

Have you ever heard....

*Yours is not to reason why,
just invert and multiply!*

8th Grade, NAEP 2007

In which of the following are the three fractions arranged from least to greatest?

A. $\frac{2}{7}, \frac{1}{2}, \frac{5}{9}$

D. $\frac{5}{9}, \frac{1}{2}, \frac{2}{7}$

B. $\frac{1}{2}, \frac{2}{7}, \frac{5}{9}$

E. $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

C. $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$

8th Grade, NAEP 2007

In which of the following are the three fractions arranged from least to greatest?

A. $\frac{2}{2}, \frac{1}{7}, \frac{5}{9}$

D. $\frac{5}{9}, \frac{1}{7}, \frac{2}{2}$

Just under one half of students tested chose the correct response.

B. $\frac{1}{2}, \frac{5}{7}, \frac{2}{9}$

E. $\frac{5}{9}, \frac{2}{7}, \frac{1}{2}$

C. $\frac{1}{2}, \frac{5}{9}, \frac{2}{7}$

If we want students to talk...

*we have to give them
something to talk about!*



Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Standards for Mathematical Content (CCSS)

Grade 3: Number and Operations – Fractions

Develop understanding of fractions as numbers.

Standards for Mathematical Content (CCSS)

Grade 4: Number and Operations – Fractions

Extend understanding of fraction equivalence and ordering.

Build fractions from unit fractions by **applying and extending previous understandings** of operations on whole numbers.

Understand decimal notation for fractions, and compare decimal fractions.

Standards for Mathematical Content (CCSS) Grade 5: Number and Operations – Fractions

Use equivalent fractions as a **strategy** to add and subtract fractions.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standards for Mathematical Content (CCSS)

Grade 6: The Number System

Apply and extend previous understandings
of multiplication and division to divide
fractions by fractions.

Apply and extend previous understandings
of numbers to the system of rational
numbers.

Standards for Mathematical Content (CCSS)

Grade 7: The Number System

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Standards for Mathematical Content (CCSS)

Grade 8: The Number System

Know that there are numbers that are not rational, and **approximate** them by rational numbers.

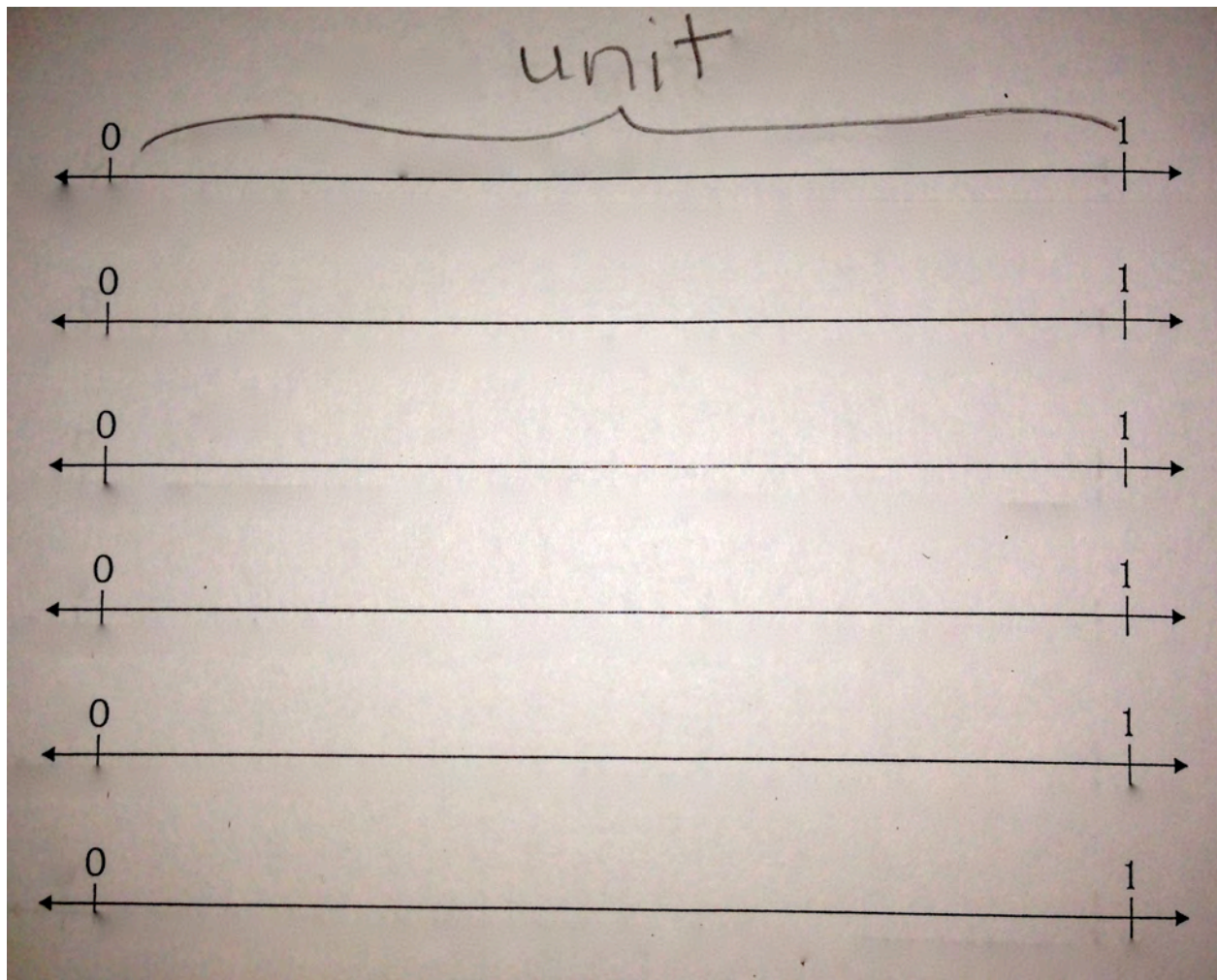
Strategies for supporting talk

- Building visual models

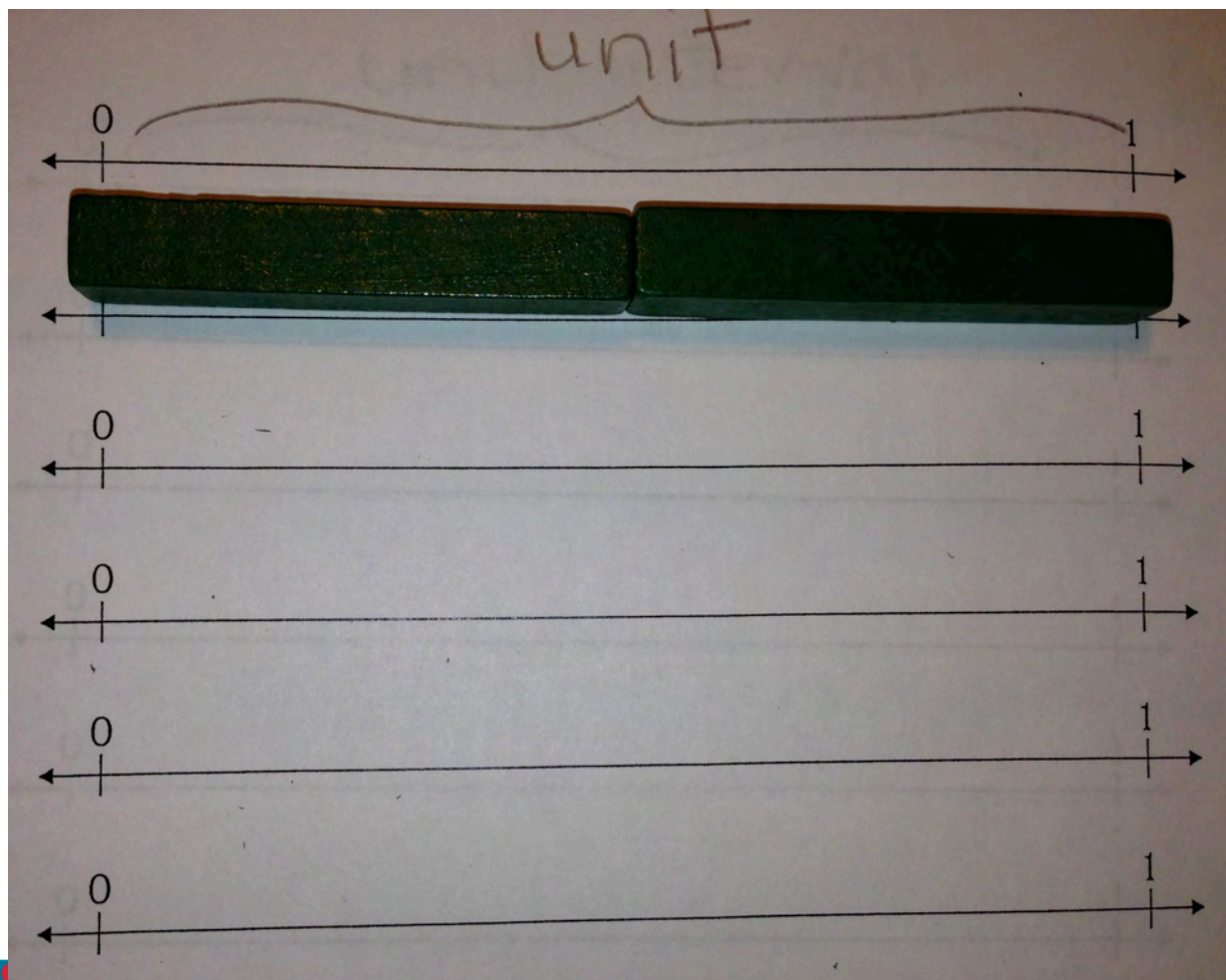
12-cm Number Lines



Identify the Unit Interval

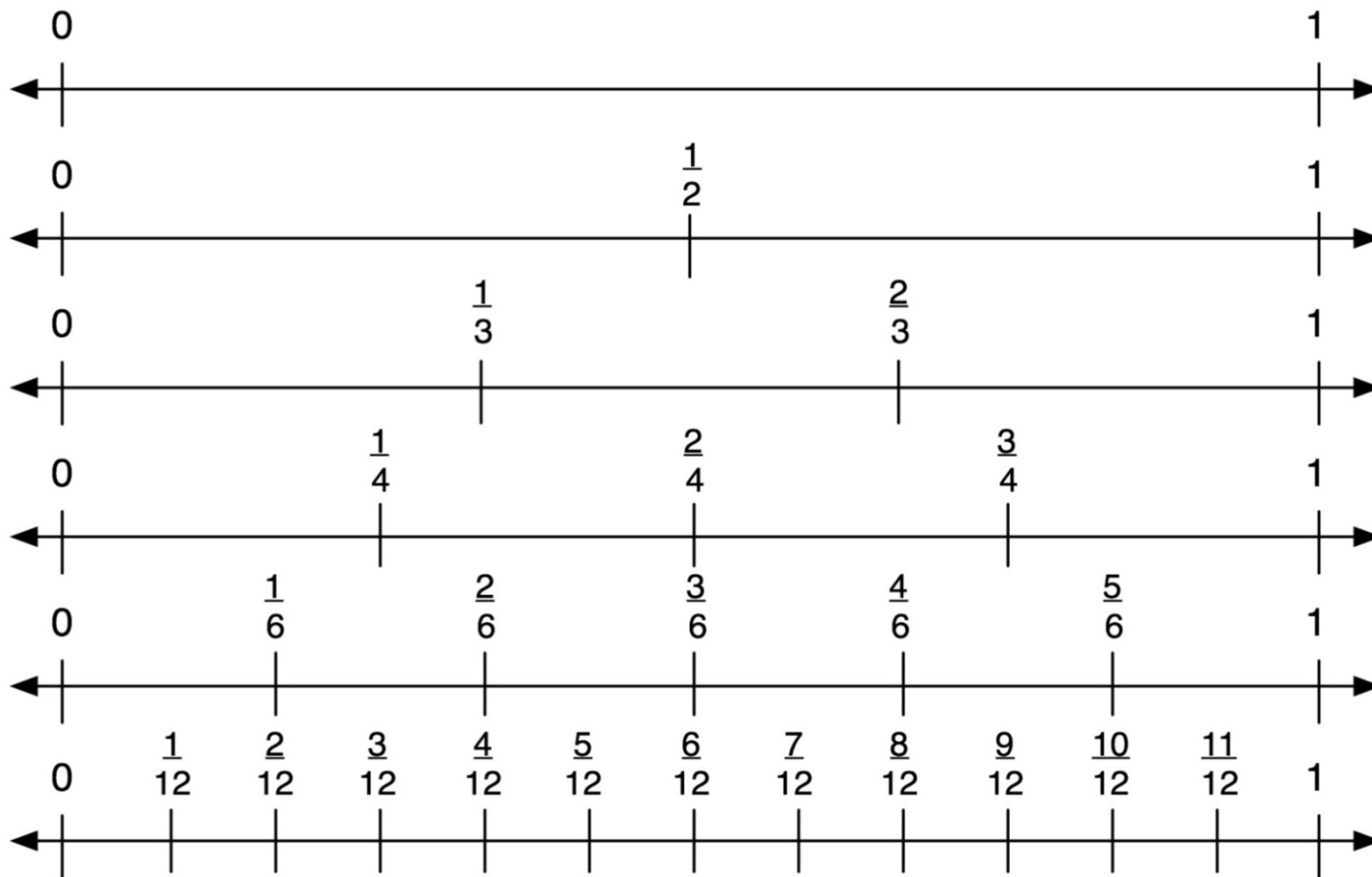


Find a rod that fits on the number line exactly two times.



Partitioning the Number Lines





Reasoning with the number line

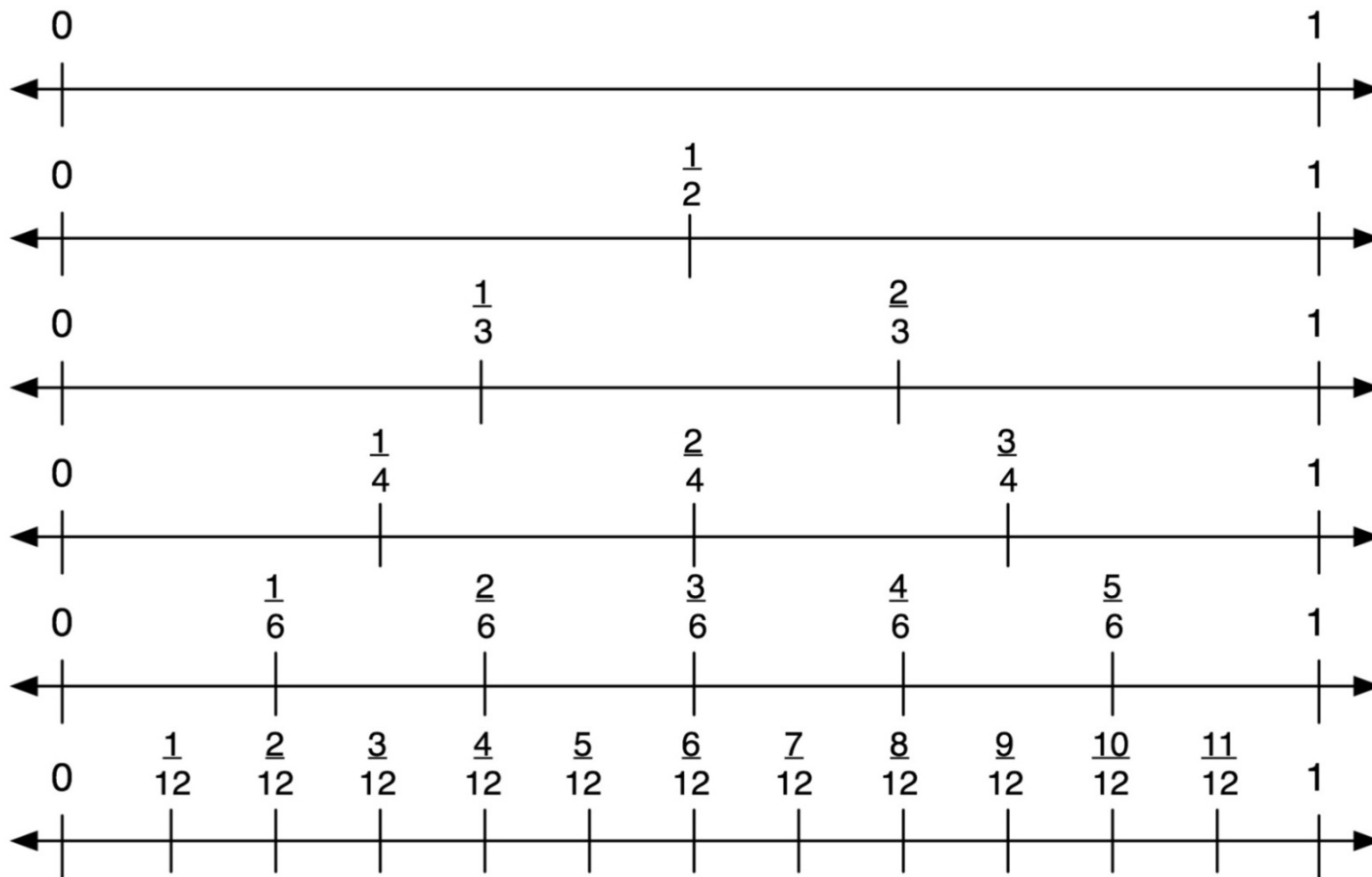
- Why are sixths smaller than fourths?
- What numbers are the same distance from zero as two-thirds?
- What number is halfway between zero and one-half?
- What would you call a number halfway between zero and one-twelfth?

Strategies for supporting talk

- Building visual models
- Reasoning with benchmarks

What do you notice about all of the fractions that equal $\frac{1}{2}$?

Jot down three statements and
share with your neighbor



Fractions that equal $\frac{1}{2}$

- The numerator is half the denominator
- The denominator is double the numerator
- You can divide the denominator by two to get the numerator ($d \div 2 = n$)
- You take half the denominator to get the numerator ($\frac{1}{2}d = n$)

Which is Greater?

$$\begin{array}{r} 3 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 5 \\ \hline 6 \end{array}$$

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Watch Malcolm

[https://mathreasoninginventory.com/Home/
VideoLibrary](https://mathreasoninginventory.com/Home/VideoLibrary)

Which is Greater?

$$\begin{array}{r} 5 \\ \underline{} \\ 12 \end{array}$$

$$\begin{array}{r} 5 \\ \underline{} \\ 8 \end{array}$$

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Watch Alberto

<https://mathreasoninginventory.com/Home/Practice>

Fractions  Alberto

Greater or Less Than 1?

$$\frac{11}{12} + \frac{1}{5} =$$

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Watch Alberto

<https://mathreasoninginventory.com/Home/VideoLibrary>

Turn and Talk

How did the students'
reasoning with benchmarks
support talk?

Strategies for supporting talk

- Building visual models
- Reasoning with benchmarks
- Strategic tasks

Order the following fractions
from least to greatest

$$\frac{7}{2}$$

$$\frac{7}{8}$$

$$3\frac{5}{9}$$

$$\frac{13}{4}$$

Video not available

Comparing $5/9$ to $1/2$

“Since 9 is an odd number I found it hard to compare it ($5/9$) to $1/2$, so I multiplied 9 and 5 by 2 to get $10/18$, & half of 18 is 9 and 10 is more than 9.”

Order the following fractions
from least to greatest.

$$\frac{7}{2}$$

$$\frac{7}{8}$$

$$3\frac{5}{9}$$

$$\frac{13}{4}$$

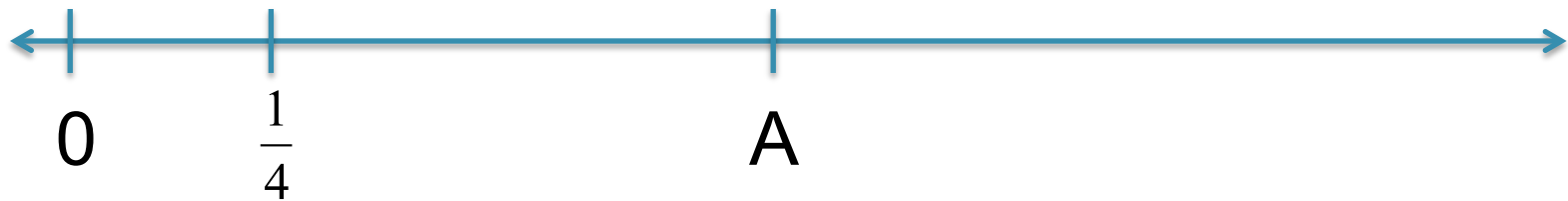
Turn and Talk

How did the numbers in the task encourage Jonah's reasoning and talk ?

Strategies for supporting talk

- Building visual models
- Reasoning with benchmarks
- Strategic tasks
- Teacher moves

Which is a better choice, $\frac{3}{5}$ or $\frac{7}{8}$, for the location marked A on the number line?



Placing Fractions on a Number Line

Clip 5.2 from *Classroom Discussions: Seeing Math Discourse in Action*, by Nancy Anderson, Suzanne Chapin, and Cathy O'Connor, Math Solutions, 2011

Turn and Talk

How did the teacher's moves
encourage talk?

Strategies for supporting talk

- Building visual models
- Reasoning with benchmarks
- Strategic tasks
- Teacher moves
- Records of thinking

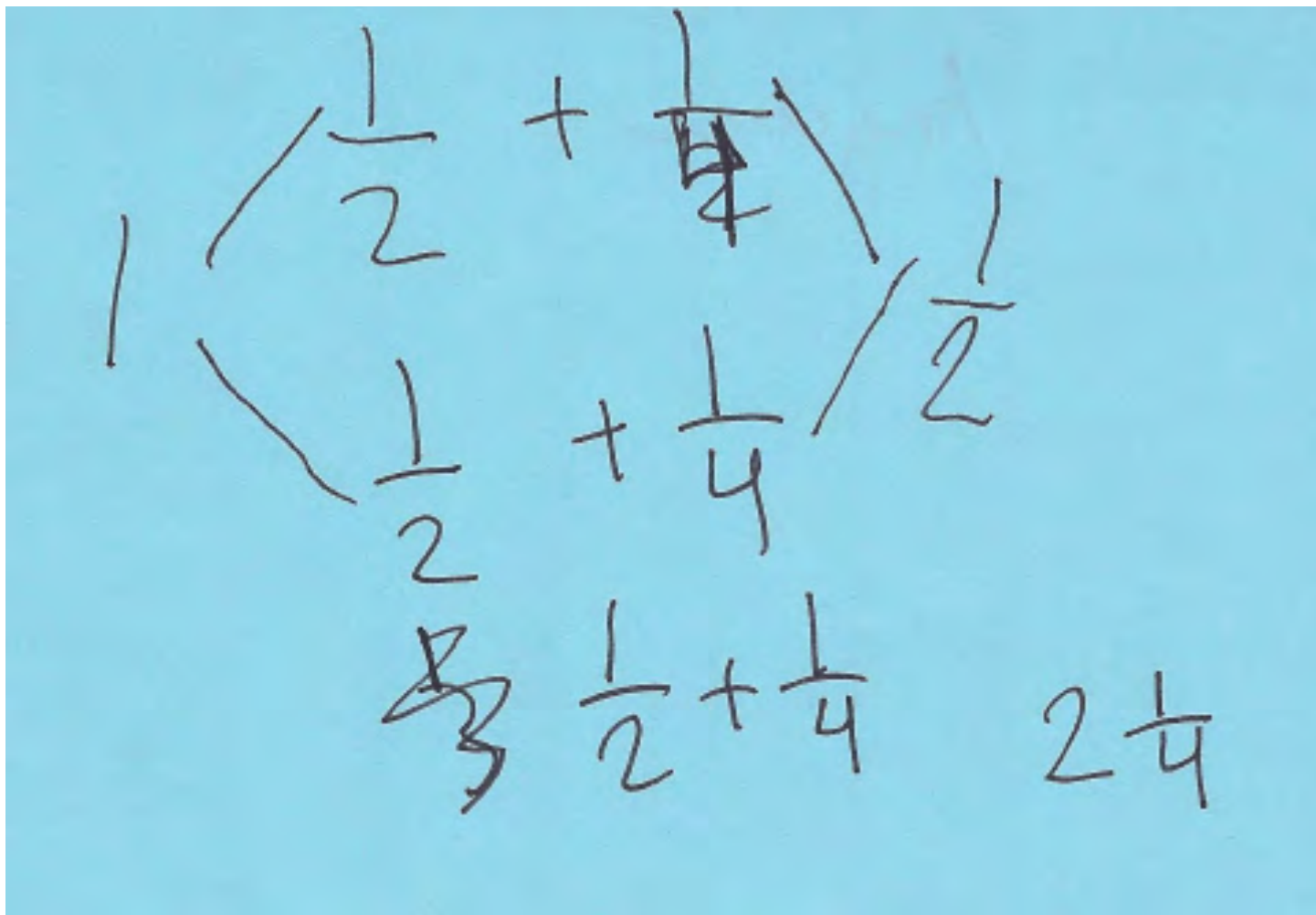
Multiplication

James baked three batches of cookies.
He used $\frac{3}{4}$ cup of sugar in each batch.
How much sugar did he use?

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Video not available

Understanding Multiplication



Handwritten mathematical work on a blue background showing the multiplication of $\frac{1}{2}$ by 2. The work includes several steps:

- $\frac{1}{2} + \frac{1}{2}$ (with a crossed-out $\frac{1}{4}$ above the second $\frac{1}{2}$)
- $\frac{1}{2} + \frac{1}{4}$ (with a $\frac{1}{2}$ to the right)
- $\frac{1}{2} + \frac{1}{4}$ (with a large bracket to the left and $2\frac{1}{4}$ to the right)

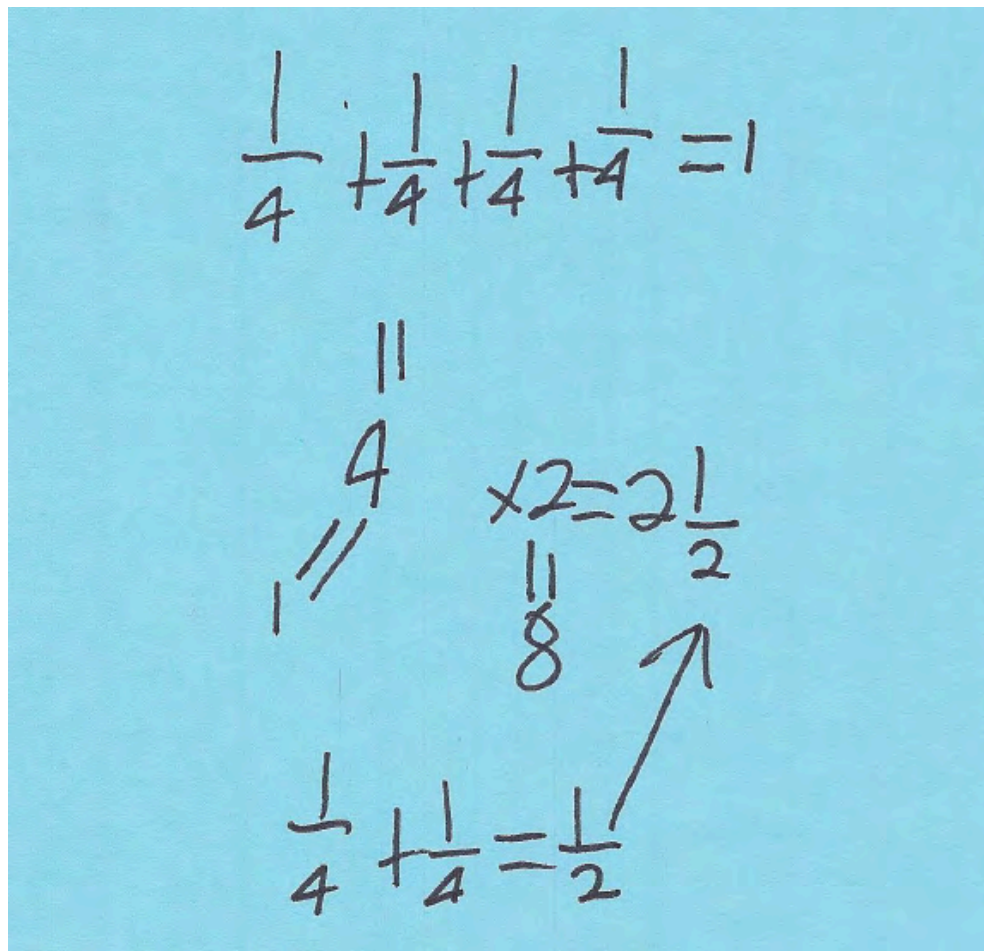
Division

I bought $2\frac{1}{2}$ pounds of hamburger meat. Each hamburger uses $\frac{1}{4}$ of a pound. How many hamburgers can I make?

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Video not available

Understanding Division



Handwritten math on a blue background:

$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$$

$$\begin{array}{r} 11 \\ 4 \overline{) 4} \\ 11 \end{array}$$

$$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$\begin{array}{r} \times 2 = 2\frac{1}{2} \\ 11 \\ 8 \end{array}$$

An arrow points from the $\frac{1}{2}$ in the bottom equation to the $\frac{1}{2}$ in the middle equation.

Turn and Talk

How did the students' records
of thinking support talk?

Strategies for supporting talk

- Building visual models
- Reasoning with benchmarks
- Strategic tasks
- Teacher moves
- Records of thinking

In Conclusion

“My definition of a good teacher has changed from “one who explains things so well that students understand” to “one who gets students to explain things so well that they can be understood.”

Steven C. Reinhart, “Never Say Anything a Kid Can Say!”
Mathematics Teaching in the Middle School 5, 8 [2000]: 478

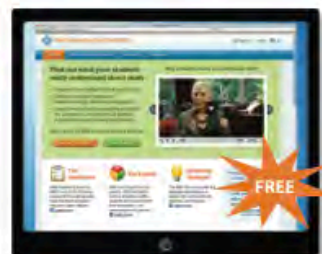
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Find Out What Your Students Really Understand About Math

 Math Reasoning Inventory™

Math Reasoning Inventory (MRI) is a **FREE** online formative assessment tool that focuses on students' numerical reasoning strategies and understandings. MRI addresses the basics that students need to be prepared for middle school math and algebra.

- Includes three **Assessments**—Whole Numbers, Decimals, and Fractions.
- Developed by **Marilyn Burns** and a team of Math Solutions master teachers.
- Asks questions that the **Common Core** expects all middle school students to answer successfully.



"MRI will certainly model what teachers should do as they follow the implementation of CCSS."

Shirley Faye,
Past President, NCTM
Mathematics Teacher and Leader

Common Core Connection

The emphasis on numerical reasoning relates directly to the processes and proficiencies in the eight **Standards for Mathematical Practice**.

- The content of the questions draw from three domains in the **Standards for Mathematical Content**: Operations & Algebraic Thinking; Number & Operations in Base Ten; and Number & Operations—Fractions.
- MRI responds to the **Common Core** recommendation for "a balanced combination of procedures and understanding."
- MRI directly addresses the **Common Core** caution that "students who lack understanding of a topic may rely on procedures too heavily."



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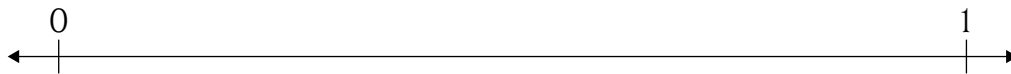
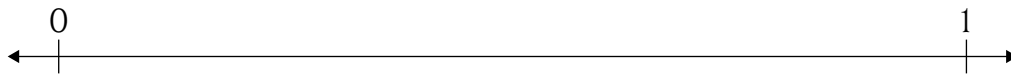
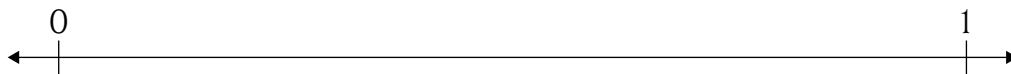
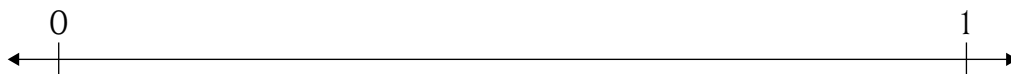
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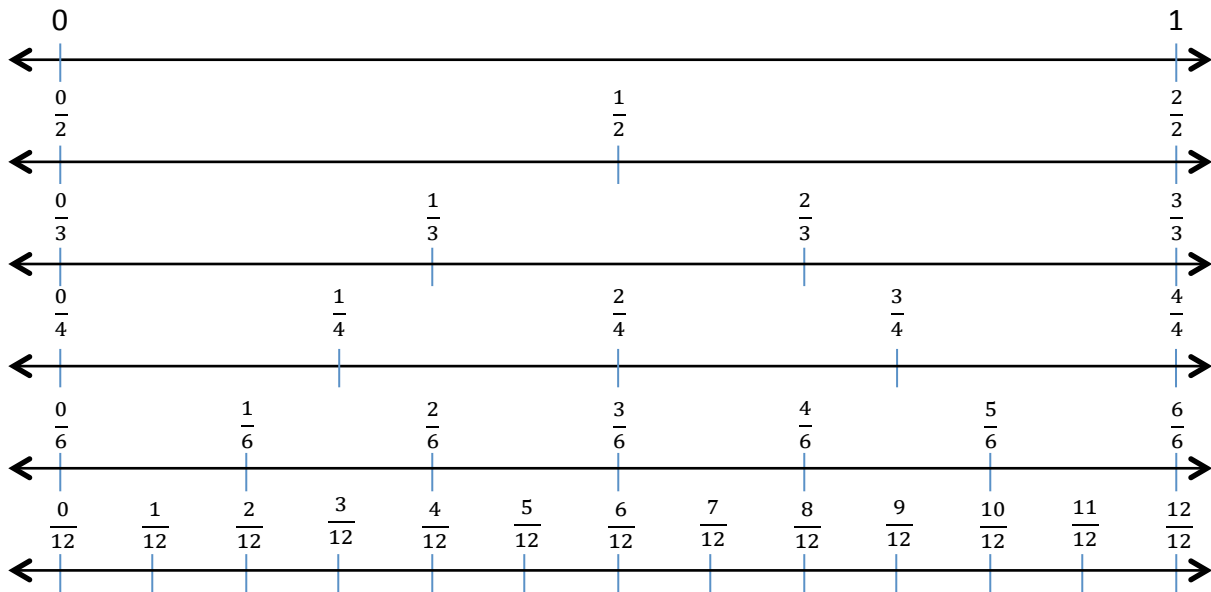
12-cm Number Lines

Name: _____

Date: _____



Fractions that Equal One Half



1. Circle all the fractions that equal $\frac{1}{2}$ and write them below:

2. What do you notice about their numerators and denominators?

3. What are some other fractions that equal $\frac{1}{2}$?

Which is a better choice, $\frac{3}{5}$ or $\frac{7}{8}$, for the location marked A on the number line?

