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Beyond Pizzas & Pies: Supporting Fraction Sense

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CMC- South/Palm Springs/November 2010

Students' Challenges with Fractions

- What are some of the biggest challenges students face with fractions?

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8th Grade, NAEP 1996

Estimate the answer to $\frac{12}{13} + \frac{7}{8}$

Answer choices were:

A.	1	7%
B.	2	24%
C.	19	28%
D.	21	27%
E.	I don't know	14%

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"The difficulty with fractions (including decimals and percents) is pervasive and is a major obstacle to further progress in mathematics. . ."

—*Report of the National Math Panel*, March 2008

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In one minute, write down everything that comes to mind when you think about or see:

$$\frac{5}{8}$$

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In one minute, write down everything that comes to mind when you think about or see:

9

Compare Your Responses

- What similarities do you see?
- What differences do you see?
- Any surprises or insights?

Adjectives vs. Nouns

(Adapted from Kathy Richardson, NCTM 2008)

- Young children initially consider numbers as adjectives or descriptors –
  - 9 *bears*
  - 6 *cookies*
  - 20 *students*
- Eventually, they come to understand numbers as nouns or concepts–
  - 9 is half of 18,
  - It is 1 less than 10,
  - It is 4.5 doubled,
  - It is 3 squared,
  - It is the square root of 81,
  - ……?

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Adjectives vs. nouns (continued)

- Students need opportunities to transition from considering fractions as adjectives –
  - 1/2 *of a pizza*
  - 3/4 *of an hour*
  - 2/3 *of a cup*
- to considering them as nouns –
  - 5/8 is a little more than 1/2, but less than 1
  - It is 3/8 less than 1
  - It is equivalent to 10/16
  - It is twice 5/16
  - It is half of 1¼
  - ……..?

"It may be surprising that, for most students, to think of a rational number as a number – as an individual entity or a single point on a number line – is a novel idea."

—*Adding it Up: Helping Children Learn Mathematics*, National Research Council, 2001

4th Grade, NAEP 2009

Which fraction has a value closest to ½?

A. 5/8
B. 1/6
C. 2/2
D. 1/5

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4th Grade, NAEP 2009

Which fraction has a value closest to ½?

A. 5/8	25%
B. 1/6	6%
C. 2/2	41%
D. 1/5	26%

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Why are fractions so hard?

- Fraction notation – numbers must be considered in new ways
- Practices that simplify and/or mask the meaning of fractions
- Overreliance on whole number knowledge
- Many meanings of fractions

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What is Fraction Sense?

"Fraction sense implies a deep and flexible understanding of fractions that is not dependent on any one context or type of problem. Fraction sense is tied to common sense: Students with fraction sense can reason about fractions and don't apply rules and procedures blindly - nor do they give nonsensical answers to problems involving fractions."

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Common Core State Standards

"Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from."

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Common Core State Standards
Number and Operations – Fractions

Grade 3: Develop understanding of fractions as numbers.

Grade 4: Extend understanding of fraction equivalence and ordering.

Grade 5: Use equivalent fractions as a strategy to add and subtract fractions.

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Beyond Pizzas & Pies: 10 Essential Strategies for Supporting Fraction Sense

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#1

Provide opportunities for students to work with irregularly partitioned, and unpartitioned, areas, lengths, and number lines.

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#2

Provide opportunities for students to investigate, assess, and refine mathematical rules and generalizations.

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#3

Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity.

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#4

Provide opportunities for students to work with changing units.

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#5

Provide opportunities for students to develop their understanding of the importance of context in fraction comparison tasks.

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#6

Provide meaningful opportunities for students to translate between fraction and decimal notation.

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#7

Provide opportunities for students to translate between different fraction representations.

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#8

- Provide students with multiple strategies for comparing and reasoning about fractions.

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#9

- Provide opportunities for students to engage in mathematical discourse and share and discuss their mathematical ideas, even those that may not be fully formed or completely accurate.

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#10

- Provide opportunities for students to build on their reasoning and sense making skills about fractions by working with a variety of manipulatives and tools, such as Cuisenaire rods, Pattern Blocks, Fraction Kits, and ordinary items from their lives.

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The Problem with Partitioning: It's Not Just About Counting the Pieces

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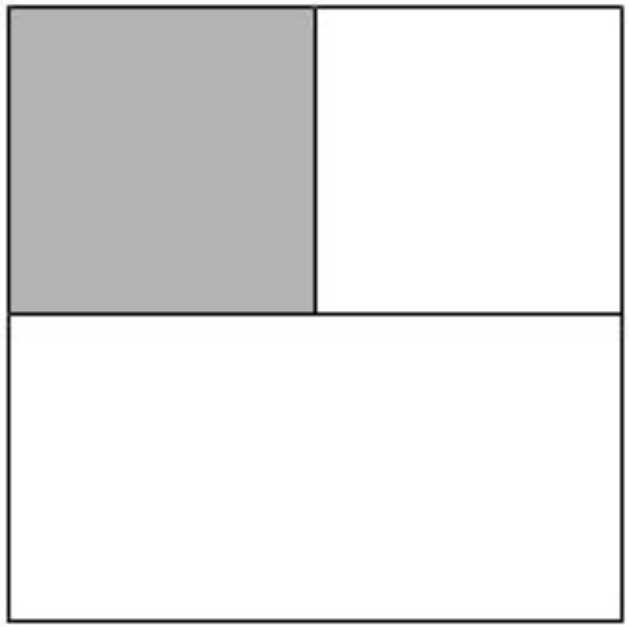
What fraction of the square is shaded? Tell me how you know.



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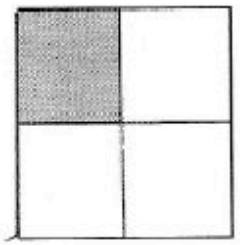
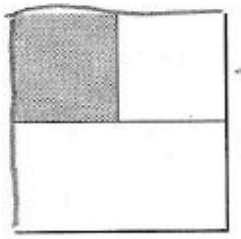
Grade 4 student - Hannah

What fraction is shaded? Tell me how you know.



$\frac{1}{4}$ Because 1 out of the 4 boxes is colored in.

What fraction is shaded? Tell me how you know.



$\frac{1}{3}$ Because 1 out of th 3 boxes is colored in.

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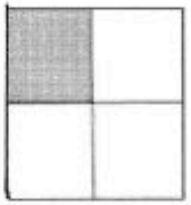
Grade 4 student - Jose

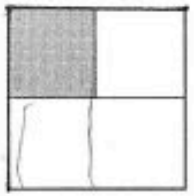
Jose

What fraction is shaded? Tell me how you know.



$\frac{1}{4}$ Because 1 is shaded out of 4

What fraction is shaded? Tell me how you know.



$\frac{1}{4}$ · they just want to trick you and not put the three pieces and the pieces have to be the same size

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What fraction is shown by B?

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What fraction is shown by B?

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#1

Provide opportunities for students to work with irregularly partitioned, and unpartitioned, areas, lengths, and number lines.

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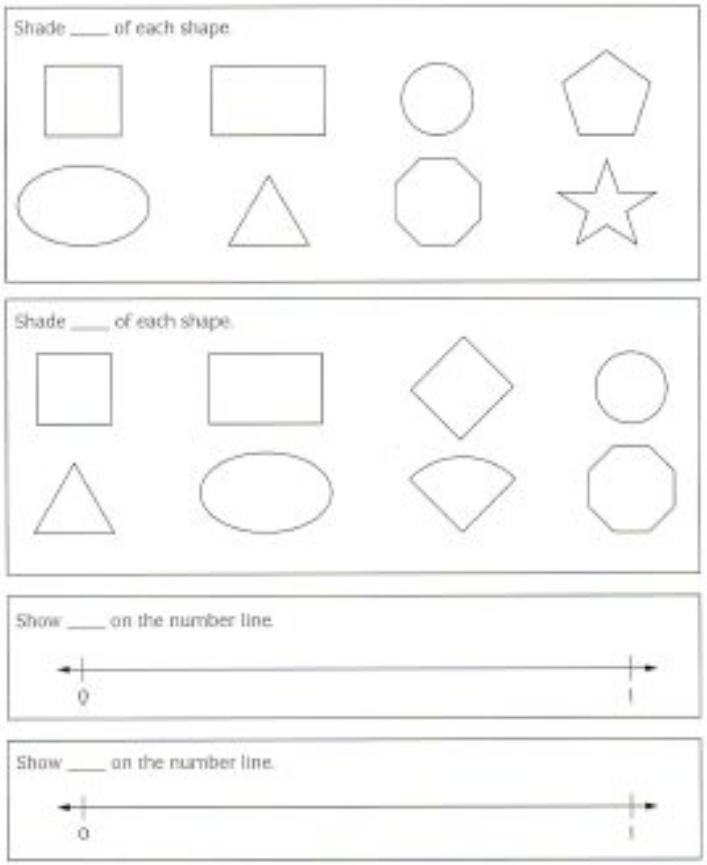
Unpartitioned areas and number lines.



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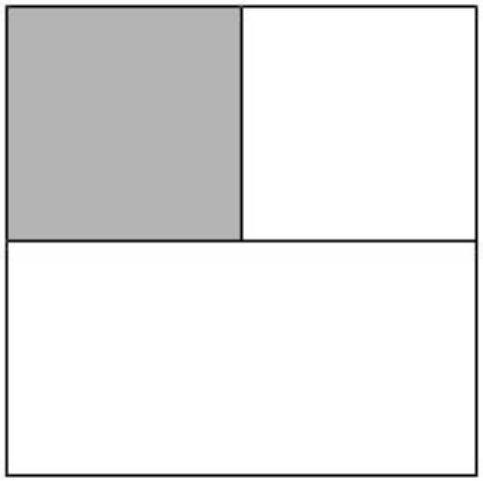
Unequally partitioned areas and number lines



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Top or Bottom:
Which One Matters?

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Circle the larger fraction.
Explain your answer.

$\frac{5}{6}$ $\frac{7}{8}$

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“If the denominator is smaller, the piece is bigger.”

1 Circle the larger fraction: $\frac{5}{6}$ $\frac{7}{8}$

Explain your answer.

If the denominator is smaller, the peice is biggen.

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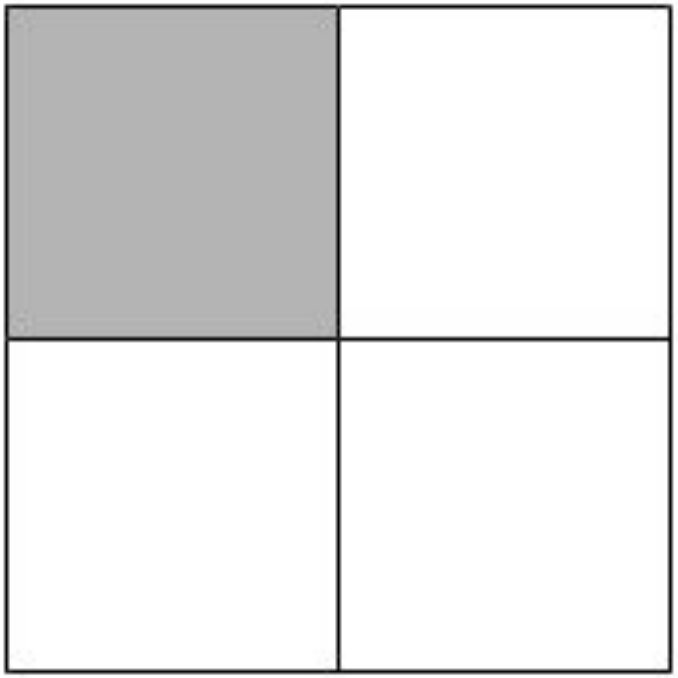
What fraction of the square is shaded? Tell me how you know.



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“This one is bigger because there is more pieces.”

1 Circle the larger fraction:

$\frac{5}{6}$ $\frac{7}{8}$

Explain your answer.

This one is biger because there is More Pices.

#2

Provide opportunities for students to investigate, assess, and refine mathematical rules and generalizations.

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- Is it always true?
- What do you think of _____’s answer?

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Understanding Equivalency: How Can Double Be the Same?

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A student does the following multiplication problem:

$$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$$

Look at the statement below:

$\frac{10}{12}$ is twice as large as $\frac{5}{6}$.

Decide whether you agree or disagree with the statement.

Agree Disagree

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A student does the following division problem:

$$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

Look at the statement below:

$\frac{3}{5}$ is half the size of $\frac{6}{10}$.

Decide whether you agree or disagree with the statement.

Agree Disagree

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#3

- Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity.

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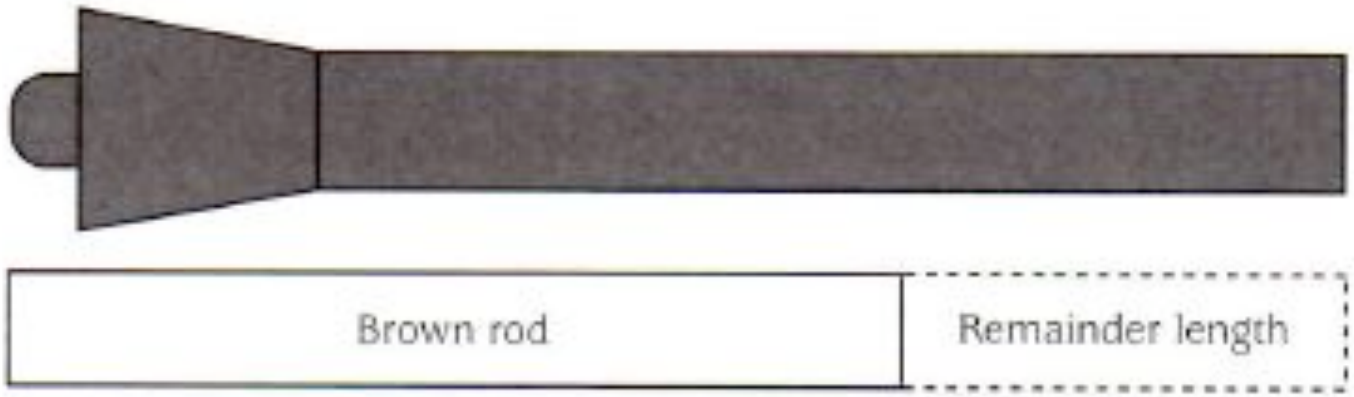
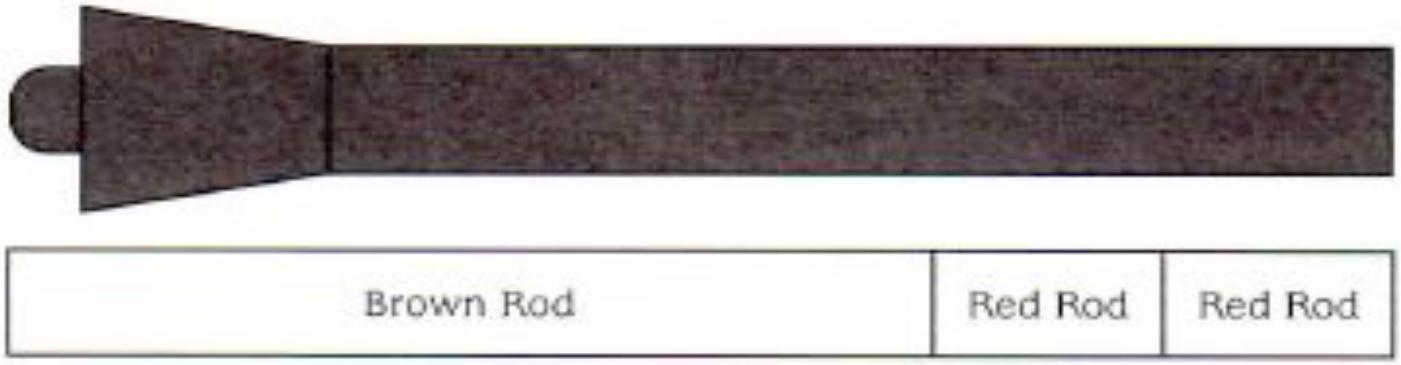
Measure the marker with brown rods



1 and 2/4 brown rods



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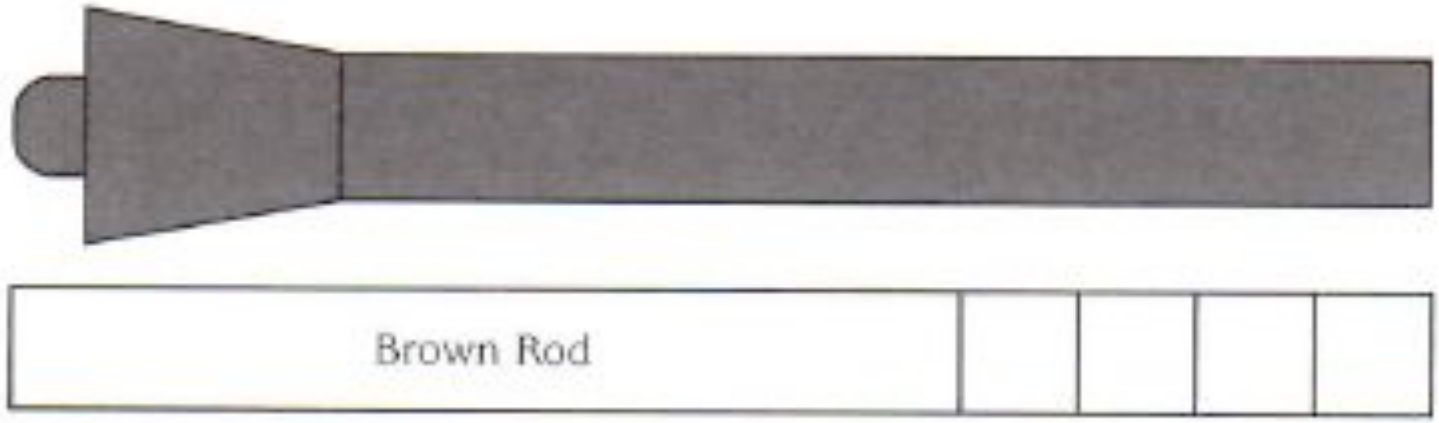
1 4/8 brown rods



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Item Being Measured	First Way	Second Way	Third Way
marker			
pencil			
book			

Comparing Fractions: Do You Always Need a Common Denominator?

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Use a common denominator to compare the fractions below.

$\frac{2}{7}$ & $\frac{3}{5}$

$\frac{3}{4}$ & $\frac{5}{6}$

$\frac{5}{6}$ & $\frac{5}{8}$

$\frac{8}{9}$ & $\frac{1}{3}$

$\frac{7}{6}$ & $\frac{6}{7}$

$\frac{1}{2}$ & $\frac{15}{30}$

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#8

- Provide students with multiple strategies for comparing and reasoning about fractions.

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Using the Cuisenaire Rods to Partition the Number Lines

Find a rod that fits on the number line exactly two times.

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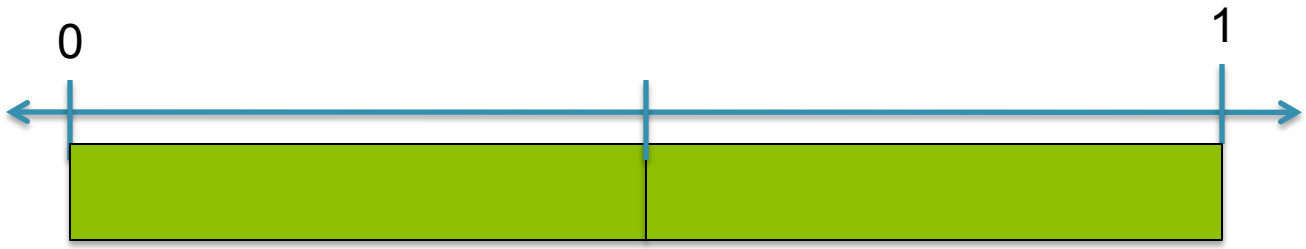
Using the Cuisenaire Rods to Partition the Number Lines



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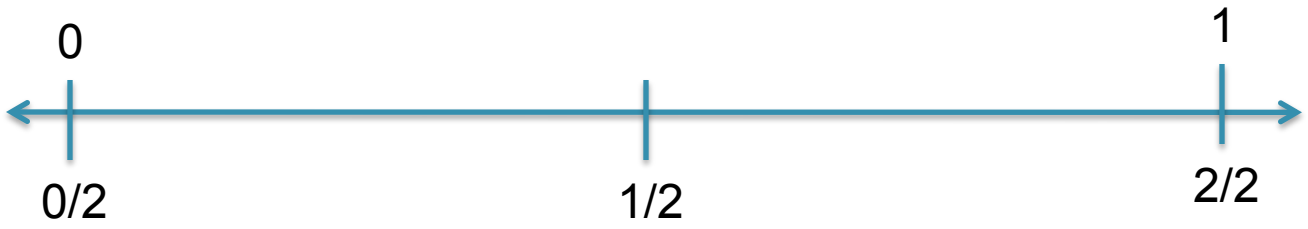
Mark ½ on the number line



Continue using the rods to mark thirds, fourths, sixths, and twelfths on the other lines.

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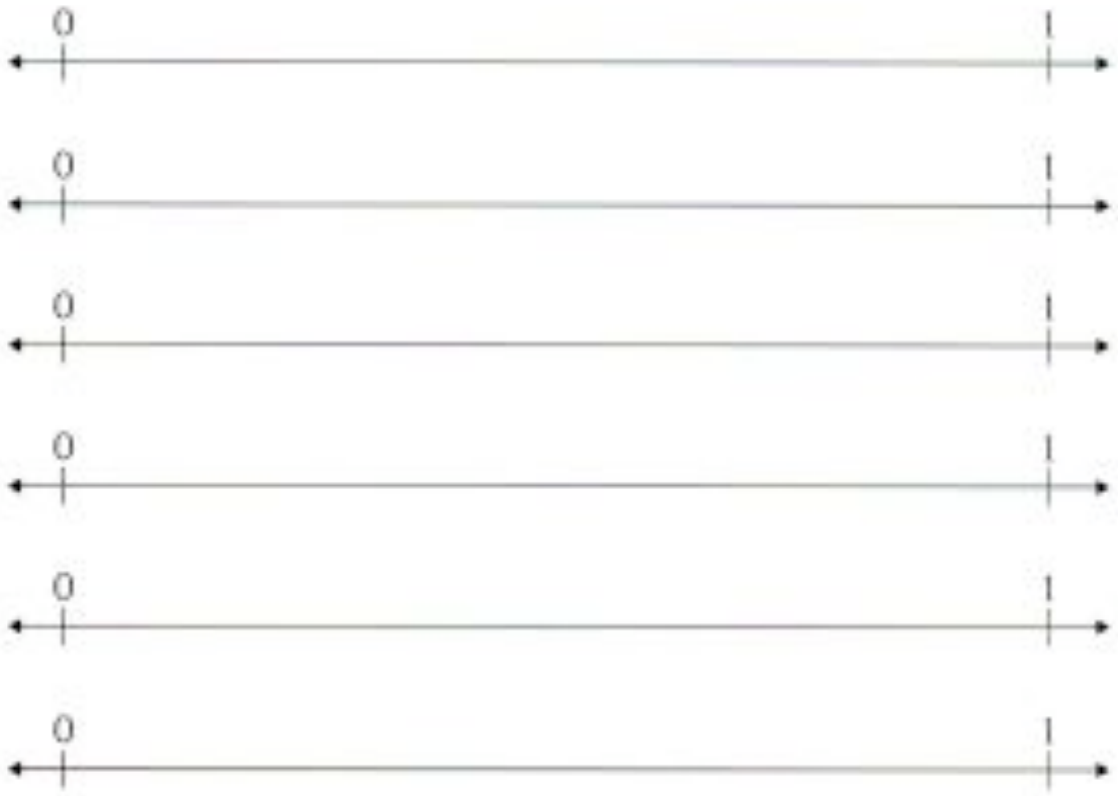
Investigating Benchmark Fractions



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Fractions Equal to ½

- What do you notice about all of the fractions that are equal to ½?
- How does the numerator compare to the denominator?

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Which of these fractions can be compared by using ½ as a benchmark?

$\frac{2}{7}$ & $\frac{3}{5}$

$\frac{8}{9}$ & $\frac{1}{3}$

$\frac{3}{4}$ & $\frac{5}{6}$

$\frac{7}{6}$ & $\frac{6}{7}$

$\frac{5}{6}$ & $\frac{5}{8}$

$\frac{1}{2}$ & $\frac{15}{30}$

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Using Benchmarks

- What other benchmarks could students use to compare 2 fractions?
- How could benchmarks help students with fraction operations?
- How does the use of benchmarks support the development of students' fraction sense?

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Fraction Sense Strategies for Comparing Fractions

Strategy	Example
More of the same-size parts (same denominators)	3/4 and 1/4 3 > 1, **so 3/4 > 1/4.**
Same number of parts but parts of different sizes (same numerators)	2/8 and 2/3 Eighths are smaller than thirds, **so 2/8 < 2/3.**
More and less than one-half or one whole	1/4 and 2/3 1/4 is less than 1/2; 2/3 is more than 1/2, **so 1/4 < 2/3.**
Closeness to one-half or one whole	3/4 and 4/5 3/4 is 1/4 away from 1 whole. 4/5 is 1/5 away from 1 whole. Fourths are > than fifths, so 4/5 is closer to one **so 3/4 < 4/5.**

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Thank you!

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