NCTM

Making Reasoning Integral to Instruction Focused on Number & Operations

Marilyn Burns Thursday, April 10, 2014



First a math problem





Lauren

99+10=109

109+7=116

Lauren/Alesandra

99+10=109

109+7= 116

Jake

9 plus any number is 1 less than the number you had.
(9+7=162)

6 is 1 loss than the T

The answer had to end in 6.

So I knew the answer had to be 116.



Shu







Lauren/Alesandra

99+10=109

Jake

9 plus any number is 1 less than the number you had.
(9+7=16)
6 is 1 loss than the 1

The answer had to end in 6.

So I knew the answer had to be 116.



Eliane

9+7=16 90+10=100 16+100=116 Jake

9 plus any number is 1 less than the number you had.
(9+7=16)
6 is 1 loss than the the 1

The answer had to end in 6.

So I knew the answer had to be 116.

Eliane

9+7=16 90+10=100 16+100=116

Lindsay 9+7=16 16+10=26 26+90=116

TEACHER

Shu Rug

99+17

Lauren/Alesandra

99+10=109

Jake

9 plus any number is 1 less than the number you had.
(9+7=16)
6 is 1 less than the 1

The answer had to end in 6.

So I knew the answer had to be 116.

Eliane

9+7=16 90+10=100 16+100=116

Lindsay 9+7=16 16+10=26 90+26=116





Lauren/Alesandra

99+10=109

Dylan 100+17=117 117-1=116 Jake

9 plus any number is 1 less than the number you had.
(9+7=16)
6 is 1 loss than the

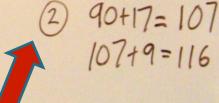
The answer had to end in 6.

So I knew the answer had to be 116.

Eliane

9+7=16 90+10=100 16+100=116

Lindsay 9+7=16 16+10=26 90+26=116



1

Lauren/Alesandra

99+10=109

Dylan 100+17=117 117-1=116

Caleb 99 is 1 less than 100. 17-1=16 99+1=100 100+16=116

Jake

9 plus any number is 1 less than the number you had.
(9+7=16)
6 is 1 loss than the 1

The answer had to end in 6.

So I knew the answer had to be 116.

Eliane

9+7=16 90+10=100 16+100=116

Lindsay 9+7=16 16+10=26 90+26=116

2 90+17=107 107+9=116

Common Core Connection

 Common Core Standards for Mathematical Practice

 Common Core Standards for Mathematical Content



Common Core Recommendation:

... a "balanced combination of procedures and understanding."

Common Core Caution:

... "students who lack understanding of a topic may rely on procedures too heavily."



Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.





Alberto



Dina



Manuel



Amir

To view videos, go to www.mathreasoninginventory.com



MRI Math Reasoning Inventory

Find out what students really understand about math

Funded by the Bill & Melinda Gates Foundation





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Find out what your students really understand about math

- · Focus on how students think and reason
- Uncover students' strategies, understandings, and misconceptions
- Learn how students respond to questions the Common Core expects all middle school students to answer successfully

Sign up for a FREE account and try it today!

SIGN UP FOR FREE

LEARN MORE

Why is Marilyn Burns so excited about MRI?



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The Assessments

Math Reasoning Inventory (MRI) is an online formative assessment tool designed to make teachers' classroom instruction more effective.

Learn More



The Reports

MRI instant reports can be used to inform instruction, monitor progress, identify students who would benefit from intervention, and communicate with parents.

Learn More



Reasoning Strategies

The MRI Interview reveals the strategies students use to reason with whole numbers, decimals, and fractions.

O Learn more

"In just a few minutes, I was able to gain valuable awareness about my math students and adjust my lessons accordingly."

Diana Jones
 Grade 6 Teacher
 SLCUSD, California

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Resources > Video Library

Video Library

The Video Library includes more than 80 video clips of students answering MRI Interview questions. These are placed throughout the website to provide examples of various MRI features. To locate specific video clips, search by Interview Question or by Student.



MRI asks questions that the Common Core expects all students entering middle school to be able to answer successfully.





Reasoning is the heart of MRI.



The *Interview* is the core of MRI.



- •We ask . . .
- •We listen . . .
- •We learn . . .



Practice Standard #3: Construct viable arguments and critique the reasoning of others.



Prompts for student writing. Grade 4:

It means . . .

Every students needs to learn . . .



17 121415 Common Core Practice -1. Standard Number 3. What it means: Every student needs to learn how to explain their thinking. It is more important to be lable to explain. If you didn't explain your thinking you would not learn that much if all you needed were answers you would not your would were answers brain would not learn.

(()5 Practice standards #3 It means that you defend your answer. It means to say your answer is right. lour answer has to make sense.

Practice Standard#3 Every student needs to learn how. to explain their thinking and have their answers make sense. Students also need learn how to ask questions if something doesn't make sense to them. They need learn how to be able to give other students feedback. I would explain my



It means that you should do something that makes sense and you should be able to critizize and guestion what the person is



CSSPractice Standard#3 What it means! I mean Stocreate Viable arguments and Critia, ue the reasoning of others. Stow your thinking



Common Core Connection

Grade 2

Number & Operations in Base Ten

Use place value understanding and properties of operations to add and subtract.

<u>CCSS.Math.Content.2.NBT.B.5</u> Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

<u>CCSS.Math.Content.2.NBT.9</u> Explain why addition and subtraction strategies work, using place value and the properties of operations.



Common Core Connection

Grade 2

Operations & Algebraic Thinking

Represent and solve problems involving addition and subtraction.

<u>CCSS.Math.Content.2.OA.A.1</u> Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions.



There are ___ girls on the playground.
There are ___ boys on the playground.
There are ___ children in all.

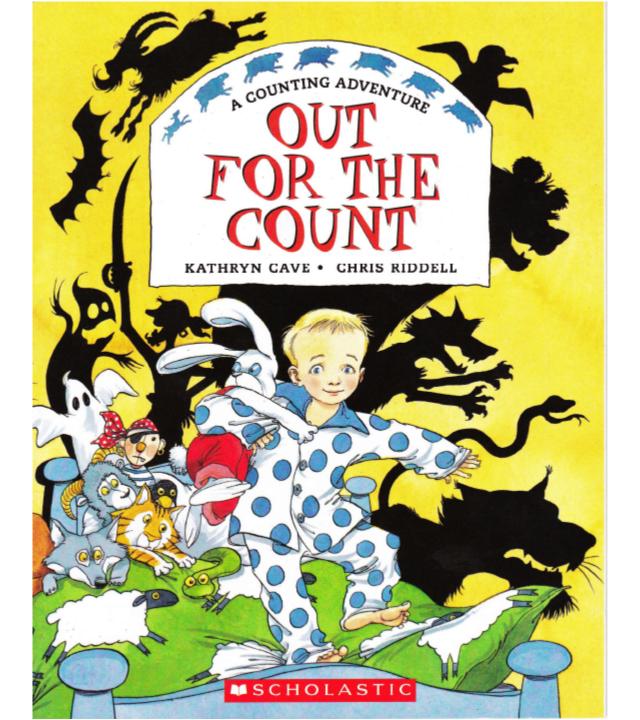


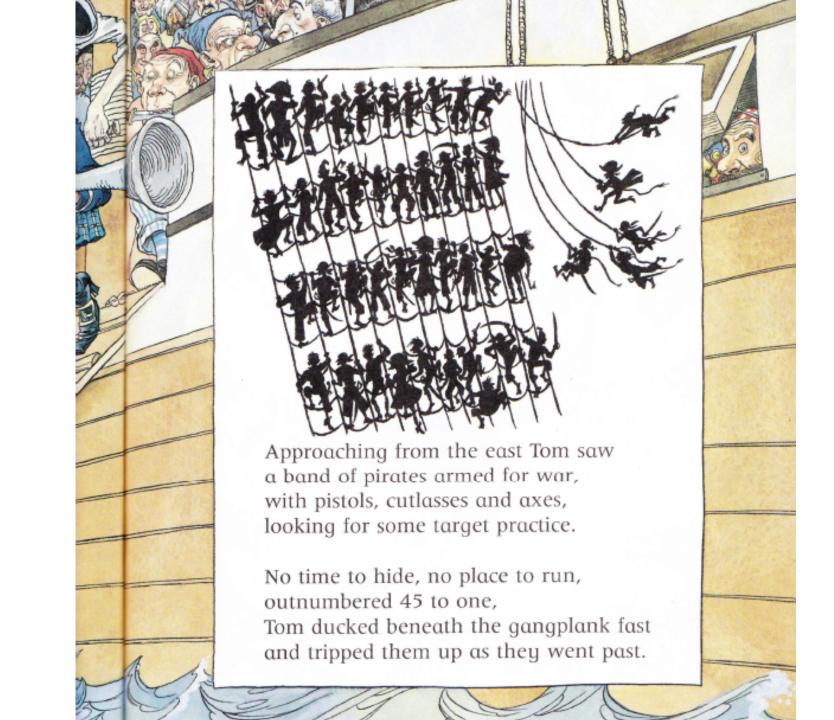
There are **7** girls on the playground. There are **15** boys on the playground. There are __ children in all.

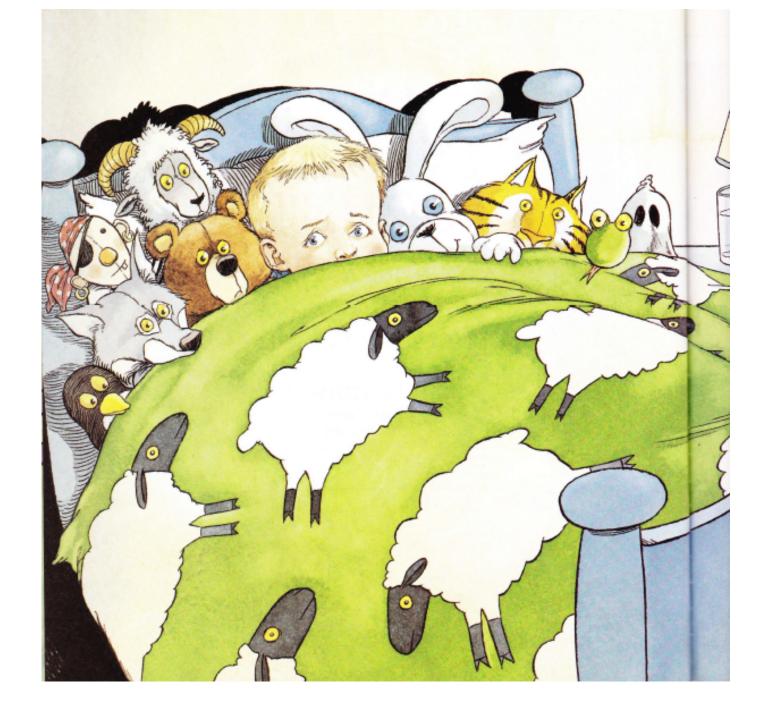


There are agins on the playground There are 15 boys on the play ground There are 50 children in all.

There are 15 girls
There are 15 boys
There are 50 in all.







STUFFED ANIMALS

Penguin \$13	Bear\$35
Sheep \$18	Wolf\$38
Ghost\$26	Pirate \$39
Python \$29	Tiger \$47
Goat\$34	Rabbit\$49



Buy Stuffed Animals



Choose two

stuffed animals. Write the problem.

3+8=11 20+11=31 13+18=31

ISabe ...

10 + 10 = 20

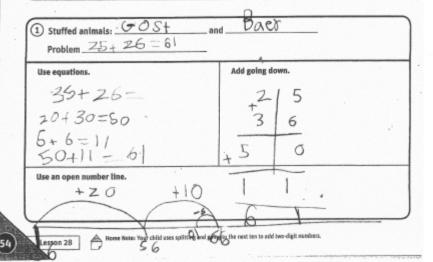
Find the total price using equations.

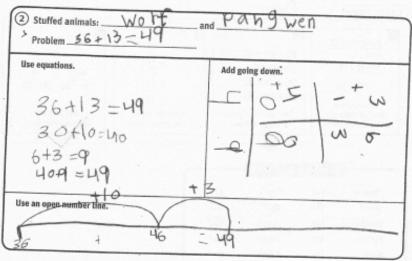
13 +18 20 +11 31 Find the total

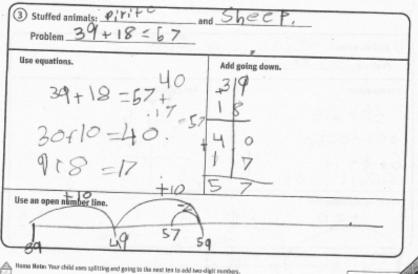
price going down.

+10 +7 +1 13 23 30 31 Add with an open number line.

| STUFFED ANIMALS | \$35 | \$35 | \$35 | \$35 | \$36 | \$36 | \$36 | \$36 | \$36 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37 | \$37



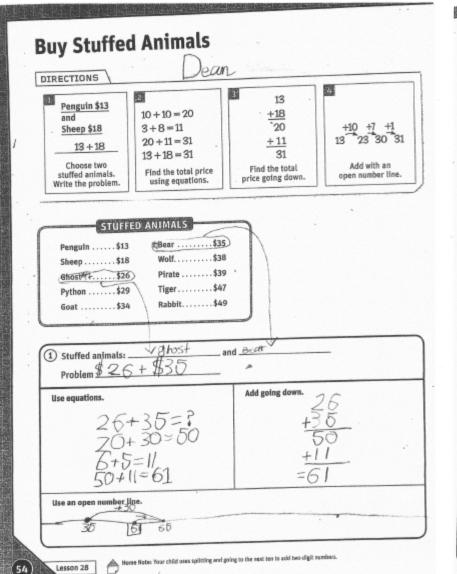


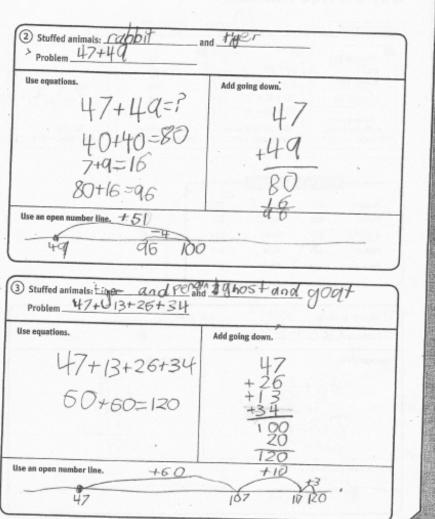




1 Stuffed animals: 608+ and Daer Problem 25+ 26 = 61		
Use equations.	Add going down.	
35+26=	215	
20+30=50	3 6	
50+11-61	+50	
Use an open number line.		
+20 +10		
-1	6	
Lesson 28 Home Note: Your child uses splitting and geme to the next ten to add two-digit numbers.		







Moree Note: Your child uses splitting and going to the next ten to add two-digit numbers.



Lesson 28

55

3 Stuffed animals:	er and Perand Ighost and	agat
Problem 47+6	13+25+34	J. L.

Use equations.

Add going down.

Use an open number line.

67 17 RO



SPEGIAL SALE!

STUFFED ANIMALS

Penguin \$13.46	Bear\$35 \$ 8
Sheep\$1849	Wolf\$38 4 10
Ghost\$26910	Pirate\$39\$6
Python\$29815	Tiger\$47\$5
Goat\$34812	Rabbit\$49 \$ 20

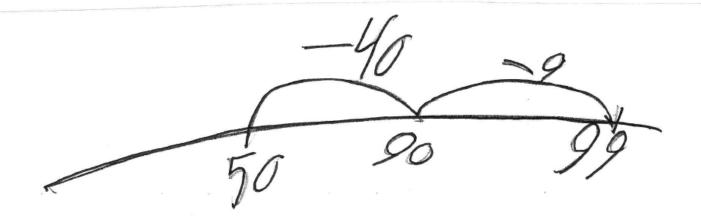


Grade 2 Benchmark Assessment

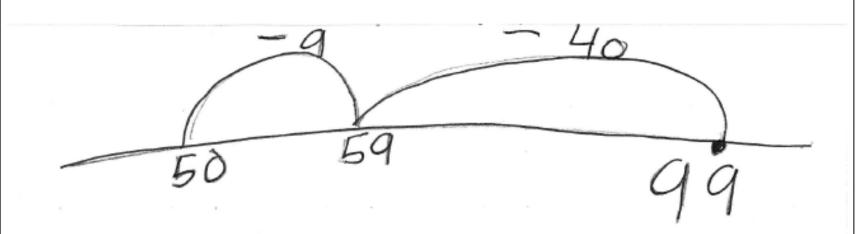
The teacher had 99 stickers. She gave stickers away. Then the teacher had 50 stickers. How many stickers did she give away?

Solve with words, numbers, or pictures. Write an equation that represents the problem.





99-49=50

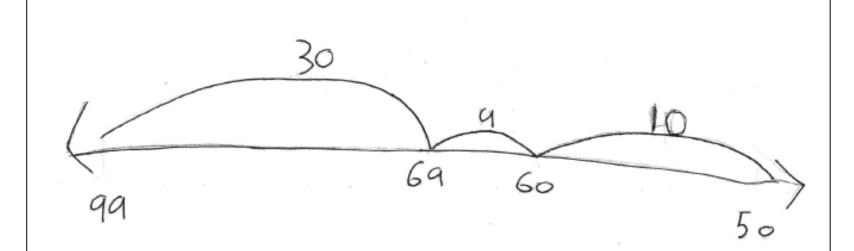


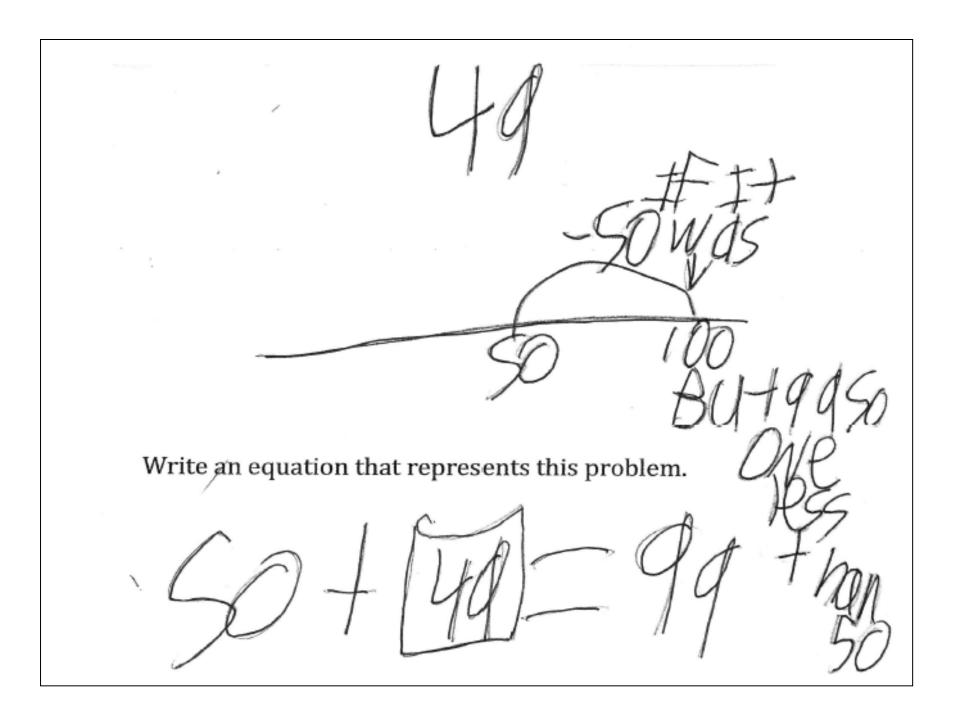
99-49=50

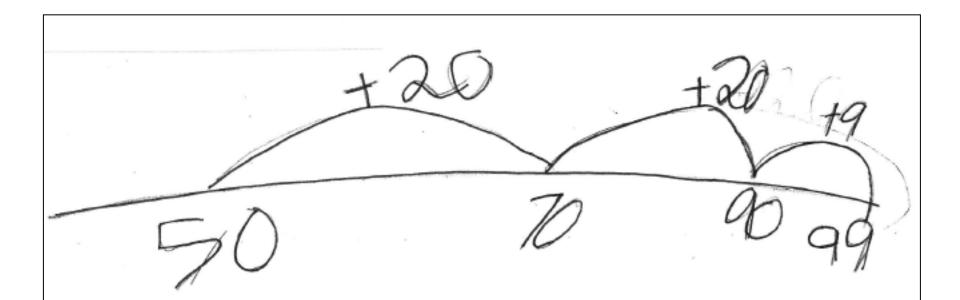
49 - 9 - 90 2 702 603 49 - 4

Write an equation that represents this problem.

99-49-50



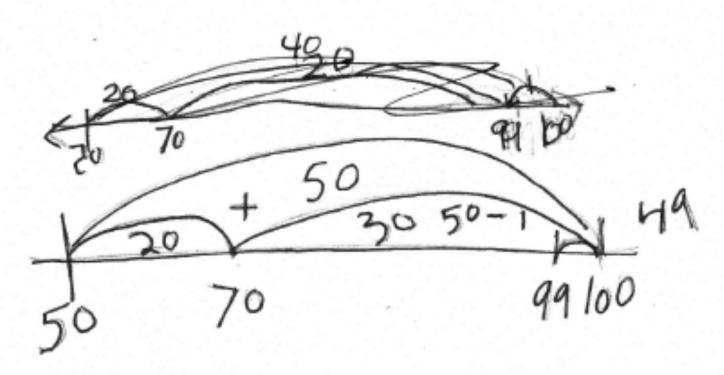




30 + 419 =94

 $\frac{40}{50} + 9 = 49$

Write an equation that represents this problem.



99 -50 -49

Write an equation that represents this problem.

99-50=49





What is 15 times 12?

 $15 \times 12 =$

- 2 Answer
- Orrect (180)
-) Incorrect
- Self-corrected (180)
- O Did Not Answer

Explanation



How did you figure out the answer?

- Used standard algorithm to multiply
- Broke 15 and/or 12 into parts and then multiplied (e.g., 15 x 10 and then 15 x 2)
- Changed to an easier problem, 30 x 6, by doubling and halving
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

Notes

record student response

■ 1 2 3 4 5 6 7 8 9 10 **■**







Current

■ Save and Exit



15 x 12



Common Core Connection

Grade 4

Number and Operations in Base Ten

Use place value understanding and properties of operations to perform multi-digit arithmetic.

CCSS.Math.Content.4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations.



Common Core Connection

Grade 5

Operations & Algebraic Thinking

Write and interpret numerical expressions.

<u>CCSS.Math.Content.5.OA.A.1</u> Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

CCSS.Math.Content.5.OA.A.2 Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as 18932 + 921, without having to calculate the indicated sum or product. Math Solutions

Common Core Connection

Grade 6

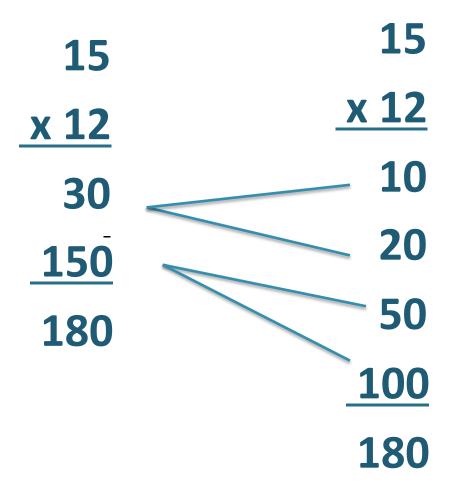
Expressions & Equations

Apply and extend previous understandings of arithmetic to algebraic expressions.

CCSS.Math.Content.6.EE.A.3 Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.



15 x 12





Distributive Property of Multiplication over Addition



15 x 12

Monica



Malcolm



Alberto



To view videos, go to www.mathreasoninginventory.co m



Monica: 15 x 12

"I did 15 times 10 and it was 150.

And then I did 15 times 2 which is 30.

And it was . . . um . . . 180."

$$12 = 10 + 2$$

$$15 \times 10 = 150$$

$$15 \times 2 = 30$$

$$150 + 30 = 180$$



Malcolm: 15 x 12

"I broke apart the 15 and did 10 plus 5.

And then I did 10 times 12 which equals 120.

And then I did 12 times 5 which equals 60

And then I added it all together and I got 180."

$$15 = 10 + 5$$

$$10 \times 12 = 120$$

$$12 \times 5 = 60$$

$$120 + 60 = 180$$



Alberto: 15 x 12

"I did 12 times 12 is 144
And then I did 3 times 12 and I got 36
And then I did 144 plus 36."

$$15 = 12 + 3$$

$$12 \times 12 = 144$$

$$3 \times 12 = 36$$

$$144 + 36 = 180$$



15 x 12

Monica



 $(15 \times 10) + (15 \times 2) = 180$

Malcolm



 $(10 \times 12) + (12 \times 5) = 180$

Alberto

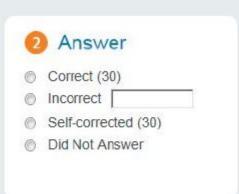


 $(12 \times 12) + (3 \times 12) = 180$





Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?







How did you figure out the answer?

- Used standard algorithm to multiply
- Multiplied 20 x 1 and then 20 x 0.5
- Multiplied 1.5 x 2 and then 3 x 10
- Multiplied 1.5 x 10 and then 15 x 2
- Multiplied 15 x 2 and then adjusted the decimal point
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation





□ Not Done □ Incomplete ■ Done □ Current



Molly Problem

Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?



Common Core Connection

Grade 5

Number and Operations in Base Ten

Perform operations with multi-digit whole numbers and with decimals to hundredths.

CCSS.Math.Content.5.NBT.B.7 Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.



Molly Problem

Molly ran 1.5 miles each day for 20 days. How many miles did she run altogether?

$$20 \times 1.5 =$$



Distributive Property of Multiplication over Addition



Sergio: Molly problem, 20 x 1.5

(View at www.mathreasoninginventory.com)

"I know that 20 times 1 is 20, so I put the 20 aside. And 20 times 5 is 100, and bloop it by one is just zero . . . 10.0. So 20 plus 10 is 30."

$$20 \times 1 = 20$$

$$20 \times 5 = 100$$
, so $20 \times .5$ is 10.0

$$20 + 10 = 30$$

$$20 \times 1.5 = (20 \times 1) + (20 \times .5)$$



Common Core Connection

Grade 2

Measurement & Data

Measure and estimate lengths in standard units.

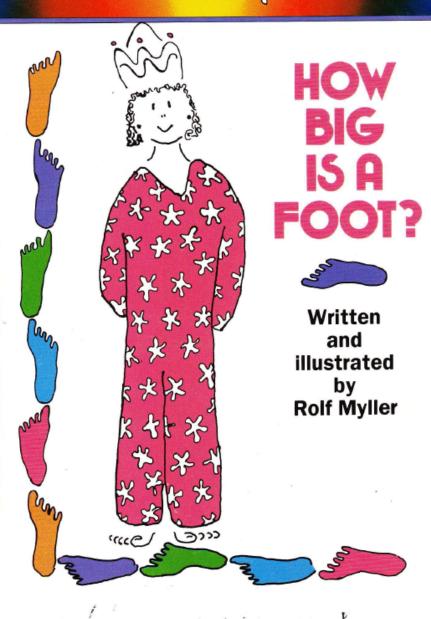
<u>CCSS.Math.Content.2MD.A.1</u> Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

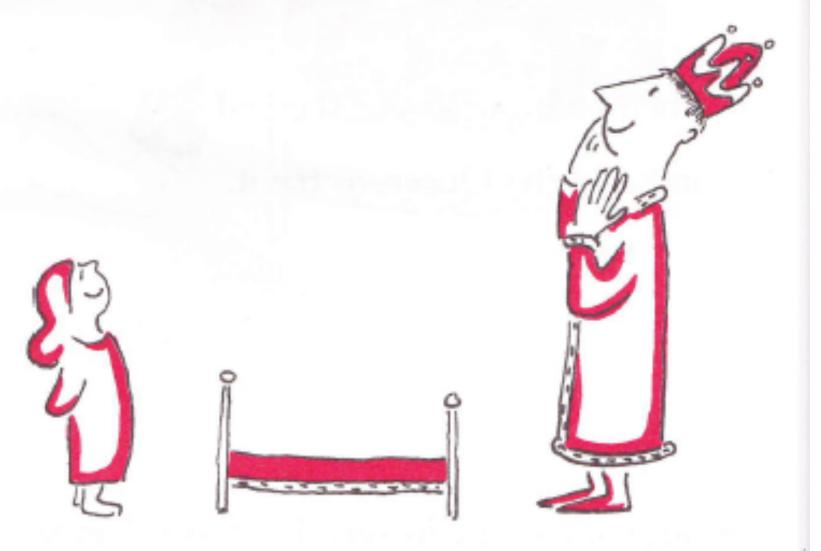
Relate addition and subtraction to length.

<u>CCSS.Math.Content.2MD.B.5.</u> Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units,.



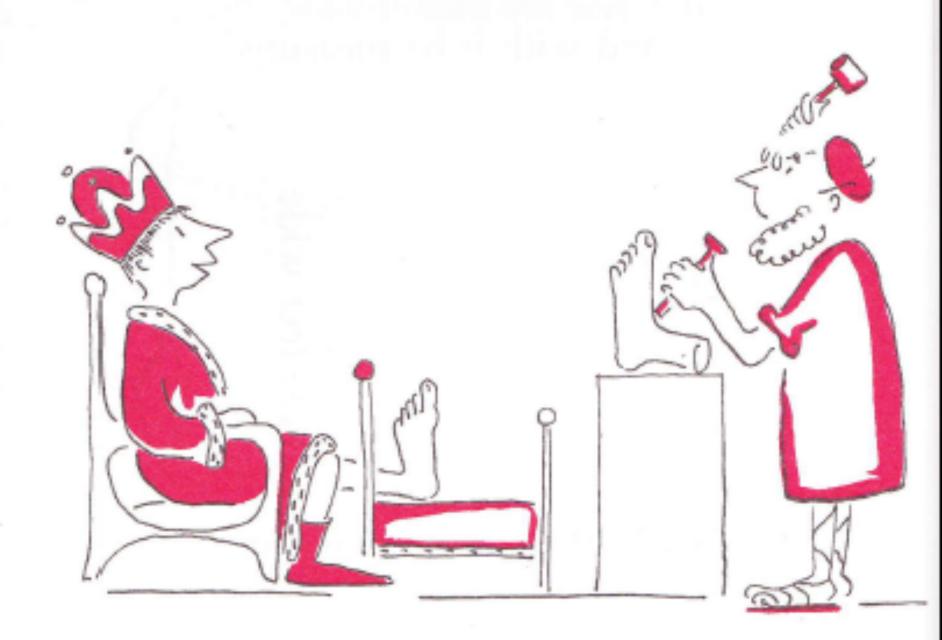




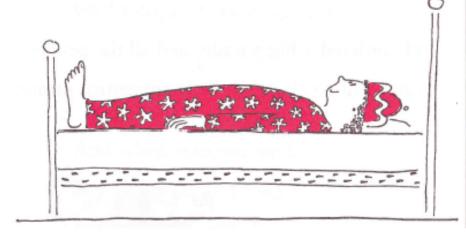


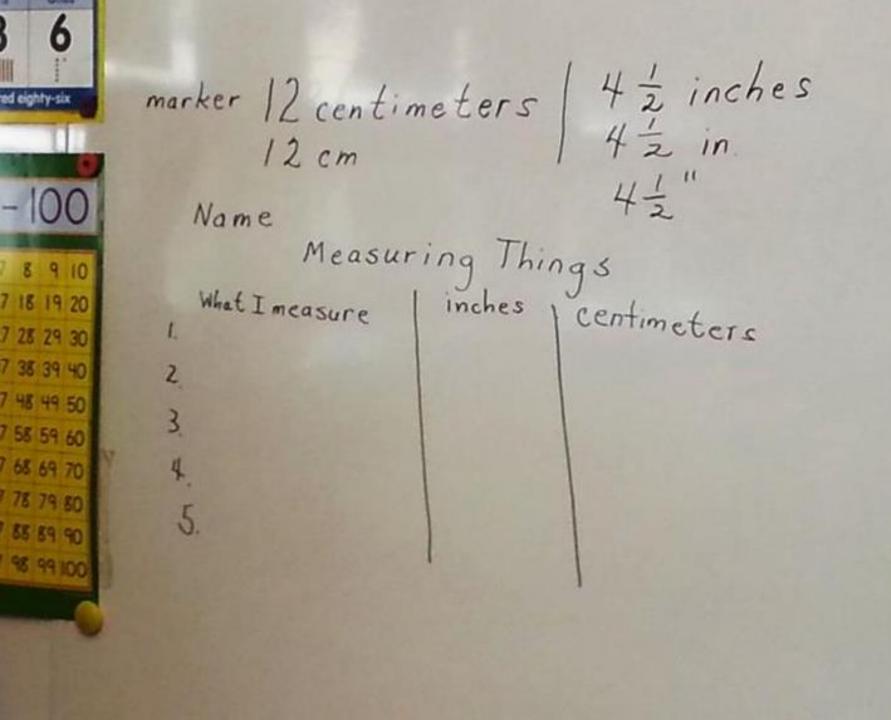












94

B

11

7

LU

STO

5

Maya hings centimeters What I measure / inches 1. 14 cm. 3.19 cm 4. 5. 2. Crayon 3. Pencil. 4. Pencil starpener

•

Mearsurin Thip95 I Mausure 1, Pencil ZICrayn 3,50155015 4.91 We Stick 5, earser

Cara Pickle Cotner 2 incl. Water Bottle Picklecotner H12cm/ 25cM hair clip2 cm hair clip ahalfainc Shapenen Einc Shapener 6cm. Sisery 12 inc Sizens 12.CM

) XM ES CUBES 2 SPE FOST 2 +* MKQ

Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in repeated reasoning.





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