Supporting
Students' Fraction Sense:
What, Why, and How?

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Welcome!

In this session we will-

- define fraction sense,

- discuss why fraction sense is important, and

- share classroom strategies for helping students develop their fraction sense.

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8th Grade, NAEP 1996

Estimate the answer to $\dfrac{12}{13} + \dfrac{7}{8}$

Answer choices were:

A.	1	7%
B.	2	24%
C.	19	28%
D.	21	27%
E.	DK/No answer	14%

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4th Grade, NAEP 2009

Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$

B. $\frac{1}{6}$

C. $\frac{2}{2}$

D. $\frac{1}{5}$

4th Grade, NAEP 2009

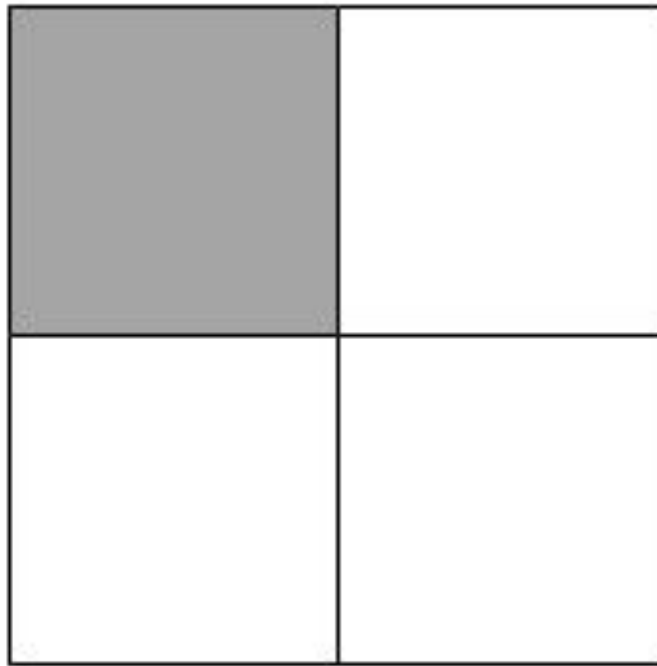Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$ 25%
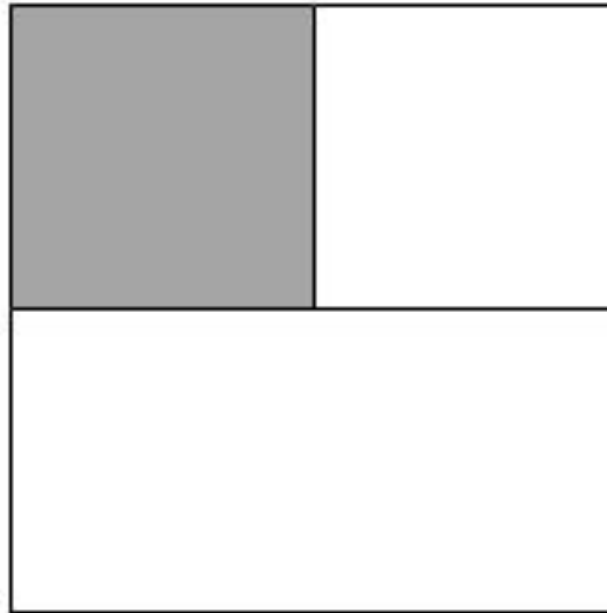
B. $\frac{1}{6}$ 6%

C. $\frac{2}{2}$ 41%

D. $\frac{1}{5}$ 26%

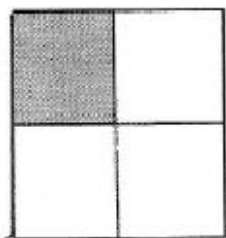# What fraction of the square is shaded? Tell me how you know.

What fraction of the square is shaded? Tell me how you know.
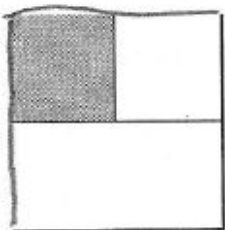
Grade 4 student - Hannah

What fraction is shaded? Tell me how you know.

$\frac{1}{4}$ Because 1 out of the 4 boxes is colored in.

What fraction is shaded? Tell me how you know.

$\frac{1}{3}$ Because 1 out of th 3 boxes is colored in.

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Grade 4 student - Jose

Jose

What fraction is shaded? Tell me how you know.
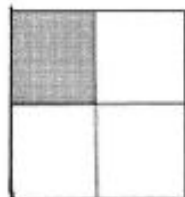
$$\frac{1}{4}$$ Becouse 1 is shaded out of 4

What fraction is shaded? Tell me how you know.
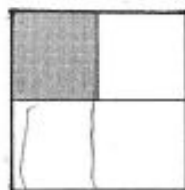
$$\frac{1}{4}$$ they Just wanto trick you and not put the three pieces and the picces have to ve the same size

What is Fraction Sense?

Fraction sense is tied to common sense: Students with fraction sense can reason about fractions and don't apply rules and procedures blindly - nor do they give nonsensical answers to problems involving fractions.

Common Core State Standards
Number and Operations – Fractions

Grade 3: Develop understanding of fractions as numbers.

Grade 4: Extend understanding of fraction equivalence and ordering.

Grade 5: Use equivalent fractions as a strategy to add and subtract fractions.

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"It may be surprising that, for most students, to think of a rational number as a number – as an individual entity or a single point on a number line – is a novel idea."

—Adding it Up: Helping Children Learn Mathematics,
National Research Council, 2001

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12

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

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In addition….

- there is a strong positive correlation between students' understanding of fractions and their overall success in mathematics (*Gomez, 2009),* and

- success in higher mathematics correlates highly with future earning potential.

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Why are fractions so hard?

- Fraction notation – numbers must be considered in new ways

- Practices that simplify and/or mask the meaning of fractions

- Overreliance on whole number knowledge

- Many meanings of fractions

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Top or Bottom:
Which One Matters?

"Students make mistakes not because they are *not* thinking, but because they are *are* thinking."

-Constance Kamii

Circle the larger fraction.
Explain your answer.

$$\frac{5}{6} \qquad \frac{7}{8}$$

40% of 4th graders and
34% of 6th graders chose 5/6.

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"If the denominator is smaller, the piece is bigger."

1 | Circle the larger fraction: (5/6)   7/8

Explain your answer.

If the denominator is smaller, the peice is bigger.

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"This one is bigger because there is more pieces."

"Students make mistakes not because they are *not* thinking, but because they *are* thinking."

-*Constance Kamii*

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Provide opportunities for students to investigate, assess, and refine mathematical rules and generalizations.

Number Line Activities with Cuisenaire Rods or Paper Strips

Using the Cuisenaire Rods to Partition the Number Lines

0   1
<———————|————————————————————————————|———————>

Find a rod that fits on the number line exactly two times.

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Using the Cuisenaire Rods to Partition the Number Lines
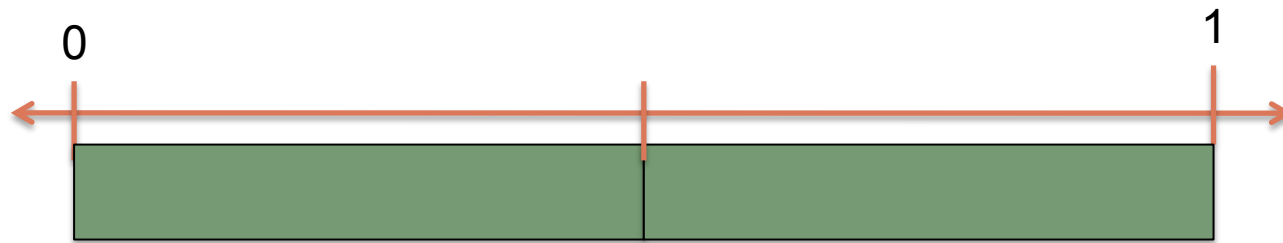
Mark $\frac{1}{2}$ on the number line

$$0 \qquad\qquad\qquad\qquad 1$$

$$\frac{0}{2} \qquad\qquad\qquad \frac{1}{2} \qquad\qquad\qquad \frac{2}{2}$$
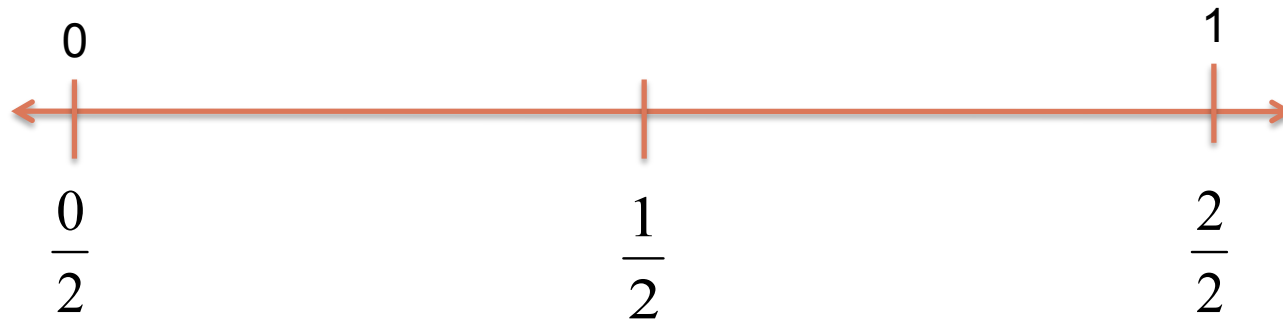
Find a rod that fits 3 times on the number line.
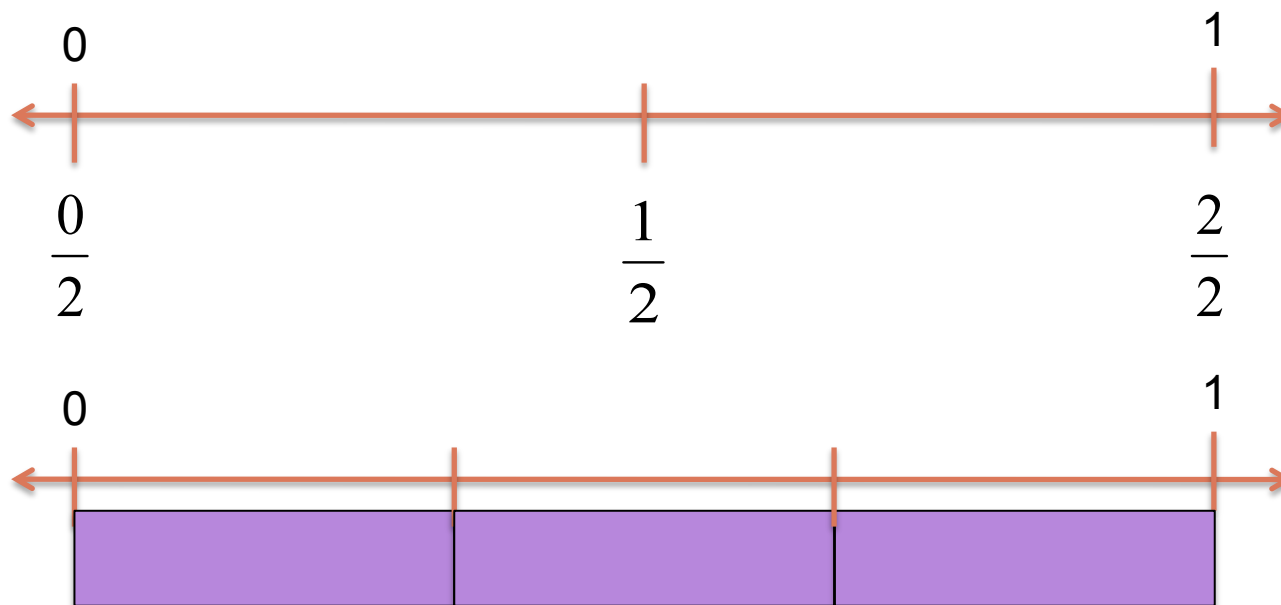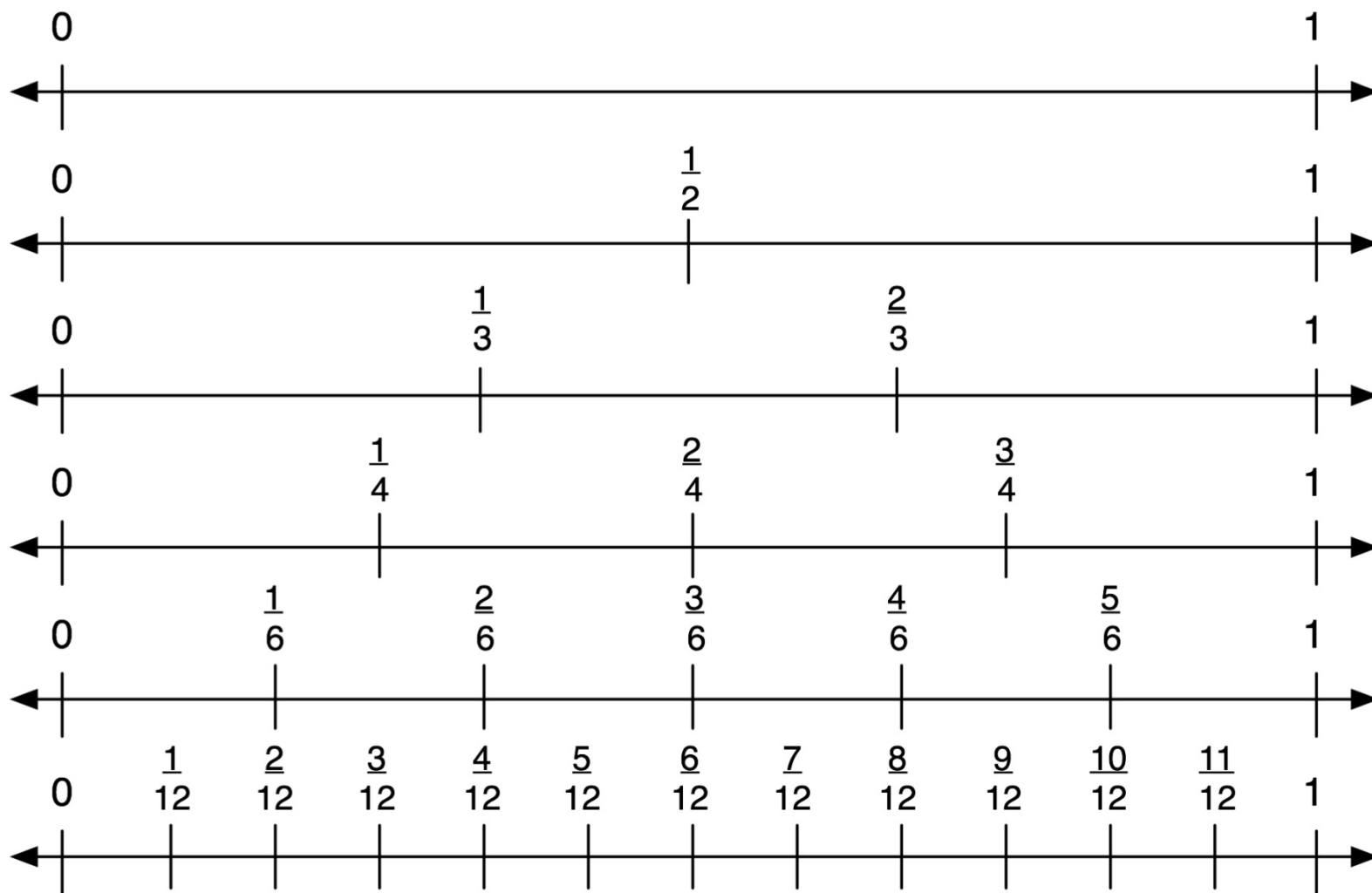
What do you think?

Use your number lines to decide if you agree or disagree with the following statements:

- "If the denominator is smaller, the piece is bigger."
- "This one (⅞)is bigger because there is more pieces."

Student A: $\dfrac{5}{6}$ $\dfrac{7}{8}$

"If the denominator is smaller, the piece is bigger."
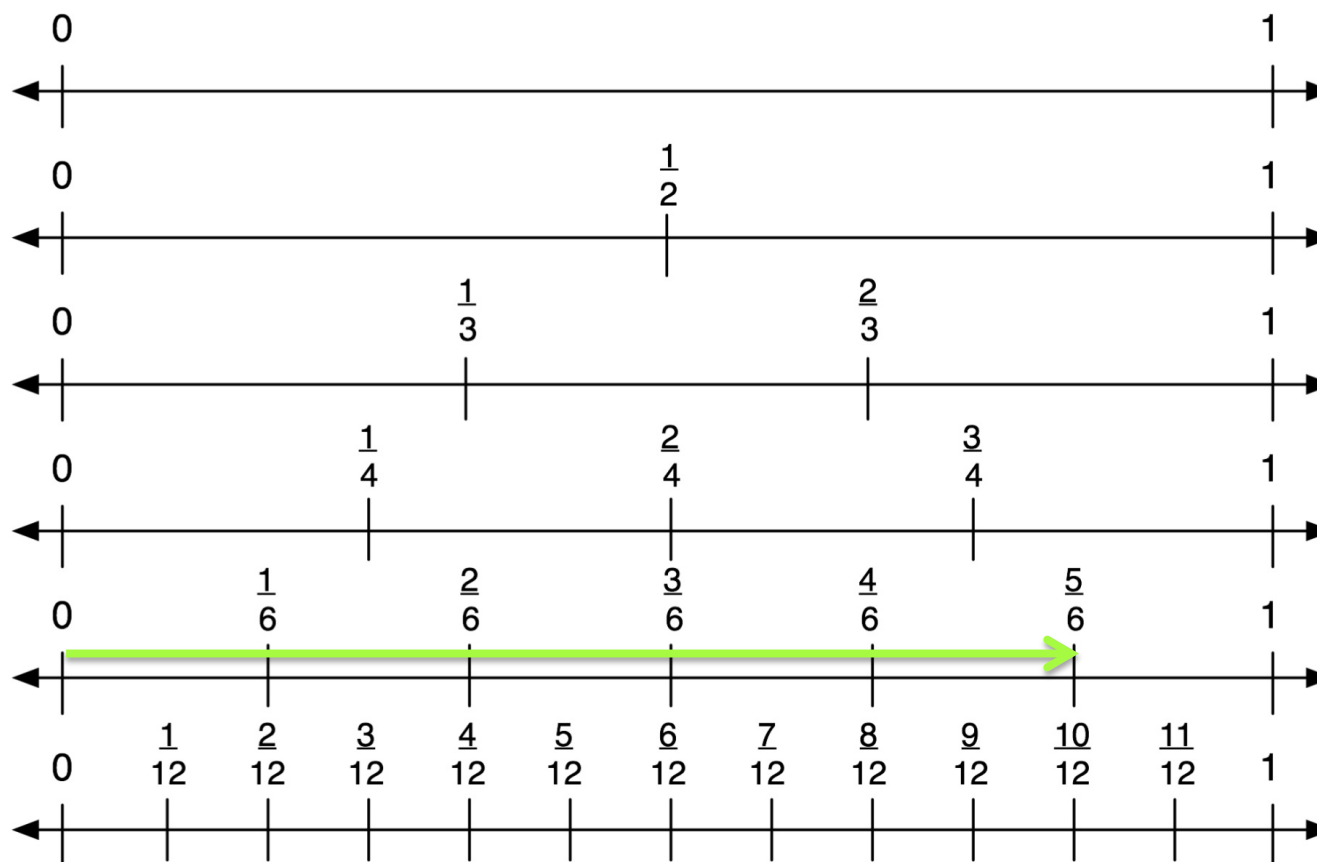
Student B: $\dfrac{5}{6}$ $\boxed{\dfrac{7}{8}}$

"This one is bigger because there is more pieces."
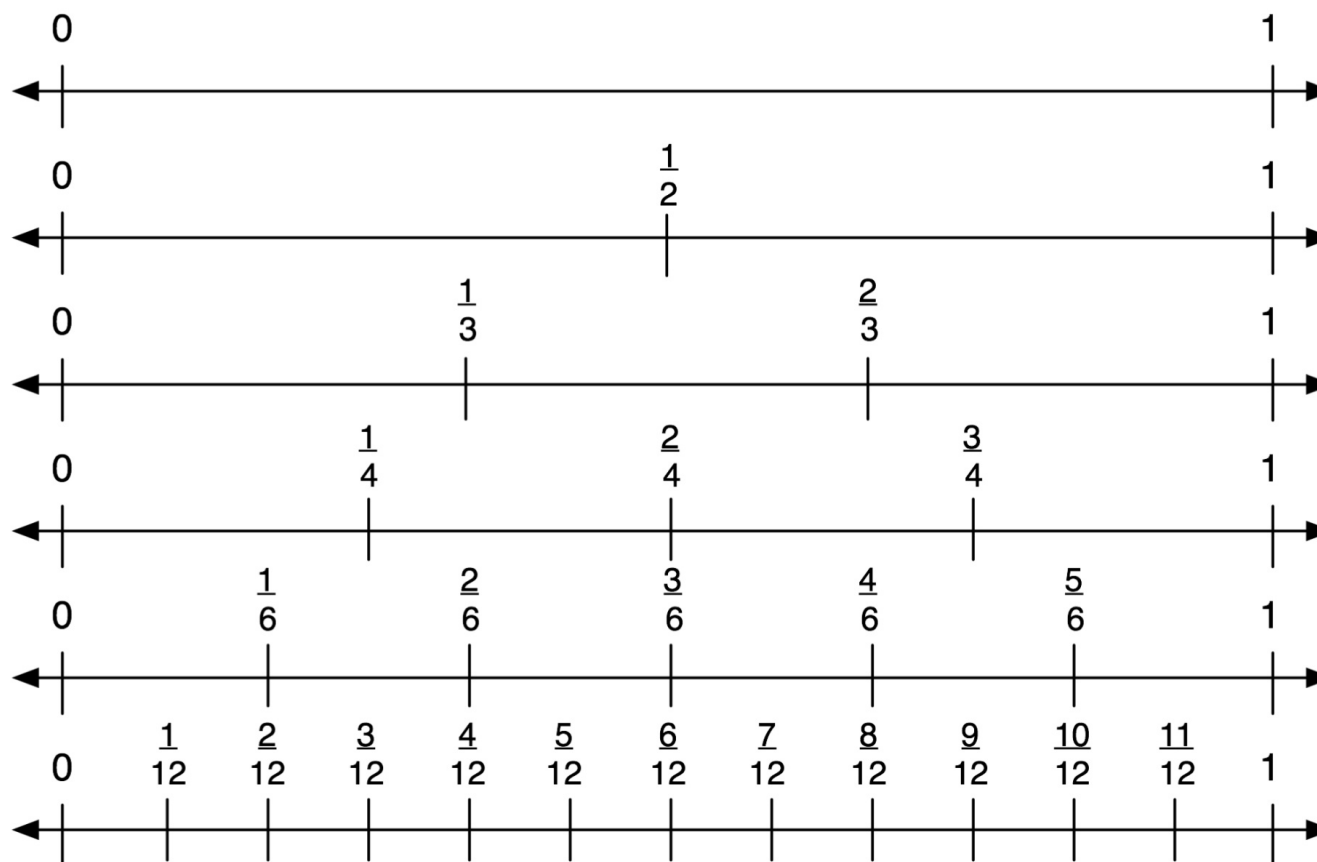
Common Core State Standards

Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from.

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Comparing Fractions: Do You Always Need a Common Denominator?

In which of the following are the three fractions arranged from least to greatest?

A. $\dfrac{2}{}\,\dfrac{1}{}\,\dfrac{5}{}$

D. $\dfrac{5}{}\,\dfrac{1}{}\,\dfrac{2}{}$

Just under one half of students tested chose the correct response.

B. $\dfrac{}{2},\dfrac{}{7},\dfrac{}{9}$

E. $\dfrac{}{9},\dfrac{}{7},\dfrac{}{2}$

C. $\dfrac{1}{2},\dfrac{5}{9},\dfrac{2}{7}$

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35

Use a common denominator to compare the fractions below.

$$\frac{2}{7} \quad \& \quad \frac{3}{5}$$

$$\frac{8}{9} \quad \& \quad \frac{1}{3}$$

$$\frac{3}{4} \quad \& \quad \frac{5}{6}$$

$$\frac{7}{6} \quad \& \quad \frac{6}{7}$$

$$\frac{5}{6} \quad \& \quad \frac{5}{8}$$

$$\frac{1}{2} \quad \& \quad \frac{15}{30}$$

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Provide students with multiple strategies for comparing and reasoning about fractions.

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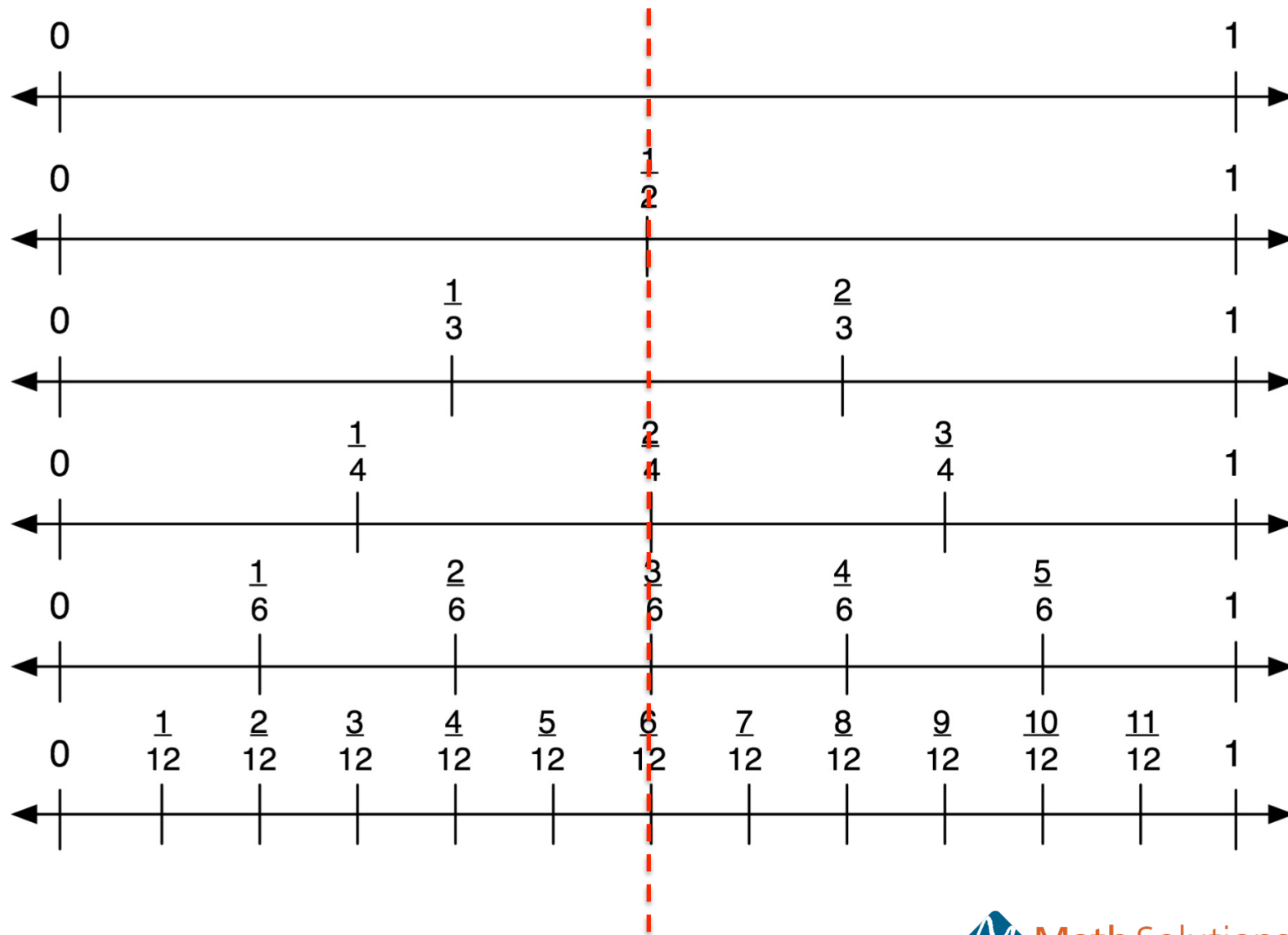
Fractions Equal to $\frac{1}{2}$

- What do you notice about all of the fractions that are equal to $\frac{1}{2}$?

- How does the numerator compare to the denominator?

What do you notice about all of the fractions that are equal to $\frac{1}{2}$?

What do you notice about all of the fractions that are Equal TO $\frac{1}{2}$?

Multiply the numerator by 2 to get the denominator

$$2N = D$$

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What do you notice about all of the fractions that are equal to $\frac{1}{2}$?

What do you notice about all of the fractions that are Equal TO $\frac{1}{2}$?

If you divide the denominator by the numerator you will get 2.

$$\frac{D}{N} = 2$$

What do you notice about all of the fractions that are equal to $\frac{1}{2}$?

What do you notice about all of the fractions that are Equal TO $\frac{1}{2}$?

The numerator has to go into the denominator twice.

$$N = \frac{D}{2}$$

Reasoning About Fractions Equal to $\frac{1}{2}$

$$2N = D$$

$$\frac{D}{N} = 2$$

$$N = \frac{D}{2}$$

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Which of these fraction pairs can be compared by using $\frac{1}{2}$ as a benchmark?

$\frac{2}{7}$ & $\frac{3}{5}$

$\frac{8}{9}$ & $\frac{1}{3}$

$\frac{3}{4}$ & $\frac{5}{6}$

$\frac{7}{6}$ & $\frac{6}{7}$

$\frac{5}{6}$ & $\frac{5}{8}$

$\frac{1}{2}$ & $\frac{15}{30}$

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In which of the following are the three fractions arranged from least to greatest?

A. $\dfrac{2}{7}, \dfrac{1}{2}, \dfrac{5}{9}$

B. $\dfrac{1}{2}, \dfrac{2}{7}, \dfrac{5}{9}$

C. $\dfrac{1}{2}, \dfrac{5}{9}, \dfrac{2}{7}$

D. $\dfrac{5}{9}, \dfrac{1}{2}, \dfrac{2}{7}$

E. $\dfrac{5}{9}, \dfrac{2}{7}, \dfrac{1}{2}$

Using Benchmarks

- What other benchmarks could students use to compare 2 fractions?

- How could benchmarks help students with fraction operations?

- How does the use of benchmarks support the development of students' fraction sense?

Closer to 0, $\frac{1}{2}$, or 1?

Where would you put $\frac{5}{15}$?

$$\longleftarrow \mid \rule{3cm}{0pt} \mid \rule{6cm}{0pt} \mid \longrightarrow$$

0 $\qquad\qquad\qquad\qquad$ $\frac{1}{2}$ $\qquad\qquad\qquad\qquad$ 1
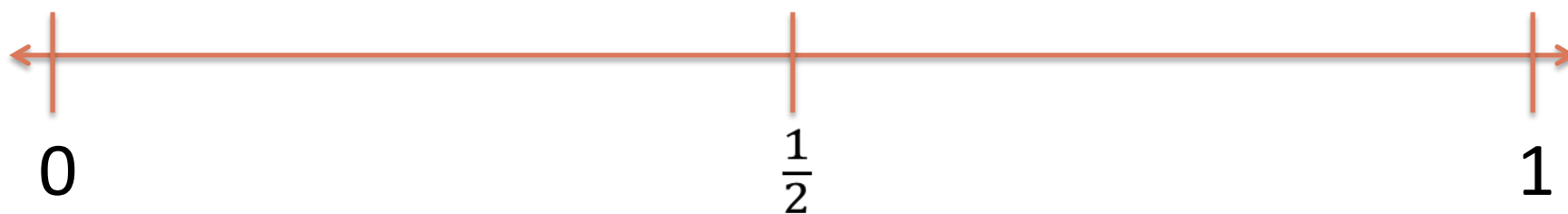
Where would you put $\frac{5}{15}$?
Closer to half
Why?
Because half is 7½ and 5 is closer to that than it is 0 or 1.

Closer to 0, $\frac{1}{2}$, or 1?

Where would you put $\frac{1}{8}$?

$$0 \quad\quad\quad\quad\quad\quad \frac{1}{2} \quad\quad\quad\quad\quad\quad 1$$
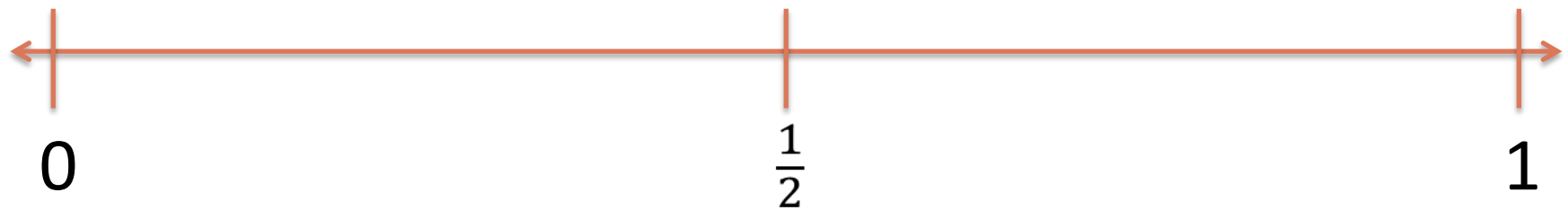
Where would you put $\frac{1}{8}$?

Why? It would be closer to 0 because it is half of $\frac{1}{4}$ and $\frac{1}{4}$ is less than $\frac{1}{2}$.

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Fraction Sense Strategies for Comparing Fractions

- More or fewer of the same sized parts (same denominator)

$$\frac{3}{8} \quad \& \quad \frac{5}{8}$$

Fraction Sense Strategies for Comparing Fractions

- Same number of parts but parts of different sizes (same numerator)

$$\frac{3}{8} \quad \& \quad \frac{3}{10}$$

Fraction Sense Strategies for Comparing Fractions

- More or less than a benchmark

$$\frac{7}{12} \quad \& \quad \frac{3}{8}$$

Fraction Sense Strategies for Comparing Fractions

- Closeness to a benchmark

$$\frac{7}{8} \quad \& \quad \frac{5}{6}$$

$$\frac{7}{2} \qquad \frac{7}{8} \qquad 3\frac{5}{9} \qquad \frac{13}{4}$$

Writing to Explain Place the numbers below in order from least to greatest. Start by expressing each number as a fraction or as a mixed number. Then find a common denominator for all the fractions. Show your work.

enVision Grade 5: Lesson 10.5

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Understanding Equivalency:
How Can Double Be the Same?

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A student does the following multiplication problem:

$$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$$

Look at the statement below:

$\frac{10}{12}$ is twice as large as $\frac{5}{6}$.

60% of 4ᵗʰ graders and 51% of 6ᵗʰ graders agreed with the statement.

A student does the following division problem:

$$\frac{6}{10} \div \frac{2}{2} = \frac{3}{5}$$

Look at the statement below:

$\frac{3}{5}$ is half the size of $\frac{6}{10}$.

73% of 4th graders and 57% of 6th graders agreed with the statement.

Provide opportunities for students to recognize equivalent fractions as different ways to name the same quantity.

Understanding Fractions through Measurement

- Fractions occur naturally in measurement activities

- Fractions are presented as units of measure

- Highlights the importance of the unit

- Supports connections between a fraction and a point on a number line

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Measure the marker with brown rods

Brown Rod | Purple Rod

Purple Rod | Purple Rod

$1\frac{1}{2}$ Brown rods

Measure the marker with brown rods

$1\frac{2}{4}$ Brown rods

Measure the marker with brown rods

Brown Rod

$1\frac{4}{8}$ Brown rods

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Measure the marker with brown rods

Brown Rod | Purple Rod $1\frac{1}{2}$ Brown rods

Brown Rod | Red Rod | Red Rod $1\frac{2}{4}$ Brown rods

Brown Rod $1\frac{4}{8}$ Brown rods

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Item	First Way	Second Way	Third Way
Marker	1 ½ brown rods	1 2/4 brown rods	1 4/8 brown rods
Name Tag			
Program			

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Fraction Kits: Friend or Foe?

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"The context in which any vehicle – concrete or pictorial – is used is as important as the material itself. By context, I mean the ways in which students work with the material, toward what purposes, with what kinds of talk and interaction."

-Deborah Ball, Dean of the School of Education,
University of Michigan
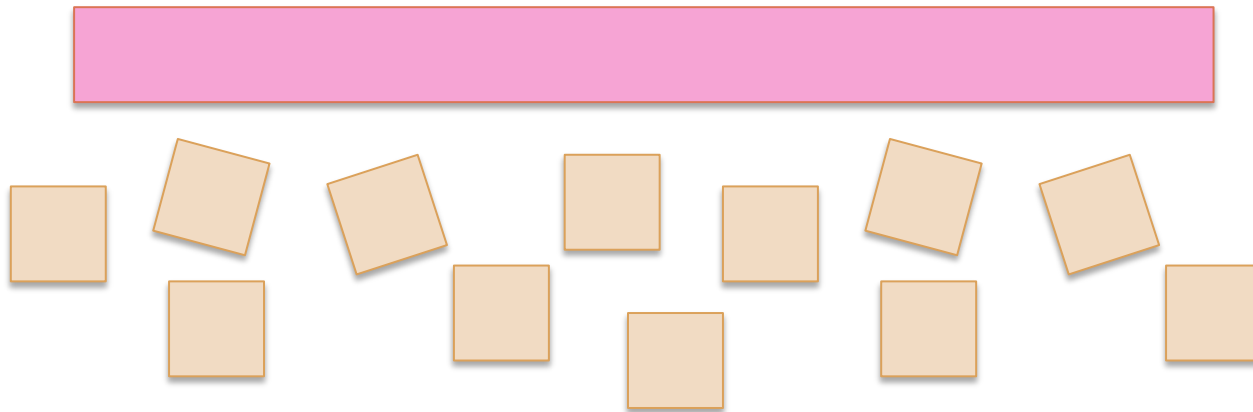
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What do you call one tan rod?

"One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve. One twelfth, 'cause there's twelve of the tans."

-Trevor, Grade 5

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What do you call one tan rod?

"Because last time you said one seventh was the tan block."

-Salim, Grade 5

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Provide opportunities for students
to work with changing units.

Item	First Way Brown Rod	Second Way Dk. Green Rod	Third Way ?
Marker	1 ½ brown rods	2 green rods	
Name Tag			
Program			

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What do you call one tan/white rod?

The tan/white rod is
$\frac{1}{8}$ *of the brown rod*.

The tan/white rod is
$\frac{1}{6}$ *of the dk. green rod*.

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What do you call the _____?

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If the black rod is the whole…

- What do you call the white/tan rod?

- What do you call the light green rod?

- What do you call the purple rod?

- What do you call the dark green rod?

- What do you call the orange rod?
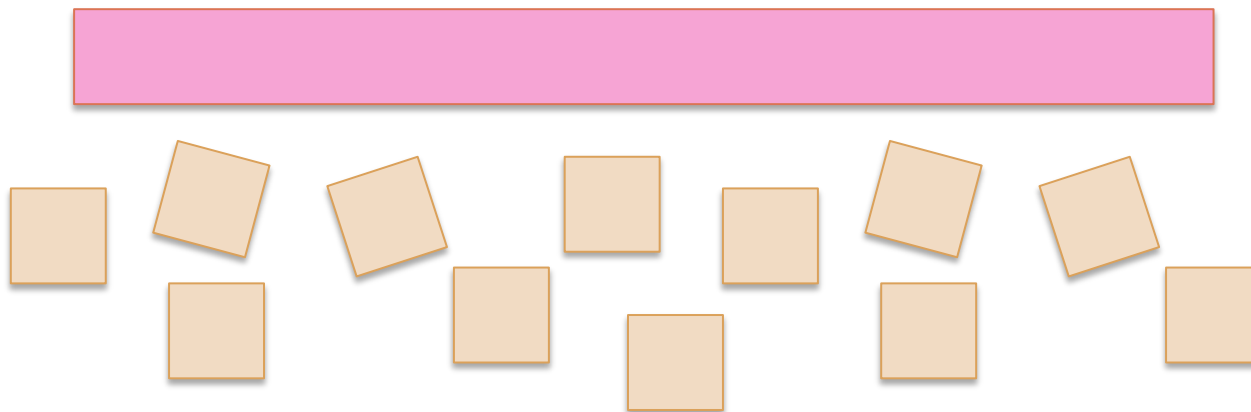
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If the yellow rod is the whole…

- What do you call the white/tan rod?

- What do you call the light green rod?

- What do you call the purple rod?

- What do you call the dark green rod?

- What do you call the orange rod?

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What do you call one tan rod?

"One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve. One twelfth, 'cause there's twelve of the tans."

-Trevor, Grade 5

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What do you call one tan rod?

"How many of these does it take to fill up?"

-Salim, Grade 5

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What is Fraction Sense?

Fraction sense is tied to common sense: Students with fraction sense can reason about fractions and don't apply rules and procedures blindly - nor do they give nonsensical answers to problems involving fractions.

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8th Grade, NAEP 1996

Estimate the answer to

Answer choices were:

$$\frac{12}{13} + \frac{7}{8}$$

A.   1

B.   2

C.   19

D.   21

E.   DK/No answer

How could fraction sense help a student answer this question?

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Which fraction has a value closest to $\frac{1}{2}$?

A. $\frac{5}{8}$

B. $\frac{1}{6}$

C. $\frac{2}{2}$

D. $\frac{1}{5}$

How could fraction sense help a student answer this question?

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8th Grade, NAEP 2007

In which of the following are the three fractions arranged from least to greatest?

A. $\dfrac{2}{7}, \dfrac{1}{2}, \dfrac{5}{9}$

B. $\dfrac{1}{2}, \dfrac{2}{7}, \dfrac{5}{9}$

C. $\dfrac{1}{2}, \dfrac{5}{9}, \dfrac{2}{7}$

How could fraction sense help a student answer this question?

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"Children who are successful at making sense of mathematics are those who believe that mathematics makes sense."

-Lauren Resnick

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Thank you!!

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mathsolutions.com/presentations

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