Number Sense and the Common Core

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MRI Math Reasoning Inventory

Funded by the Bill & Melinda Gates Foundation



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Formative Assessment

Find out what students really understand about math



First a math problem



I left Syracuse in 1964.

Figure out in your head how many years I've been living in California.



Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.







Jonathan: "Just circle 10s and then say 10, 20, 30, 40, . . ."









Cena:

"Look, you've got 4 tens and you, like, put a 4 right there. And you have 9 stars left over, and then you put 9 right there. It's 49."



Standards for Mathematical Content

Grade 2

- Domain: Number & Operations in Base Ten Cluster: Understand place value
- **Standard:** Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.



Standards for Mathematical Content

Grade 1

- Domain: Number & Operations in Base Ten Cluster: Understand place value
- **Standard:** Understand that the two digits of a two-digit number represent amounts of tens and ones



Standards for Mathematical Practice

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- 3. Construct viable arguments and critique the reasoning of others.
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Cena with tiles in groups of 10





I was intrigued. I became hooked.













So . . . what is MRI?





Formative assessment tool



"When the cook tastes the soup, it's Formative Assessment. When the guest tastes the soup, it's Summative Assessment."

Jeane M. Joyner and Mari Muri. 2011.INFORMative Assessment. Sausalito, CA: Math Solutions Publications. [Robert Stake. 2004. *Standards-Based and Responsive Evaluation*. Thousand Oaks, CA: Sage Publications.]





- Formative assessment tool
- Addresses the Common Core



MRI asks questions that the **Common Core expects all** students entering middle school to be able to answer successfully.



15% & 20% TIP TABLE®

Check	15%	20%	Check	15%	20%
\$1.00	\$.15	\$.20	\$26.00	\$3.90	\$5.20
2.00	.30	.40	27.00	4.05	5.40
3.00	.45	.60	28.00	4.20	5.60
4.00	.60	.80	29.00	4.35	5.80
5.00	.75	1.00	30.00	4.50	6.00
6.00	.90	1.20	31.00	4.65	6.20
7.00	1.05	1.40	32.00	4.80	6.40
8.00	1.20	1.60	33.00	4.95	6.60
9.00	1.35	1.80	34.00	5.10	6.80
10.00	1.50	2.00	35.00	5.25	7.00
11.00	1.65	2.20	36.00	5.40	7.20
12.00	1.80	2.40	37.00	5.55	7.40
13.00	1.95	2.60	38.00	5.70	7.60
14.00	2.10	2.80	39.00	5.85	7.80
15.00	2.25	3.00	40.00	6.00	8.00
16.00	2.40	3.20	41.00	6.15	8.20
17.00	2.55	3.40	42.00	6.30	8.40
18.00	2.70	3.60	43.00	6.45	8.60
19.00	2.85	3.80	44.00	6.60	8.80
20.00	3.00	4.00	45.00	6.75	9.00
21.00	3.15	4.20	46.00	6.90	9.20
22.00	3.30	4.40	47.00	7.05	9.40
23.00	3.45	4.60	48.00	7.20	9.60
24.00	3.60	4.80	49.00	7.35	9.80
25.00	3.75	5.00	50.00	7.50	10.00





- Formative assessment tool
- Addresses the Common Core
- Reveals what students do know, do not know, and should know





- Formative assessment tool
- Addresses the Common Core
- Reveals what students do know, do not know, and should know
- Available online, free of charge, to all teachers






The *Interview* is the core of MRI.



Reasoning is the heart of MRI.







ando

How man



We ask . . .We listen . . .We learn . . .







School Bus Problem

There are 295 students in the school. School buses hold 25 students. How many school buses are needed to fit all of the students?



Marisa: School Bus Problem





Marisa





Ellen Grade 3

100 - 98



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Ellen: 100 – 98





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1000 - 998



Ana: 1000 – 998





1000 - 998

What % of students answered incorrectly? 18%

(and these do not include the Ana's)



What % of students answered the School Bus problem incorrectly?

39%



Common Core Connection

 Common Core Standards for Mathematical Practice

 Common Core Standards for Mathematical Content



Common Core Recommendation: ... a "balanced combination of procedures and understanding."

Common Core Caution:

... "students who lack understanding of a topic may rely on procedures too heavily."



Molly Problem

Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?



Natasha: Molly Problem





Standards for Mathematical Practice

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15 x 12







- o Used standard algorithm to multiply
- Broke 15 and/or 12 into parts and then multiplied
 (e.g., 15 x 10 and then 15 x 2)
- o Changed to an easier problem, 30 x 6, by doubling and halving
- o Gave other reasonable explanation
- o Guessed, did not explain, or gave faulty explanation



15 x 12



Malcolm



Alberto



Cecilia



- o Used standard algorithm to multiply
- Broke 15 and/or 12 into parts and then multiplied
 (e.g., 15 x 10 and then 15 x 2)
- o Changed to an easier problem, 30 x 6, by doubling and halving
- o Gave other reasonable explanation
- o Guessed, did not explain, or gave faulty explanation



15 x 12

What % of students answered incorrectly? 52%



MRI Whole Number Interview Questions

- W1 1000-998
- W2 99 + 17
- W3 100 18
- W4 15 + ____ = 200
- W5 20 x 15 = 300, 21 x 15 = ____
- W6 60 x 40
- W7 15 x 12
- W8 7000 ÷ 70
- W9 18 x 21: Which is closest—100, 400, 700, or 1000?

W10 School Bus Problem (295 students, 25 in each bus) [p&p OK]



MRI Whole Number Written Computation Questions



MRI Whole Number Written Computation Questions

- 5000 328 **22%**
- 842 x 35 36%
- 3423 ÷ 6 55% answered incorrectly
- 275 ÷ 22 41%



MRI Whole Number Interview Questions

<u>% Inc</u>

- W1 **18%** 1000 998
- W2 **19%** 99 + 17
- W3 **21%** 100 18
- W4 **20%** 15 + ___ = 200
- W5 38% 20 x 15 = 300, 21 x 15 =
- W6 31% 60 x 40
- W7 52% 15 x 12
- W8 36% 7000 ÷ 70
- W9 26% 18 x 21: Which is closest—100, 400, 700, or 1000?

W10 39% School Bus Problem (295 students, 25 in each bus)



As we worked on MRI, essential numerical reasoning **Strategies** and **Understandings** emerged.


Strategies Not Appropriate to the Numbers at Hand

Whole Numbers: Multiplying & Dividing Mentally

Uses standard algorithm to multiply or divide

Figures exact answer when asked to estimate



Strategies Appropriate to the Numbers at Hand

Whole Numbers: Multiplying & Dividing Mentally



Uses known facts and place value to multiply or divide



Breaks numbers apart to multiply or divide



Uses benchmark numbers to make estimates



Whole Number Strategies: Multiplying and Dividing Mentally

- Uses known facts and place value to multiply or divide
- Breaks numbers apart to multiply or divide
- Uses benchmark numbers to make estimates
- Uses standard algorithm to multiply or divide
 - Figures exact answer when asked to estimate



Whole Numbers Applying Understanding

- Models with mathematics to solve problems in context
- Uses inverse relationship of addition and subtraction
- Uses distributive property



MRI provides instant reports

- Inform instruction
- Monitor progress
- Identify students in need of intervention
- Communicate with parents







Individual Report: Whole Numbers

MR, E

Legend

Appropriate for the numbers at hand

😑 Not Appropriate for the numbers at hand

Interview

Date Completed: 17-Nov-2011

Category	Strategies or Understandings Demonstrated	Appropriate Strategies or Understandings Not Demonstrated
Adding and Subtracting Mentally	 Breaks numbers apart to add or subtract 100 – 18 Gives other reasonable explanation 1000 – 998 Uses standard algorithm to add or subtract 99 + 17 	 Uses addition to solve subtraction problems Uses benchmark numbers to add or subtract
Multiplying and Dividing Mentally	 Uses known facts and place value to multiply or divide 7000 ÷ 70 Breaks numbers apart to multiply or divide 15 × 12 Gives other reasonable explanation 60 × 40 Guesses, does not explain, or gives faulty explanation Estimate 18 × 21 	Uses benchmark numbers to make estimates
Applying Understanding	 Uses distributive property 15 × 12 20 × 15 = 300, 21 × 15 = 	 Models with mathematics to solve problems in context Uses inverse relationship of addition and subtraction

Written Computation

Date Completed: 17-Nov-2011

Category	Demonstrated	Not Demonstrated	
Computing Accurately with Paper and Pencil	5000 – 328 842 × 35 3423 ÷ 6 275 ÷ 22		

Group Report: Whole Numbers

All Students

Start Date: 15-Aug-2011 End Date: 25-Nov-2011

Interview

Students: 8

Legend

Appropriate for the numbers at hand

Not Appropriate for the numbers at hand

Category	Consistently Demonstrated (75–100% of Students)	Often Demonstrated (50–74% of Students)	Sometimes Demonstrated (25–49% of Students)	Rarely Demonstrated (0–24% of Students)
Adding and Subtracting Mentally	Breaks numbers apart to add or subtract (75%)	 Uses benchmark numbers to add or subtract (62%) Uses standard algorithm to add or subtract (50%) 	 Uses addition to solve subtraction problems (38%) 	 Gives other reasonable explanation (12%) Counts by 1s (12%)
Multiplying and Dividing Mentally		 Uses known facts and place value to multiply or divide (50%) Breaks numbers apart to multiply or divide (50%) Gives other reasonable explanation (50%) Uses standard algorithm to multiply or divide (62%) 	 Uses benchmark numbers to make estimates (25%) Figures exact answer when asked to estimate (25%) 	
Applying Understanding		 Uses distributive property (62%) Models with mathematics to solve problems in context (50%) 	Uses inverse relationship of addition and subtraction (38%)	

Written Computation

Students: 7

Item Analysis: Whole Numbers

Period 1

Start Date: 30-Sep-2011 End Date: 28-Oct-2011

Interview

Students: 7

Legend

Appropriate for the numbers at hand

Not Appropriate for the numbers at hand

	Question	Correct / Self-corrected	Incorrect	Did Not Answer	Strategies Used by Students Who Gave Correct Answers
1	1000 – 998	100%	0%	0%	 Uses addition to solve subtraction problems (6/7) Guesses, does not explain, or gives faulty explanation (1/7)
2	99 + 17	43%	57%	0%	 Breaks numbers apart to add or subtract (1/3) Uses standard algorithm to add or subtract (2/3)
3	100 – 18	57%	29%	14%	 Breaks numbers apart to add or subtract (2/4) Uses standard algorithm to add or subtract (1/4) Counts by 1s (1/4)
4	15 + = 200	71%	14%	14%	 Breaks numbers into parts to add or subtract (4/5) Uses standard algorithm to add or subtract (1/5)
5	20 × 15 = 300, 21 × 15 =	0%	43%	57%	No correct answers given
6	60 × 40	43%	57%	0%	Uses known facts and place value to multiply or divide (3/3)
7	15 × 12	29%	43%	29%	Uses standard algorithm to multiply or divide (2/2)
8	7000 ÷ 70	57%	14%	29%	Uses known facts and place value to multiply or divide (4/4)
9	Estimate 18 × 21	57%	43%	0%	 Relates to benchmark number to make estimates (1/4) Figures exact answer when asked to estimate (1/4) Guesses, does not explain, or gives faulty explanation (2/4)
10	295 students, 25 on each bus	0%	43%	57%	No correct answers given

Written Computation

Students: 7

	Question	Correct / Self-corrected	Incorrect	Did Not Answer
1	5000 – 328	100%	0%	0%
2	842 × 35	57%	29%	14%
3	3423 ÷ 6	71%	14%	14%
4	275 ÷ 22	0%	43%	57%



11/12 + 1/5

Is the answer greater than or less than 1?



3 Explanations for 11/12 + 1/5

- o Converted to common denominators
- o Explained that 11/12 is 1/12 away from 1 and 1/5 is greater than 1/12
- o Reasoned with decimals or percents
- o Gave other reasonable explanation
- o Guessed, did not explain, or gave faulty explanation



Is 11/12 + 1/5 greater or less than 1?



Manuel



Amir



Jennifer



Jada

11/12 + 1/5

(30% incorrect)



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MRI Fraction Interview Questions

- F1 Which is greater, 3/8 or 5/6?
- F2 Which is greater, 5/12 or 5/8?
- F3 Which is greater, 3/5 or 5/12?
- F4 11/12 + 1/5: Is the answer greater or less than 1?
- F5 8/9 + 12/13: Which is closest 1/2, 1, 2, or 8?
- F6 13/8 4/5: Is the answer greater or less than 1?
- F7 3/4 + ___ = 2 1/2
- F8 3 1/2 x 2
- F9 1 1/2 x ____ = 6
- F10 Carlos/Terrell (6/8 mile and 3/4 mile, same distance?)
- F11 Cookies (3 cookie recipes, 3/4 cup sugar each) [p&p OK]
- F12 Hamburgers (2 1/2 lbs, 1/4 lb each burger) [p&p OK]



Fractions Strategies: Comparing Mentally

- Uses relationships between numerators and denominators to compare
- Converts to common denominators to compare



÷

Fractions Strategies: Computing Mentally





Extends understanding of operations with whole numbers to operations with fractions



Uses standard algorithm to compute

Figures exact answer when asked to estimate



Fractions Applying Understanding



Models with mathematics to solve problems in context



Understands equivalence in context



12.6 x 10





12.6 x 10

- 1. What percent of students do you think answered incorrectly?
- 2. How would you like to hear students explain the answer?
- 3. What do you think was the most common incorrect answer?



3 Explanations for 12.6 x 10

- o Used standard algorithm to multiply
- o Used rule of adjusting the decimal point to multiply by 10
- o Multiplied 12 x 10 and 0.6 x 10
- o Gave other reasonable explanation
- o Guessed, did not explain, or gave faulty explanation



Amir: 12.6 x 10





12.6 x 10



Jennifer



Luisa



Craig



Natasha



(MRI) is an online formative assessment tool designed to make teachers' classroom instruction more effective. Learn More

used to inform instruction. monitor progress, identify students who would benefit from intervention, and communicate with parents.

Learn More

strategies students use to reason with whole numbers. decimals, and fractions.

Learn more

math students and adjust my lessons accordingly."

> - Diana Jones Grade 6 Teacher SLCUSD, California







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