematics, addressed the importance of teachers having deep content knowledge in her article "Knowing What We Teach and Teaching What We Know." She wrote:

> Our own content knowledge affects how we interpret the content goals we are expected to reach with our students. It affects the way we hear and respond to our students and their questions. It affects our ability to explain clearly and to ask good questions. It affects our ability to approach a mathematical idea flexibly with our students and to make connections. It affects our ability to push each student at that special moment when he or she is ready or curious. And it affects our ability to make those moments happen more often for our students.[3]

A friend of mine, also a math teacher, has a T-shirt with the following message: *Those who can, do. Those who understand, teach.* I agree with this message. Even with elementary math topics that seem fairly uncomplicated and easy to understand, unexpected twists and turns can emerge during classroom teaching. But if our math knowledge as teachers is robust enough, we can treat these surprises not as difficulties but as opportunities to guide students in uncovering their understanding of mathematics.

Endnotes

[1] I was first introduced to this idea by Annie Fetter's talk "Ever Wonder What They'd Notice?" at the National Council of Teachers of Mathematics conference in Indianapolis, Indiana, which can be viewed at www.youtube.com/watch?v=WFvYZDR4OeY.)

[2] National Governors Association for Best Practices & Council of Chief State School Officers. (2010). *Standards for Mathematical Practice.* Washington, DC: Authors. Retrieved from www.corestandards.org/Math/Practice.

[3] Lappan, G. (1999, November). Knowing what we teach and teaching what we know. *NCTM News Bulletin* (online newsletter). Retrieved from www.nctm.org/about/content.aspx?id=998.

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