



Math Solutions[®]

FOUNDED BY MARILYN BURNS

Linking Arithmetic and Algebraic Thinking

Genni Steele

NCTM Annual San Diego

Thursday April 22, 2010

9:30 – 10:30

NCTM Principles and Standards for School Mathematics

- Understand patterns, relations, and functions
- Represent and analyze mathematical situations and structures using algebraic symbols
- Use mathematical models to represent and understand quantitative relationships
- Analyze change in various contexts

NCTM Principles and Standards for School Mathematics

Process Standards

- Problem Solving
- Communication
- Reasoning and Proof
- Connections
- Representation

What is Algebraic Thinking?

“... Algebraic reasoning is not a separate topic; it is integrally bound with learning arithmetic, and it can make the learning of arithmetic easier and richer.”

-Thomas Carpenter

Big Ideas

- Equivalence
- Properties
- Arithmetic tasks linking to algebraic thinking

How would students answer?

$$8 + 4 = \square + 5$$

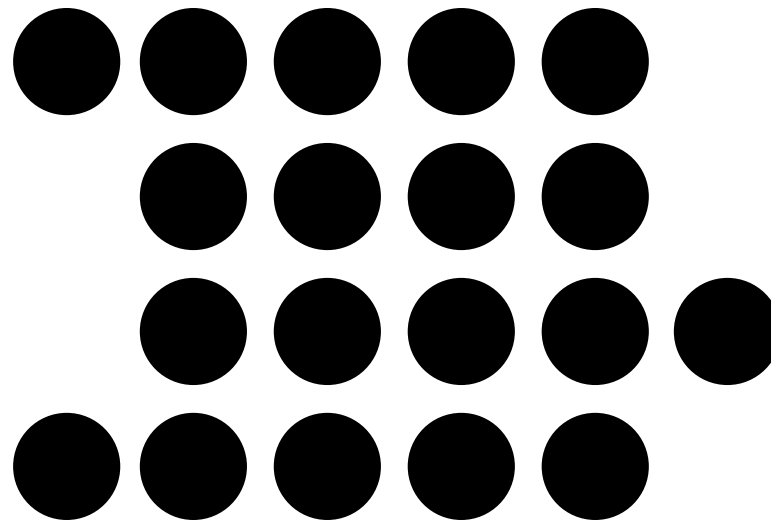
Grade	Answers Given (% students)				
	7	12	17	12 & 17	Other
3	10	60	20	5	5
4	7	9	44	30	11
5	7	48	45	0	0
6	0	84	14	2	0

Carpenter, et al – NTCM, *Teaching Children Mathematics* Vol. 6 December 99

The Grapes of Math by Greg Tang

- A springboard to properties

How Many Dots?



True, False and how do you know?

$$7 = 3 + 4$$

$$8 = 8$$

$$5 + 8 = 8 + 5$$

$$8 = 5 + 13$$

True, False and how do you know?

$$27 + 48 - 48 = 27$$

$$74 + 56 - 59 = 71$$

$$345 + 63 = 347 + 61$$

True, False and how do you know?

$$3 \times 7 = 7 + 7 + 7$$

$$6 \times 4 = 4 + 4 + 4 + 4$$

$$3 \times 8 = 2 \times 8 + 8$$

$$8 \times 6 = 8 \times 5 + 6$$

$$9 \times 7 = 10 \times 7 - 7$$

What's Missing?

$$12 + 9 = 10 + 8 + \underline{\quad}$$

$$345 + 576 = 342 + 574 + \underline{\quad}$$

$$46 + 28 = 27 + 50 - \underline{\quad}$$

The Distributive Property

$$57 \times 23$$

$$(50 + 7)(20 + 3)$$

$$50 \times 20 + 50 \times 3 + 7 \times 20 + 7 \times 3$$

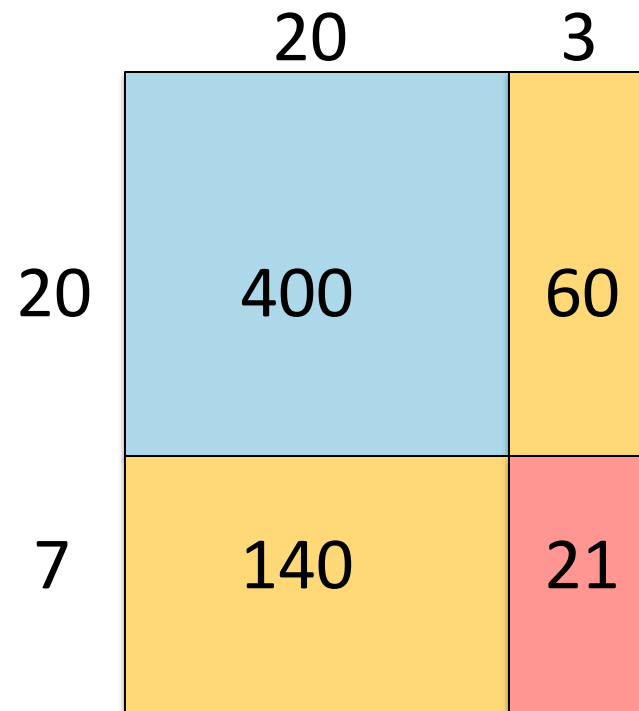
The Distributive Property

An Area Model

$$27 \times 23$$

$$(20 + 7)(20 + 3)$$

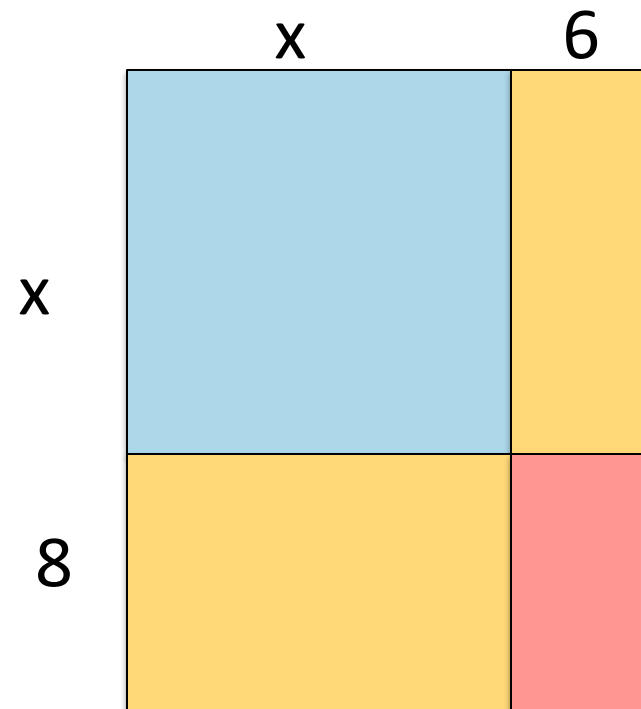
$$621$$



The Distributive Property

An Area Model

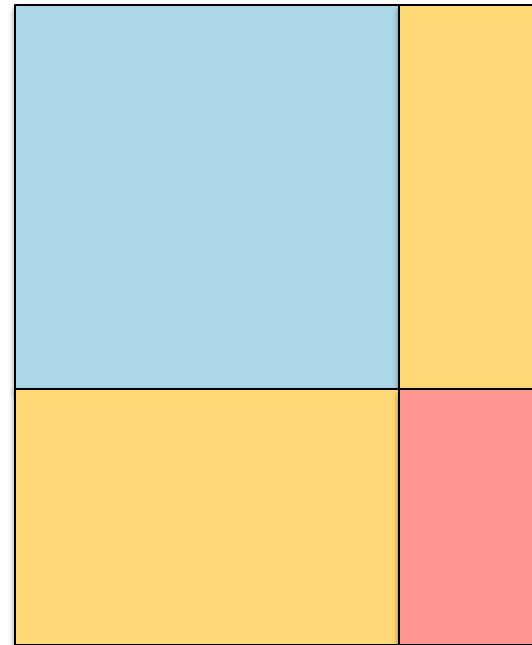
$$(x + 8)(x + 6)$$



Factoring

$$x^2 + 5x + 4$$

$$(x + 1)(x + 4)$$



Extending Arithmetic Tasks to Algebraic Thinking Tasks

- Organize the data
- Look for patterns
- Describe the patterns in words or pictures
- Make predictions
- Generalize

Algebraic Thinking in Unexpected Places

A Remainder of One

Elinor J. Pinczes

Remainder of One

- For the 25th squadron, we would write:

$$25 \div 2 = 12 \text{ R } 1$$

$$25 \div 3 = 8 \text{ R } 1$$

$$25 \div 4 = 6 \text{ R } 1$$

$$25 \div 5 = 5 \text{ R } 0$$

Remainder of One

- Investigate division problems with a remainder of one.

Record, i.e. $8 \div 7$ or $5 \div 2$

Study the division problems, how are they alike and how are they different?

Can you make some generalizations?

Let's gather a collection

$$8 \div 7$$

$$5 \div 2$$

$$7 \div 2$$

$$10 \div 9$$

$$13 \div 6$$

$$13 \div 4$$

$$14 \div 13$$

$$27 \div 13$$

$$34 \div 11$$

$$26 \div 25$$

$$33 \div 16$$

$$49 \div 16$$

What is the link to algebraic thinking?

- What are possible squadron sizes when divided by 3, there is a remainder of 2?

5, 8, 11, 14, 17

What is the link to algebraic thinking?

- What are possible squadron sizes when divided by 3, there is a remainder of 2?

5

Algebraic expression

8

$3n + 2$

11

14

17

What is the link to algebraic thinking?

- What are possible squadron sizes when divided by 3, there is a remainder of 2?

$$3n + 2 = 5$$

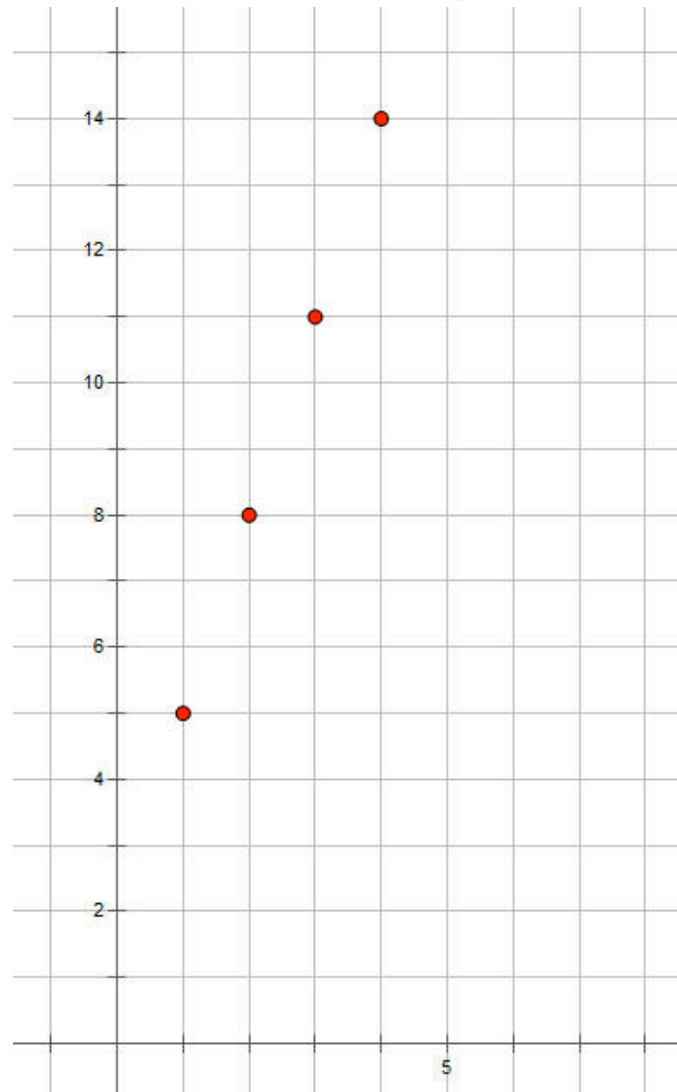
$$3n + 2 = 8$$

$$3n + 2 = 11$$

$$3n + 2 = 14$$

$$3n + 2 = 17$$

What is the link to algebraic thinking?



Think about...

- How is the task algebraic?
- Strategies
- Challenges
- Questions
- Generalizations
- Justify
- Model



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gsteele@mathsolutions.com

Booth # 1833

mathsolutions.com

800.868.9092