Lessons Learned from Interviews about Numerical Reasoning

Marilyn Burns

NCTM 2013, Denver, CO
Thursday, April 18, 2013
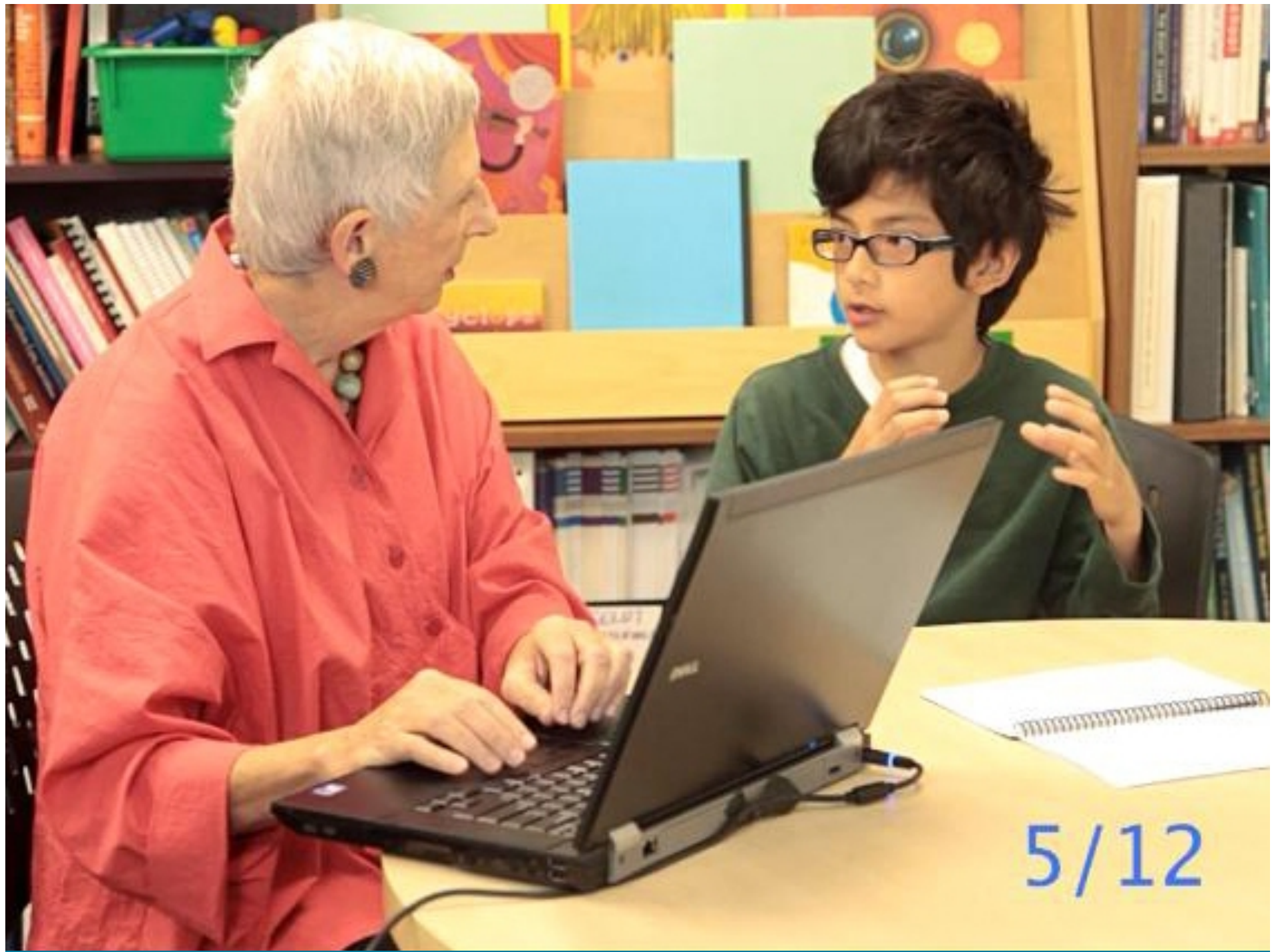11 am–noon

Math Solutions.
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MRI
Math Reasoning Inventory

Find out what students really understand about math

Funded by the Bill & Melinda Gates Foundation

Connecting MRI
and
Classroom Instruction

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So . . . what is MRI?

MRI

- Formative assessment tool

"When the cook tastes the soup, it's Formative Assessment. When the guest tastes the soup, it's Summative Assessment."

Jeane M. Joyner and Mari Muri. *2011.INFORMative Assessment*. Sausalito, CA: Math Solutions Publications. [Robert Stake. 2004. *Standards-Based and Responsive Evaluation*. Thousand Oaks, CA: Sage Publications.]

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MRI

- Formative assessment tool

- Addresses the Common Core

MRI asks questions that the Common Core expects all students entering middle school to be able to answer successfully.

15% & 20% TIP TABLE®

Check	15%	20%	Check	15%	20%
$1.00	$.15	$.20	$26.00	$3.90	$5.20
2.00	.30	.40	27.00	4.05	5.40
3.00	.45	.60	28.00	4.20	5.60
4.00	.60	.80	29.00	4.35	5.80
5.00	.75	1.00	30.00	4.50	6.00
6.00	.90	1.20	31.00	4.65	6.20
7.00	1.05	1.40	32.00	4.80	6.40
8.00	1.20	1.60	33.00	4.95	6.60
9.00	1.35	1.80	34.00	5.10	6.80
10.00	1.50	2.00	35.00	5.25	7.00
11.00	1.65	2.20	36.00	5.40	7.20
12.00	1.80	2.40	37.00	5.55	7.40
13.00	1.95	2.60	38.00	5.70	7.60
14.00	2.10	2.80	39.00	5.85	7.80
15.00	2.25	3.00	40.00	6.00	8.00
16.00	2.40	3.20	41.00	6.15	8.20
17.00	2.55	3.40	42.00	6.30	8.40
18.00	2.70	3.60	43.00	6.45	8.60
19.00	2.85	3.80	44.00	6.60	8.80
20.00	3.00	4.00	45.00	6.75	9.00
21.00	3.15	4.20	46.00	6.90	9.20
22.00	3.30	4.40	47.00	7.05	9.40
23.00	3.45	4.60	48.00	7.20	9.60
24.00	3.60	4.80	49.00	7.35	9.80
25.00	3.75	5.00	50.00	7.50	10.00

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MRI

- Formative assessment tool

- Addresses the Common Core

- Reveals what students do know, do not know, and should know

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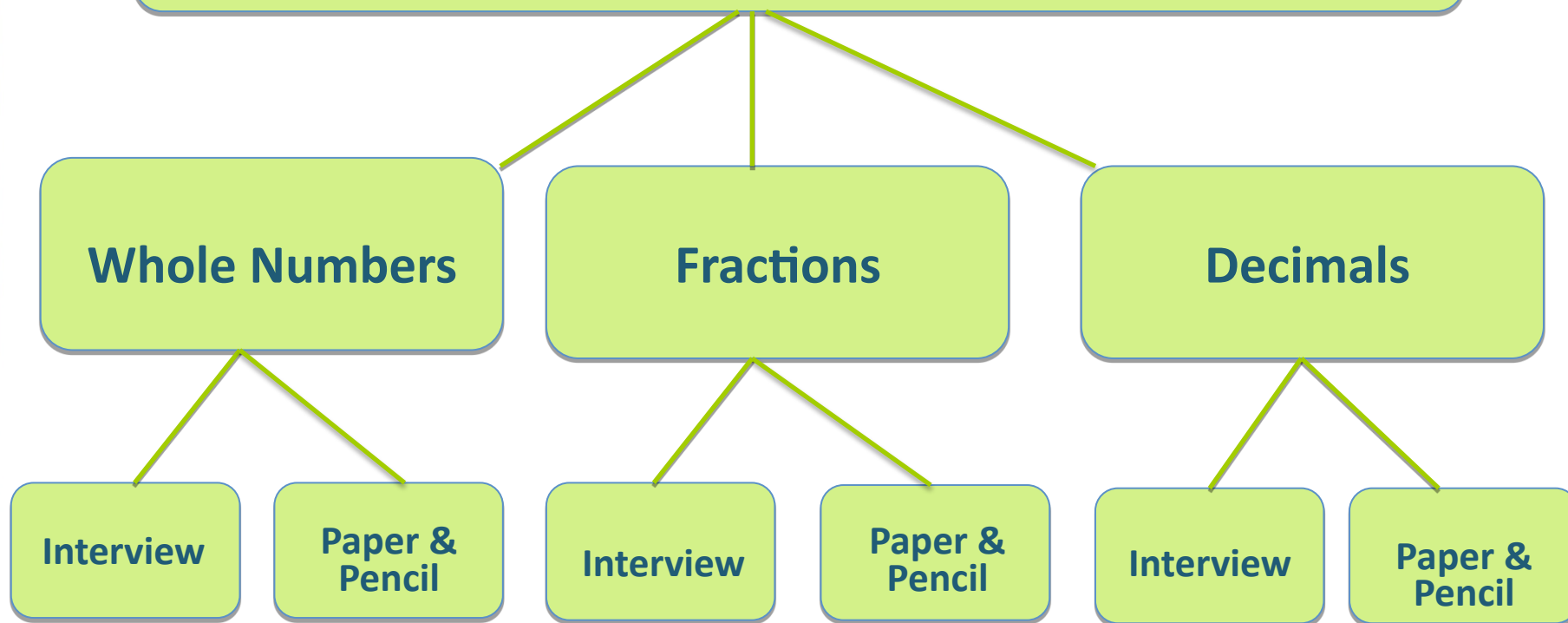- Available online, free of charge, to all teachers

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MRI provides instant reports

- Inform instruction

- Monitor progress

- Identify students in need of intervention

- Communicate with parents

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Individual Report: Whole Numbers

MR, E

Interview

Date Completed: 17-Nov-2011

Category	Strategies or Understandings Demonstrated	Appropriate Strategies or Understandings Not Demonstrated
Adding and Subtracting Mentally	➕ Breaks numbers apart to add or subtract $100 - 18$ ➕ Gives other reasonable explanation $1000 - 998$ ➖ Uses standard algorithm to add or subtract $99 + 17$	• Uses addition to solve subtraction problems • Uses benchmark numbers to add or subtract
Multiplying and Dividing Mentally	➕ Uses known facts and place value to multiply or divide $7000 \div 70$ ➕ Breaks numbers apart to multiply or divide 15×12 ➕ Gives other reasonable explanation 60×40 ➖ Guesses, does not explain, or gives faulty explanation Estimate 18×21	• Uses benchmark numbers to make estimates
Applying Understanding	➕ Uses distributive property 15×12 $20 \times 15 = 300, 21 \times 15 = \_\_\_$	• Models with mathematics to solve problems in context • Uses inverse relationship of addition and subtraction

Written Computation

Date Completed: 17-Nov-2011

Category	Demonstrated	Not Demonstrated
Computing Accurately with Paper and Pencil	$5000 - 328$ 842×35 $3423 \div 6$ $275 \div 22$	

Group Report: Whole Numbers

All Students

Start Date: 15-Aug-2011
End Date: 25-Nov-2011

Legend

➕ Appropriate for the numbers at hand
➖ Not Appropriate for the numbers at hand

Interview

Students: 8

Category	Consistently Demonstrated (75–100% of Students)	Often Demonstrated (50–74% of Students)	Sometimes Demonstrated (25–49% of Students)	Rarely Demonstrated (0–24% of Students)
Adding and Subtracting Mentally	➕ Breaks numbers apart to add or subtract (75%)	➕ Uses benchmark numbers to add or subtract (62%) ➖ Uses standard algorithm to add or subtract (50%)	➕ Uses addition to solve subtraction problems (38%)	➕ Gives other reasonable explanation (12%) ➖ Counts by 1s (12%)
Multiplying and Dividing Mentally		➕ Uses known facts and place value to multiply or divide (50%) ➕ Breaks numbers apart to multiply or divide (50%) ➕ Gives other reasonable explanation (50%) ➖ Uses standard algorithm to multiply or divide (62%)	➕ Uses benchmark numbers to make estimates (25%) ➖ Figures exact answer when asked to estimate (25%)	
Applying Understanding		➕ Uses distributive property (62%) ➕ Models with mathematics to solve problems in context (50%)	➕ Uses inverse relationship of addition and subtraction (38%)	

Written Computation

Students: 7

Category	75–100% of Students	50–74% of Students	25–49% of Students	0–24% of Students
Computing Accurately with Paper and Pencil	$5000 - 328$ (86%) 842×35 (86%) $3423 \div 6$ (86%)	$275 \div 22$ (71%)		

Item Analysis: Whole Numbers

Period 1

Start Date: 30-Sep-2011
End Date: 28-Oct-2011

Interview

Students: 7

	Question	Correct / Self-corrected	Incorrect	Did Not Answer	Strategies Used by Students Who Gave Correct Answers
1	1000 – 998	100%	0%	0%	➕ Uses addition to solve subtraction problems (6/7) ➖ Guesses, does not explain, or gives faulty explanation (1/7)
2	99 + 17	43%	57%	0%	➕ Breaks numbers apart to add or subtract (1/3) ➖ Uses standard algorithm to add or subtract (2/3)
3	100 – 18	57%	29%	14%	➕ Breaks numbers apart to add or subtract (2/4) ➖ Uses standard algorithm to add or subtract (1/4) ➖ Counts by 1s (1/4)
4	15 + ___ = 200	71%	14%	14%	➕ Breaks numbers into parts to add or subtract (4/5) ➖ Uses standard algorithm to add or subtract (1/5)
5	20 × 15 = 300, 21 × 15 = ____	0%	43%	57%	No correct answers given
6	60 × 40	43%	57%	0%	➕ Uses known facts and place value to multiply or divide (3/3)
7	15 × 12	29%	43%	29%	➖ Uses standard algorithm to multiply or divide (2/2)
8	7000 ÷ 70	57%	14%	29%	➕ Uses known facts and place value to multiply or divide (4/4)
9	Estimate 18 × 21	57%	43%	0%	➕ Relates to benchmark number to make estimates (1/4) ➖ Figures exact answer when asked to estimate (1/4) ➖ Guesses, does not explain, or gives faulty explanation (2/4)
10	295 students, 25 on each bus	0%	43%	57%	No correct answers given

Written Computation

Students: 7

	Question	Correct / Self-corrected	Incorrect	Did Not Answer
1	5000 – 328	100%	0%	0%
2	842 × 35	57%	29%	14%
3	3423 ÷ 6	71%	14%	14%
4	275 ÷ 22	0%	43%	57%

Solutions.
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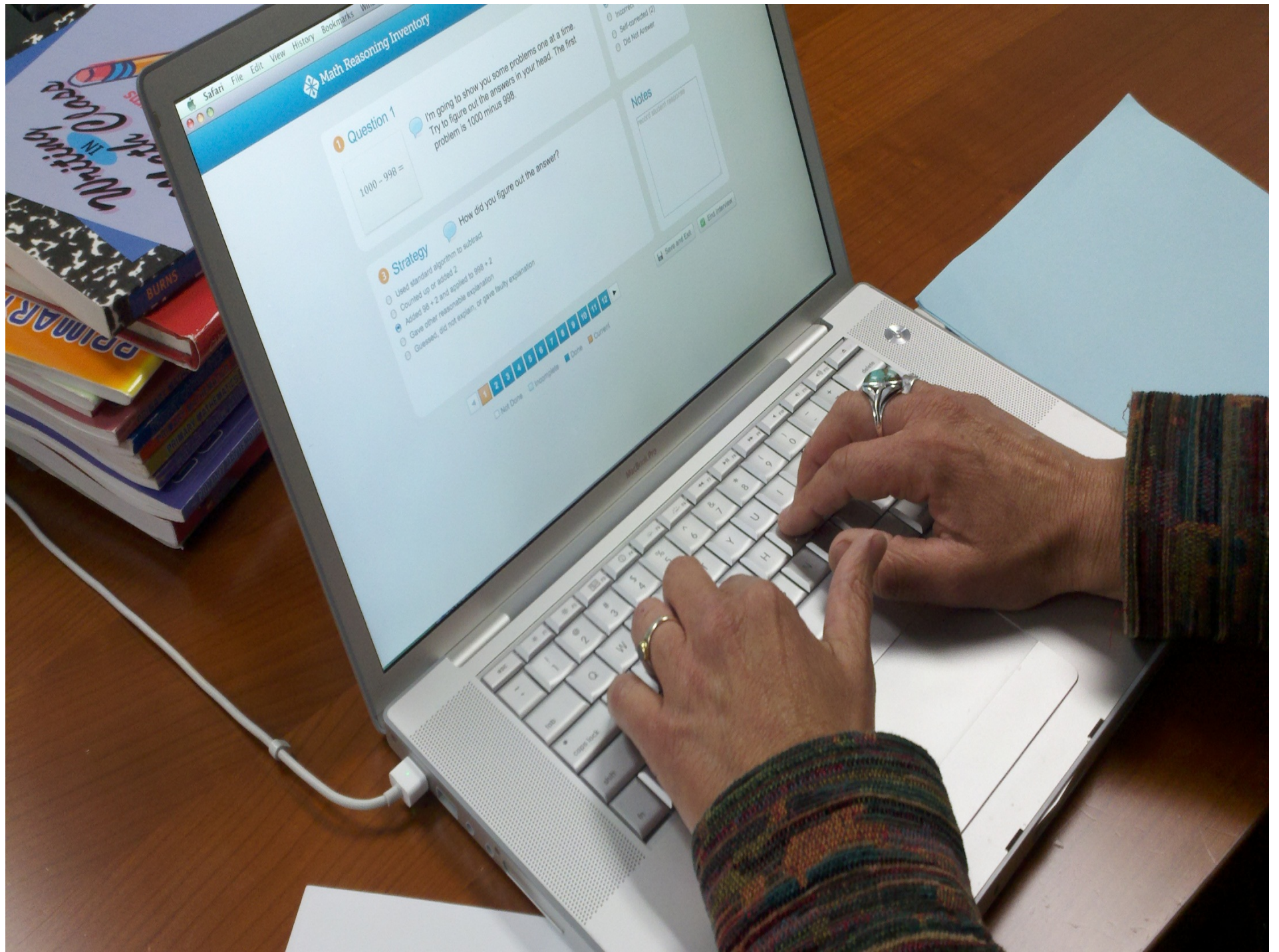# *Reasoning* is the heart of MRI.

The *Interview* is the core of MRI.

- We ask . . .

- We listen . . .

- We learn . . .

Math Solutions.
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MRI and Classroom Instruction

1. **99 + 17 Number Talk**

2. Introducing Multiplication

3. 15 x 12 and the Distributive Property

4. 12.6 x 10

5. MRI K-5

6. A Math Investigation

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Math Reasoning Inventory

Question 1

I'm going to show you some problems one at a time. Try to figure out the answers in your head. The first problem is 1000 minus 998.

1000 − 998 =

How did you figure out the answer?

Strategy

- Used standard algorithm to subtract
- Counted up or added 2
- Added 98 + 2 and applied to 998 + 2
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

- Incorrect
- Self-corrected (2)
- Did not Answer

Notes

Record student response

Save and Exit   End Interview

1 2 3 4 5 6 7 8 9 10 11 12 ▶

☐ Not Done   ☐ Incomplete   ■ Done   ■ Current

1 Question 2

What is 99 plus 17?

$$99 + 17$$

2 Answer

- Correct (116)
- Incorrect [ ]
- Self-corrected (116)
- Did Not Answer

3 Explanation   How did you figure out the answer?

- Counted on by 1s
- Used standard algorithm to add
- Added 90 + 10, 9 + 7, and then 100 + 16
- Added 99 + 10 and then 109 + 7
- Added 100 + 17 and then subtracted 1
- Changed problem to 100 + 16
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

Notes

record student response

◄ 1 **2** 3 4 5 6 7 8 9 10 ►

☐ Not Done   ☐ Incomplete   ■ Done   ■ Current

💾 Save and Exit   ✅ End Interview

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99 + 17

99 + 17

Lauren / Alessandra

$99 + 10 = 109$

$109 + 7 = 116$

Dylan

$100 + 17 = 117$

$117 - 1 = 116$

Caleb

99 is 1 less than 100.

$17 - 1 = 16$

$99 + 1 = 100$

$100 + 16 = 116$

Jake

9 plus any number
is 1 less than the
number you had.

$(9 + 7 = 16)$

6 is 1 less than
the 7

The answer had to
end in 6.

So I knew the
answer had to be 116.

Eliane

$9 + 7 = 16$

$90 + 10 = 100$

$16 + 100 = 116$

Lindsay

① $9 + 7 = 16$

$16 + 10 = 26$

$90 + 26 = 116$

② $90 + 17 = 107$

$107 + 9 = 116$

99 + 17
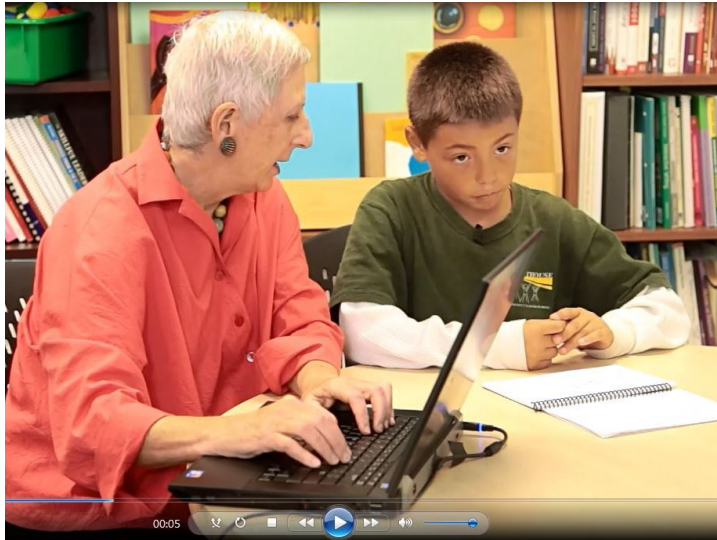
Alberto

Manuel

Dina
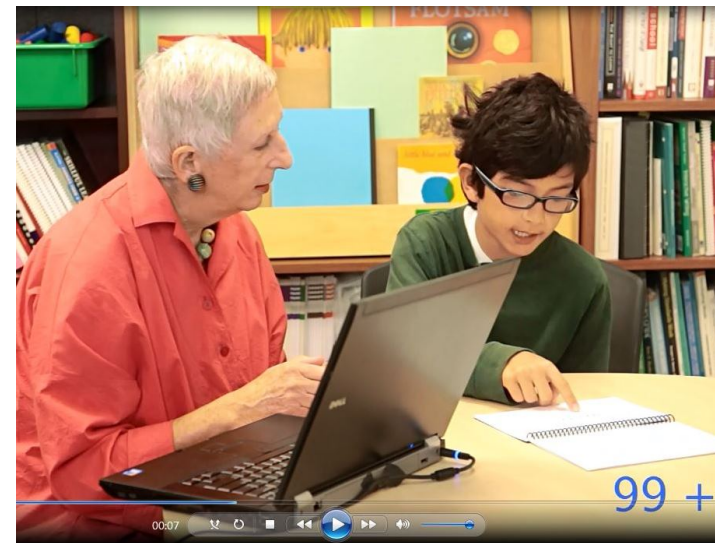
Amir

Alberto

- Video of Alberto solving 99 + 17 on MRI site

$$99 + 17$$

Lauren/Alessandra

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Dylan

$100 + 17 = 117$
$117 - 1 = 116$

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- Video of Dina solving 99 + 17 on MRI site

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$107 + 9 = 116$

Common Core Connection

Grade 3

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

CCSS.Math.Content.3.NBT.A.2 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. **Construct viable arguments and critique the reasoning of others.**
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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Using MRI in Classroom Instruction

1. 99 + 17 Number Talk

2. **Teaching Multiplication**

3. 15 x 12 and the Distributive Property

4. 12.6 x 10

5. MRI K-5

6. An Investigation

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Teaching Arithmetic INTRODUCING MULTIPLICATION, Grade 3

Chapter Four: Multiplication Stories

Estimate

✓10
✓20
✓30
✓40
50
60
70
80
90
100
more than 100
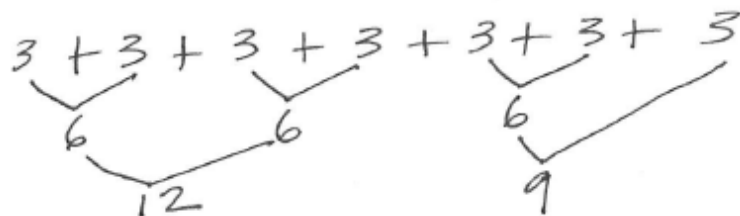
Equation: $7 \times 3 = \square$

Problem: There are 7 tricycles. How many wheels are there altogether?

Figuring

1. (OOO) (OOO) (OOO) (OOO) (OOO) (OOO) (OOO)

2. 3, 6, 9, 12, 15, 18, 21

3. $3 \times 3 = 9$ (3 tricycles).
   $3 \times 3 = 9$ (3 more tricycles)
   $9 + 9 = 18$ (6 tricycles)
   $18 + 3 = 21$

4. 3 + 3 + 3 + 3 + 3 + 3 + 3
   6   6   6
        12   9

   $12 + 10 = 22$
   $12 + 9 = 21$

   $7 \times 3 = 21$

Multiplication

Estimate

Equation: $8 \times 4 = \square$

Problem:

10
20
30
40
50
60
70
80
90
100
>100

Figuring:

There was the usual confusion that happens when students do anything for the first time.

Multiplication

Nove, 21, 2000

Estimate

Equation 8 x 4 [32]

Problem There was 8 dogs and they all had 4 legs. How many legs all together?

Figuring

100

20

√30

40

50

60

70

80

90

100

more than
100

$4+4+4+4=16 + 4 + 4 + 4$

$16+4+4+4+4=32$

$4 \times 4 + 8 = 1$

$4 \times 4 + 8 = 1$

4+4 4+4 4+4 4+4

8   8   8   8
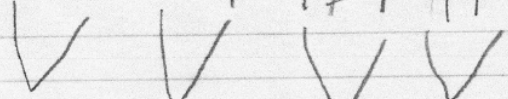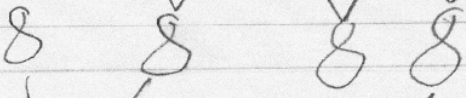
16   16

$(16+4=16$
$=20)$
$20+4=24$
$24+4=28$
$28+4=32$

$\begin{array}{r} 16 \\ +16 \\ \hline 32 \end{array}$

16

$\begin{array}{r} 16 \\ 16 \ 12 \\ \hline 32 \end{array}$

$\begin{array}{r} +16 \\ 16 \ 12 \\ \hline 32 \end{array}$

estimate **Multiqlicqtoh** NOV. 21,2

○ 10

equation 8×4=□

20 **problem**

There were 8 cars. How
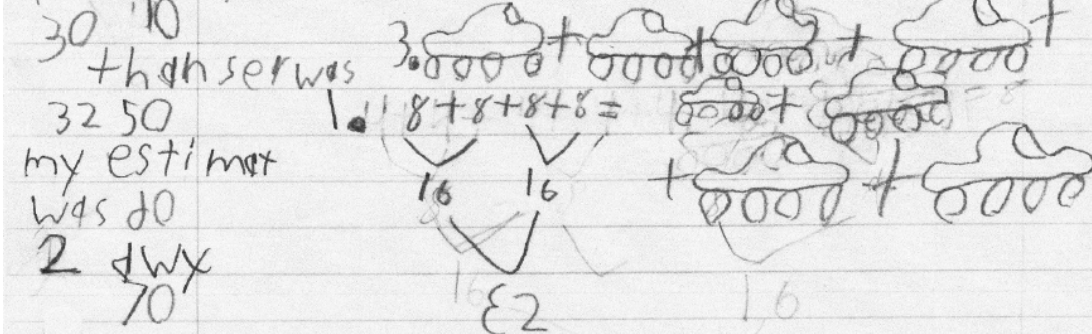
∨ 30 meny wels were thar

I estimated

30 '10

thah serwas 3. ⬜⬜⬜⬜ + ⬜⬜⬜⬜ ⬜⬜⬜⬜ + ⬜⬜⬜⬜ +

32 50   1. 8+8+8+8 =   ⬜⬜⬜⬜ + ⬜⬜⬜⬜

my estimat

was 20   16   16   + ⬜⬜⬜⬜ + ⬜⬜⬜⬜

2 ∠wx

70   82   16

80

PO   2. 4+4+4+4+4+4+4+4 =

100   4+4=8 8   8   8   8

> 100   4+4=8 16

4+4=8   16

4+4=8

8+8=16   32

8+8=16

16+16=32

Multiplication

$8 \times 4 = \square$

Estimate

Probolom

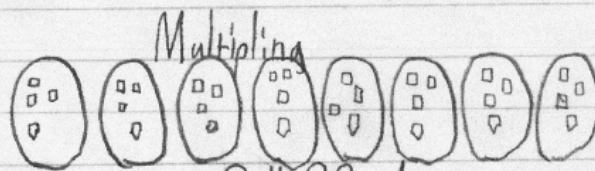There were eight baseball dimonds. 4 bases on each dimond. How many bases in all? figuring

10

20

Counting Countings

√30* 1.

 = 32 bases

40

2. ### Multipling
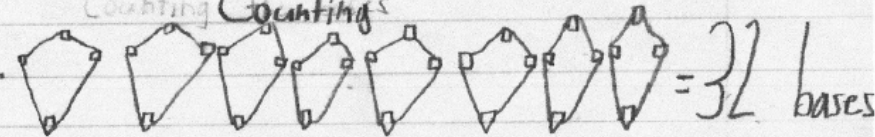
50

60 $8 \times 4 = 32$ bases in all circle
$4 \times 8 = 32$ $4 \times 8 = 32$

70 3. ### Adding
$4+4+4+4+4+4+4+4=32$ $4+4+4+4+16=32$

80 $8+8+8+8=32$

4. ### Figuring
$8 \times 4 = 32$ $4 \times 8 = 32$ $4+4+4+4+16=32$

90 $16+16=32$ $4+4+4+4+4+4+4+4=32$ $8+8+8+8=32$

100 all of the ways I figured out the
more than 100 problom every ansur was 32.

* My estamate was 2 off the ansur.

Standards for Mathematical Practice

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Common Core Connection

Multiplication, Grade 3

Introduction

Students develop an understanding of the meaning of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models.

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Common Core: Multiplication, Grade 3

Operations & Algebraic Thinking

Represent and solve problems involving multiplication and division.

- OA.A.1 **Interpret products** of whole numbers; e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.*

- OA.A.3 Use multiplication and division within 100 to **solve word problems** in situations involving equal groups, arrays, and measurement quantities; e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

- OA.A.4 **Determine the unknown** whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = __ ÷ 3, 6 x 6 = ?*

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Using MRI in Classroom Instruction

1. 99 + 17 Number Talk

2. Teaching Multiplication

3. **15 x 12 and the Distributive Property**

4. 12.6 x 10

5. MRI K-5

6. An Investigation

Common Core Connection

Grade 4

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

CCSS.Math.Content.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations.

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Common Core Connection

Grade 4

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Use place value understanding and properties of operations to perform multi-digit arithmetic.

CCSS.Math.Content.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and **multiply two two-digit numbers, using strategies based on place value and the properties of operations.**

Distributive Property of Multiplication over Addition

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Common Core Connection

Grade 5

Operations & Algebraic Thinking

Write and interpret numerical expressions.

CCSS.Math.Content.5.OA.A.1 Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

CCSS.Math.Content.5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product*.

Math Solutions®
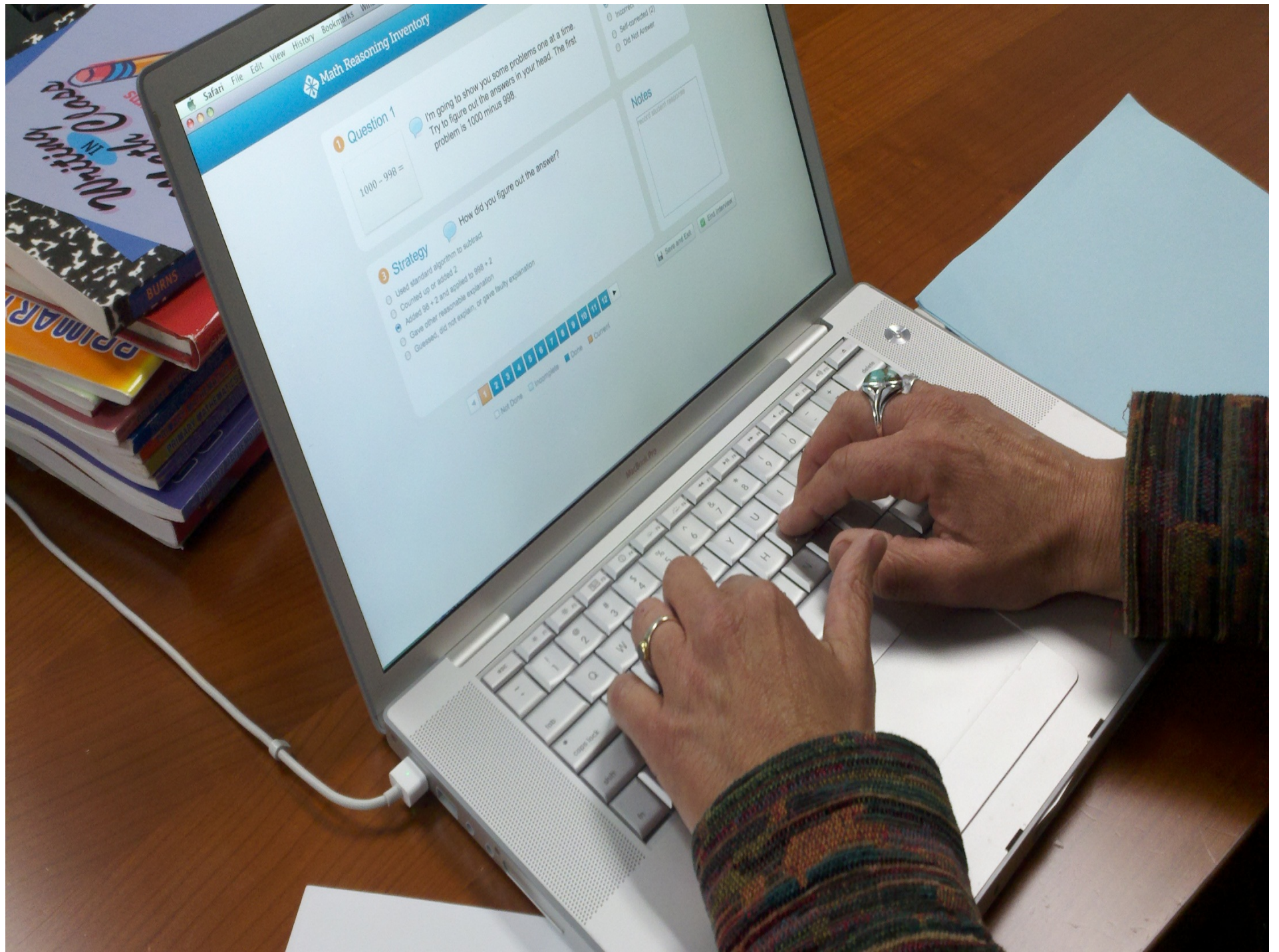FOUNDED BY MARILYN BURNS

Common Core Connection

Grade 6

Expressions & Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

CCSS.Math.Content.6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

1 Question 7

15 × 12 =

💬 What is 15 times 12?

2 Answer

- ○ Correct (180)
- ○ Incorrect ____
- ○ Self-corrected (180)
- ○ Did Not Answer

3 Explanation 💬 How did you figure out the answer?

- ○ Used standard algorithm to multiply
- ○ Broke 15 and/or 12 into parts and then multiplied (e.g., 15 x 10 and then 15 x 2)
- ○ Changed to an easier problem, 30 x 6, by doubling and halving
- ○ Gave other reasonable explanation
- ○ Guessed, did not explain, or gave faulty explanation

Notes

record student response

◄ 1 2 3 4 5 6 **7** 8 9 10 ►

☐ Not Done ☐ Incomplete ■ Done ■ Current

💾 Save and Exit ✅ End Interview

15 x 12

15 x 12

Monica

Malcolm

Alberto

Monica: 15 x 12

- Video on MRI site

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Monica: 15 x 12

"I did 15 times 10 and it was 150.

And then I did 15 times 2 which is 30.

And it was . . . um . . . 180."

Monica: 15 x 12

"I did 15 times 10 and it was 150.

And then I did 15 times 2 which is 30.

And it was . . . um . . . 180."

12 = 10 + 2

15 x 10 = 150

15 x 2 = 30

150 + 30 = 180

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Malcolm: 15 x 12

- Video on MRI site

Malcolm: 15 x 12

"I broke apart the 15 and did 10 plus 5.

And then I did 10 times 12 which equals 120.

And then I did 12 times 5 which equals 60

And then I added it all together and I got 180."

Malcolm: 15 x 12

"I broke apart the 15 and did 10 plus 5.

And then I did 10 times 12 which equals 120.

And then I did 12 times 5 which equals 60

And then I added it all together and I got 180."

15 = 10 + 5

10 x 12 = 120

12 x 5 = 60

120 + 60 = 180

Alberto: 15 x 12

- Video on MRI site

Alberto: 15 x 12

"I did 12 times 12 is 144

And then I did 3 times 12 and I got 36

And then I did 144 plus 36."

Alberto: 15 x 12

"I did 12 times 12 is 144

And then I did 3 times 12 and I got 36

And then I did 144 plus 36."

15 = 12 + 3

12 x 12 = 144

3 x 12 = 36

144 + 36 = 180

15 x 12

Monica

$(15 \times 10) + (15 \times 2) = 180$

Malcolm

$(10 \times 12) + (12 \times 5) = 180$

Alberto

$(12 \times 12) + (3 \times 12) = 180$

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. **Construct viable arguments** and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. **Look for and make use of structure.**

8. Look for and express regularity in repeated reasoning.

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15 x 12

Monica

Malcolm

Alberto

Cecilia

Cecilia: 15 x 12

- Video on MRI site

Cecilia: 15 x 12

"First I'm breaking it into steps and I'm doing 5 times 2. I leave the zero here and I bring the 1 up here.

Then 2 times 1 is 2, plus 1 is 3, so that's 30.

Put a zero. Five times 1 is 5, and then 1 x 1 is 1.

So then the answer is 180."

$$
\begin{array}{r}
15 \\
\times\ 12 \\
\hline
30 \\
150 \\
\hline
180
\end{array}
$$

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Common Core Connection

Grade 5

Number and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

CCSS.Math.Content.5.NBT.B.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

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Common Core Recommendation:

. . . a "balanced combination of procedures and understanding."

Common Core Caution:

. . . "students who lack understanding of a topic may rely on procedures too heavily."

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Using MRI in Classroom Instruction

1. 99 + 17 Number Talk
2. Teaching Multiplication
3. 15 x 12 and the Distributive Property
4. **12.6 x 10**
5. MRI K-5
6. An Investigation

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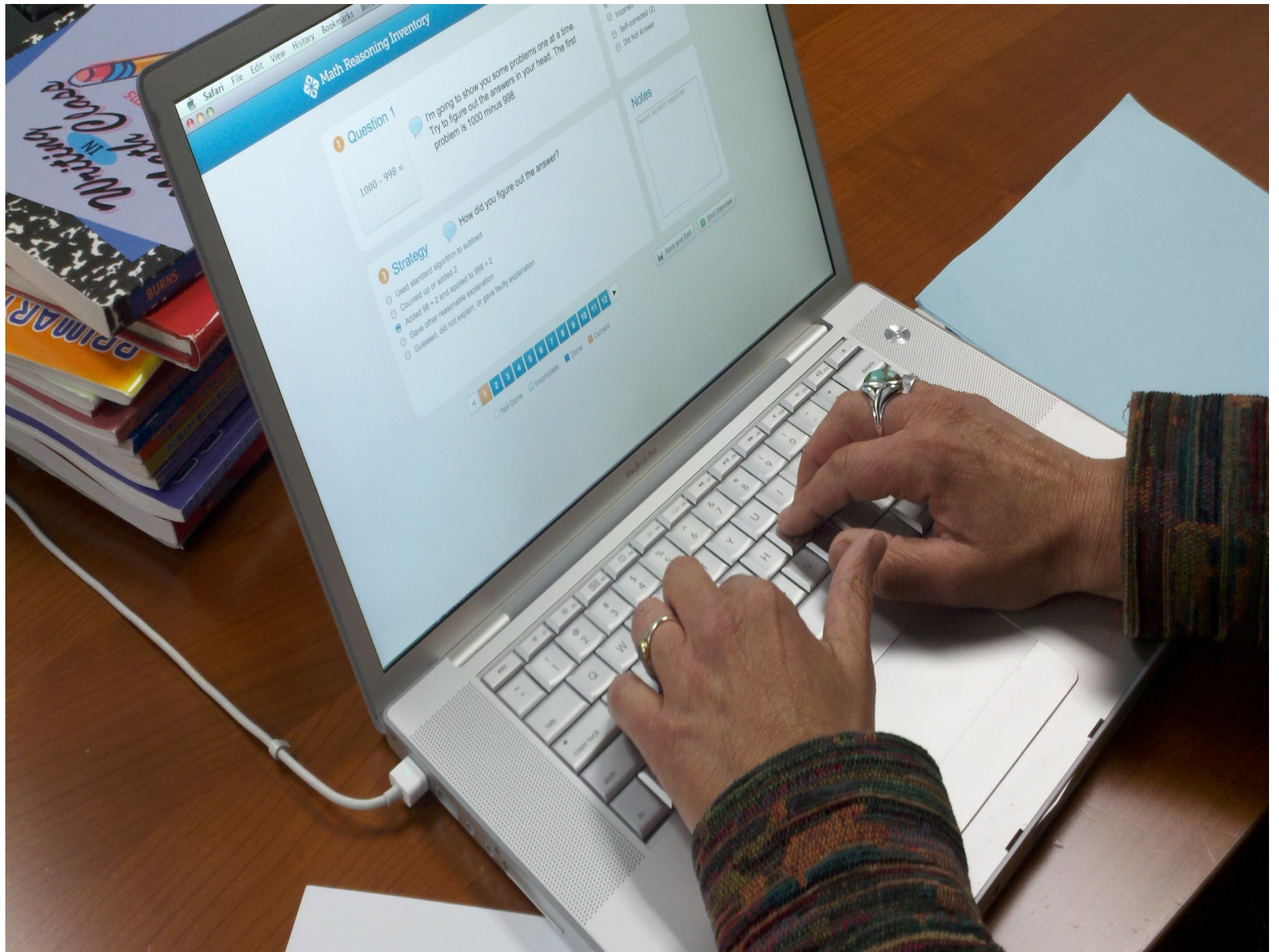
1 Question 11

Rectangular Snip

Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?

Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?

2 Answer

- ○ Correct (30)
- ○ Incorrect [ ]
- ○ Self-corrected (30)
- ○ Did Not Answer

3 Explanation   How did you figure out the answer?

- ○ Used standard algorithm to multiply
- ○ Multiplied 20 x 1 and then 20 x 0.5
- ○ Multiplied 1.5 x 2 and then 3 x 10
- ○ Multiplied 1.5 x 10 and then 15 x 2
- ○ Multiplied 15 x 2 and then adjusted the decimal point
- ○ Gave other reasonable explanation
- ○ Guessed, did not explain, or gave faulty explanation

Notes

record student response

| ◄ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | ► |

☐ Not Done   ☐ Incomplete   ■ Done   ■ Current

💾 Save and Exit   ✓ End Interview

Molly Problem

Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?

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Common Core Connection

Grade 5

Number and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

CCSS.Math.Content.5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

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Molly Problem

Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?

20 x 1.5 = ____

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20 x 1.5 = ___

How would you figure out the answer using the Distributive Property of Multiplication over Addition?

Sergio: Molly problem, 20 x 1.5

- Video on MRI site

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Sergio: Molly problem, 20 x 1.5

"I know that 20 times 1 is 20, so I put the 20 aside. And 20 times 5 is 100, and bloop it by one is just zero . . . 10.0. So 20 plus 10 is 30."

20 x 1 = 20

20 x 5 = 100, so 20 x .5 is 10.0

20 + 10 = 30

20 x 1.5 = (20 x 1) + (20 x .5)

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Using MRI in Classroom Instruction

1. 99 + 17 Number Talk

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3. 15 x 12 and the Distributive Property

4. 12.6 x 10

5. **MRI K-5**

6. An Investigation

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What we've learned from MRI

- MRI K–5

- Mental reasoning should be integral to math instruction

- Intervention is needed for students who need to catch up

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Using MRI in Classroom Instruction

1. 99 + 17 Number Talk

2. Teaching Multiplication

3. 15 x 12 and the Distributive Property

4. 12.6 x 10

5. MRI K-5

6. **An Investigation**

Multiplication

Estimate

Equation: $8 \times 4 = \square$

10
20
30
40
50
60
70
80
90
100
more than 100

Problem :

Figuring :

How I figured out my estimate.

$7 \times 3 = 21$

7 is one less than 8

and

3 is one less than 4

one more than 2 is 3

and

one more than 1 is 2

and

three and two is 32.

$8 \times 4 = 32$

7 X 3 = 21

8 X 4 = 32

7 X 3 = 21

8 X 4 = 32

6 X 2 =

7 X 3 = 21

8 X 4 = 32

6 X 2 = 12

7 X 3 = 21

8 X 4 = 32

6 X 2 = 12

7 X 3 = 23

8 X 4 = 32

"Hey. It wouldn't work. I don't get it. It worked over here."

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Teaching Arithmetic
Lessons for INTRODUCING MULTIPLICATION, Grade 3
Chapter 4

"While her reasoning had no mathematical grounding, it worked in this particular instance."

7 X 3 = 21

8 X 4 = 32

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