Understanding Fraction Computation

by Applying and Extending Previous Understandings

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NCTM 2014

Yours is not to reason why
Just invert and multiply

Standards for Mathematical Practice

1. **Make sense** of problems and persevere in solving them.
2. **Reason** abstractly and quantitatively.
3. Construct **viable** arguments and **critique** the reasoning of others.
4. **Model** with mathematics.
5. Use appropriate tools **strategically**.
6. Attend to **precision**.
7. Look for and **make use** of structure.
8. Look for and express regularity in repeated **reasoning**.

Brendan, Grade 4

1. Without computing the exact answer, decide which of these expressions would produce the answer with the least value and the greatest value.

A. Addition: $\frac{3}{4} + \frac{5}{8}$

B. Subtraction: $\frac{3}{4} - \frac{5}{8}$

C. Multiplication: $\frac{3}{4} \times \frac{5}{8}$

D. Division: $\frac{3}{4} \div \frac{5}{8}$

Least Value Division

Greatest Value multiplication

Explain your thinking below:

Division because you have tol split it up.
Multiplication because you have to add more than 2 of it.

Grade 3: Number and Operations - Fractions

Develop understanding of fractions as numbers.

- *Implications for addition/subtraction*
- *Implications for multiplication/division*

Grade 4: Number and Operations - Fractions

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

- *Implications for addition/subtraction*
- *Implications for multiplication*

Grade 5: Number and Operations - Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- *Implications for addition/subtraction*
- *Implications for multiplication/division*

Fractions as numbers…

"*In mathematics, do whatever it takes to help you learn something, provided you do not lose sight of what you are supposed to learn*. In the case of fractions, it means you may use any pictorial image you want to process your thoughts on fractions, but *at the end, you should be able to formulate logical arguments in terms of the original definition of a fraction as a point on the number line.*"

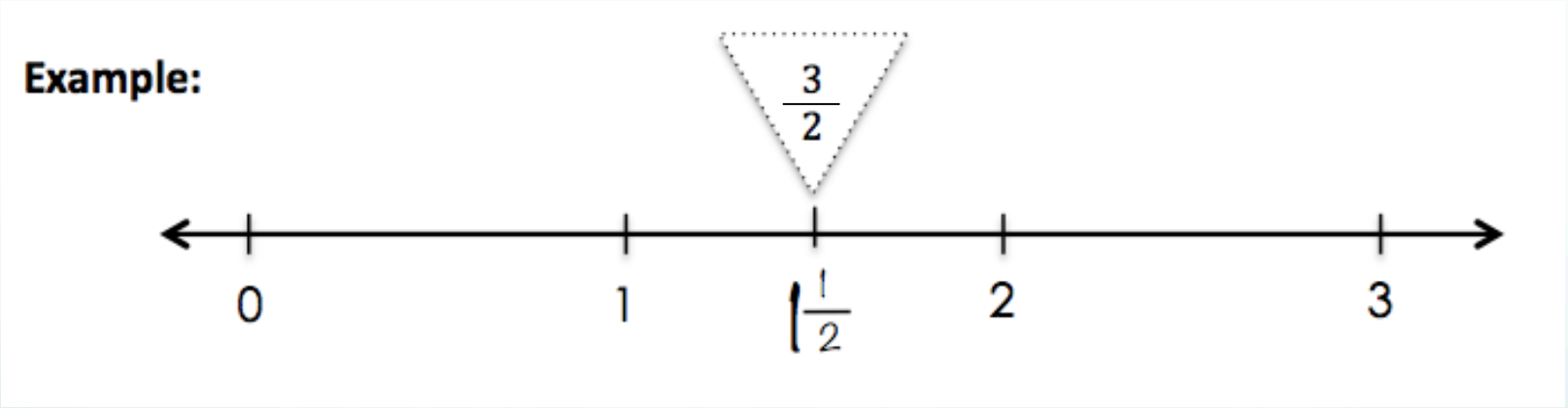
-Wu, 2002, p. 13

Grade 3: Number and Operations - Fractions

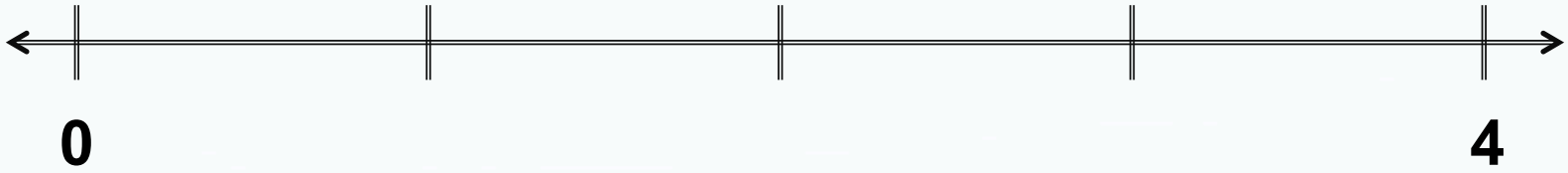
Develop understanding of fractions as numbers.

Fractions Greater than One

- Placing mixed numbers and fraction equivalents on the number line



Placing $\frac{1}{2}$ on the number line



Placing $\frac{1}{2}$ on the number line

View video: “Placing ½ on the number line”

https://mathsolutions.wistia.com/projects/r4bjpdzb31

Beyond Invert & Multiply (forthcoming)

Grade 3: Number and Operations - Fractions

Develop understanding of fractions as numbers.

Grade 4: Number and Operations - Fractions

3.b: Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation.

Whole number addition strategies

- Decomposing/recomposing
- Associative property
- Commutative property

Ways to Make

- How many ways can you make…

1

Ways to Make

- How many ways can you make…

$$\frac{1}{2}$$

Ways to Make

- How many ways can you make…

$$\frac{7}{12}$$

Get to the Whole!

- Decomposing and recomposing fractions to “get to the whole” when adding and subtracting.

$$\frac{3}{4} + \frac{3}{4}$$

Get to the Whole!

View Video:

"Three-fourths plus three-fourths: Will's strategy"

https://mathsolutions.wistia.com/projects/r4bjpdzb31

Beyond Invert & Multiply (forthcoming)

Get to the Whole!

View video:

"Three-fifths plus four-fifths: Yuli's strategy"

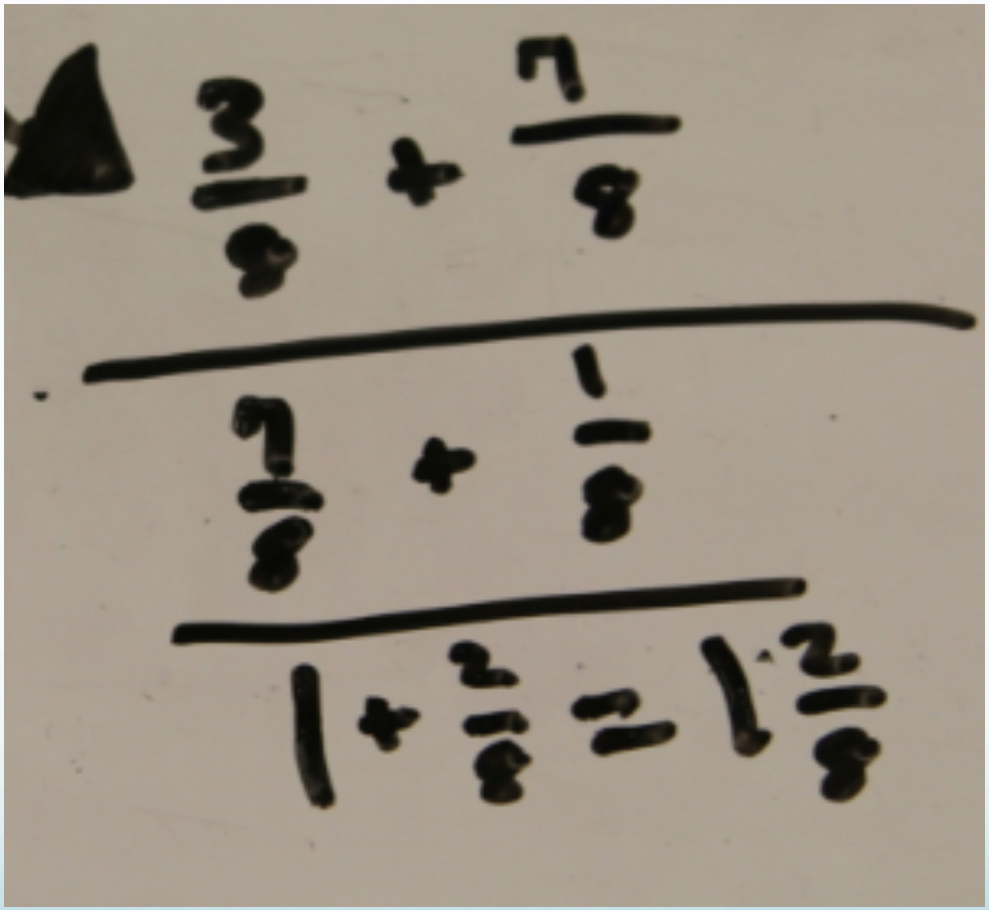
https://mathsolutions.wistia.com/projects/r4bjpdzb31

Beyond Invert & Multiply (forthcoming)

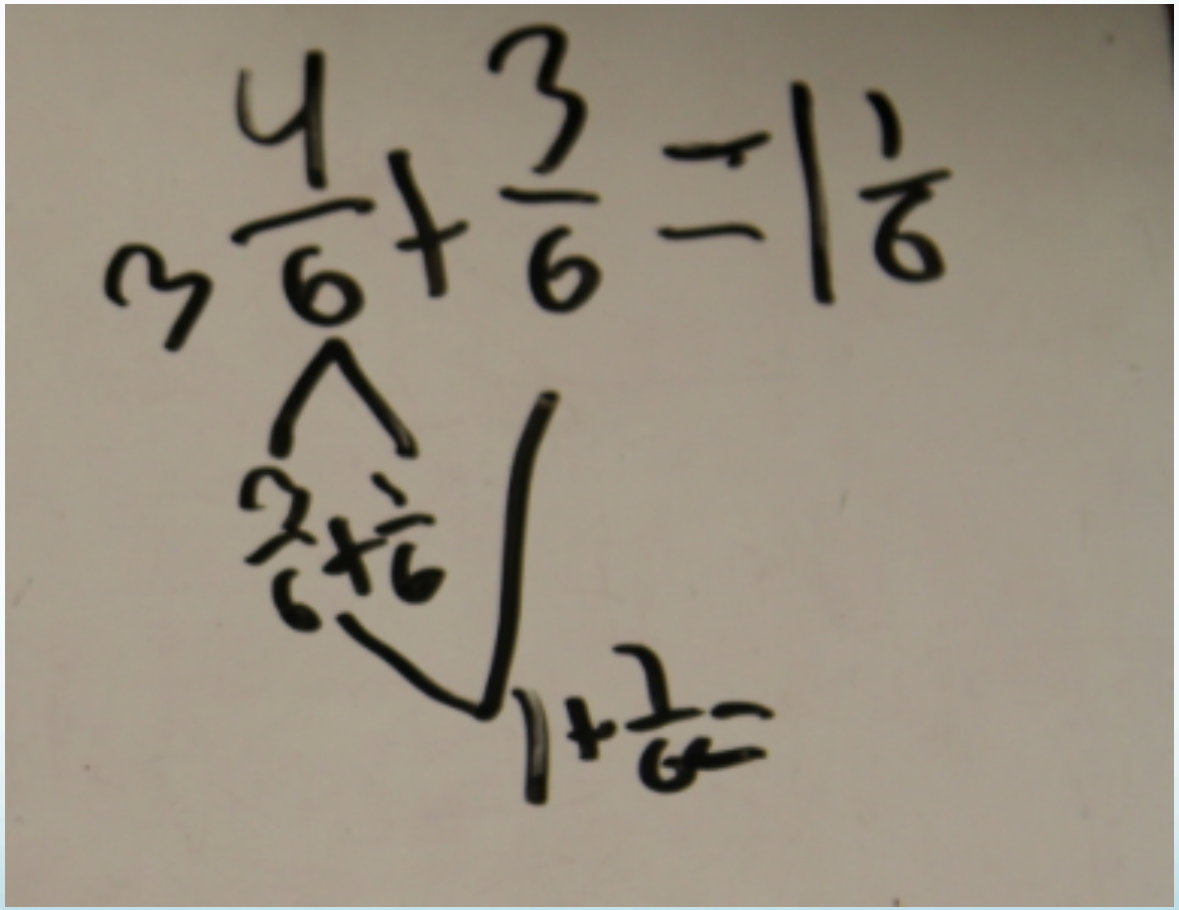
Get to the Whole!

$$\frac{3}{8} + \frac{7}{8} \qquad \frac{5}{6} + \frac{5}{12} \qquad \frac{13}{9} - \frac{8}{9}$$

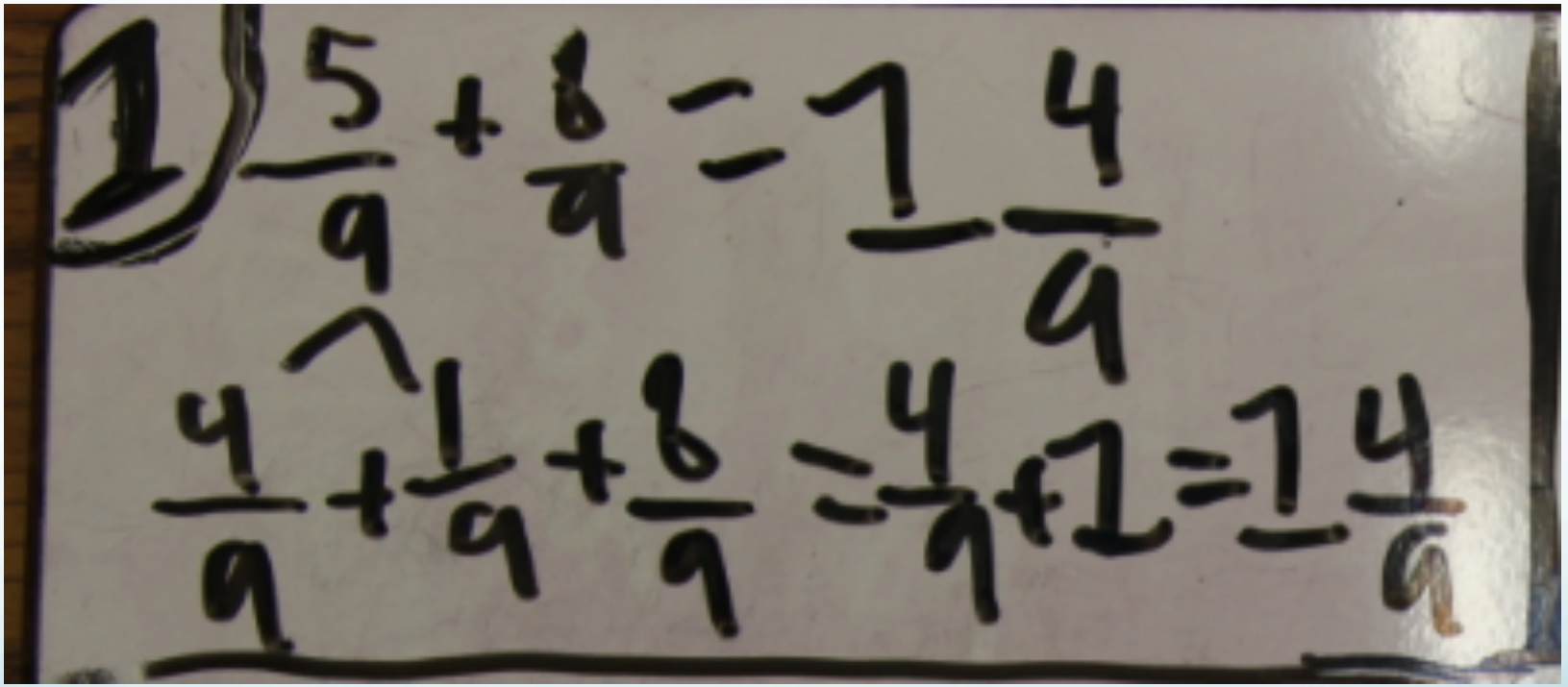
Student work



Student work



Student work



Multiplication Patterns

Students consider patterns of products in problems with factors that decrease in value.

Multiplication Patterns

$6 \times 8 = 48$

$6 \times 4 = 24$

$6 \times 2 = 12$

$6 \times 1 = 6$

$6 \times \_\_ = \_\_$

Multiplication Patterns

$6 \times 8 = 48$

$6 \times 4 = 24$

$6 \times 2 = 12$

$6 \times 1 = 6$

$6 \times \frac{1}{2} = 3$

Tell Me All You Can

Before coming up with an exact answer, consider what you know about the answer as a means of getting a sense of the “neighborhood” of the answer.

Tell Me All You Can

- The answer will be less than _____ because _____.
- The answer will be greater than _____ because _____.
- The answer will be between _____ and _____ because _____.

from Bresser and Holzman (2006)

Tell Me All You Can

$$6 \times 2\frac{1}{2}$$

Tell Me All You Can

$$4\frac{1}{2} \times 5$$

Grade 5: Number and Operations - Fractions

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- *Implications for multiplication/division*

Is $60 \div 2$ the same as $60 \div \frac{1}{2}$?

What strategies will students have to answer this question after engaging in *How Long? How Far?*

How Long? How Far?

Dividing a whole number by a fraction

Dividing a fraction by a whole number

Reasoning about $1 \div \frac{1}{4}$

- How many $\frac{1}{4}$s are in 1?

How Long? How Far? Part 1

View video: “How many one-fourths are in one?”

https://mathsolutions.wistia.com/projects/r4bjpdzb31

Beyond Invert & Multiply (forthcoming)

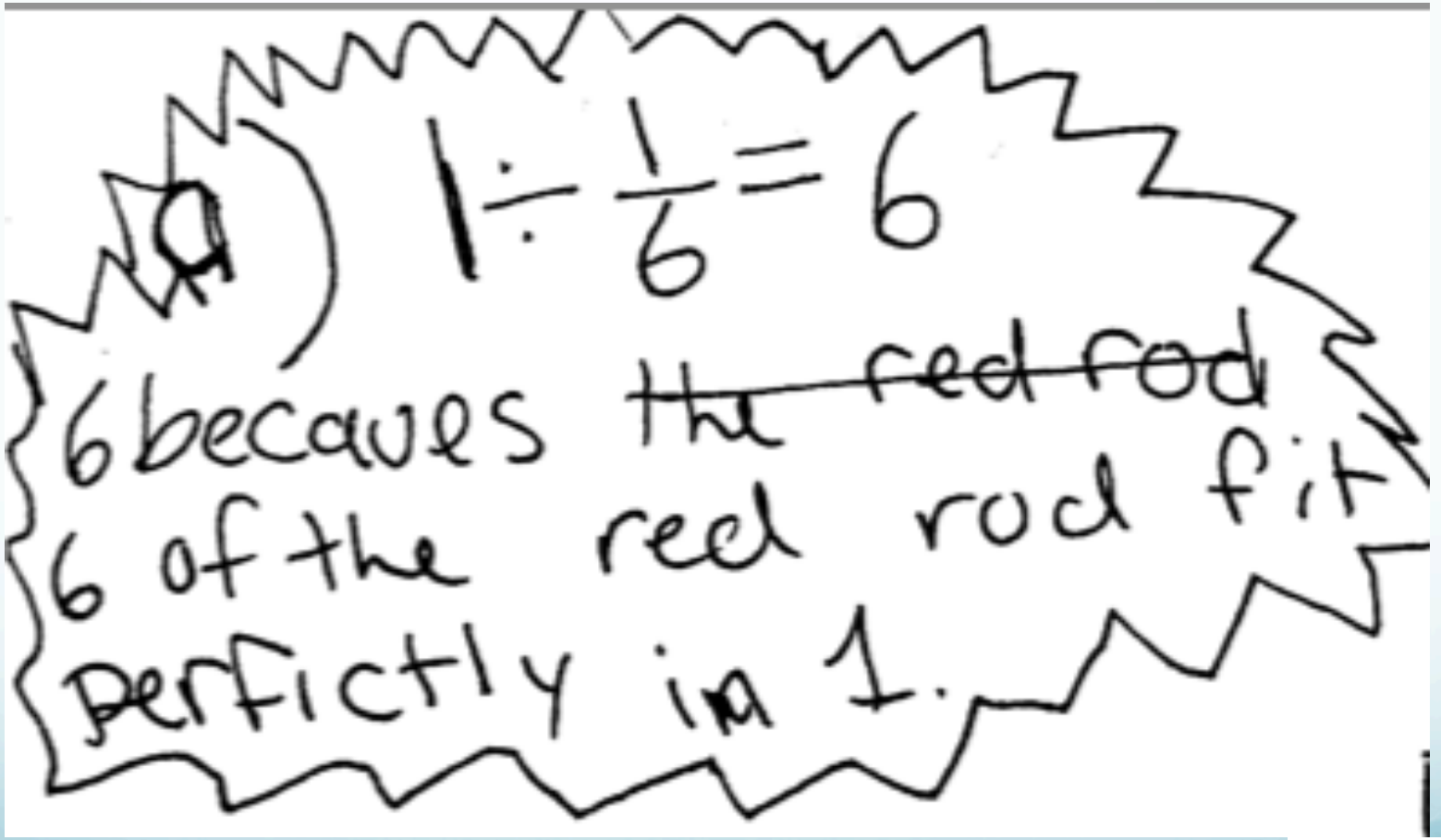
How Long? How Far? Part 1

View video: “How many one-thirds are in two?”

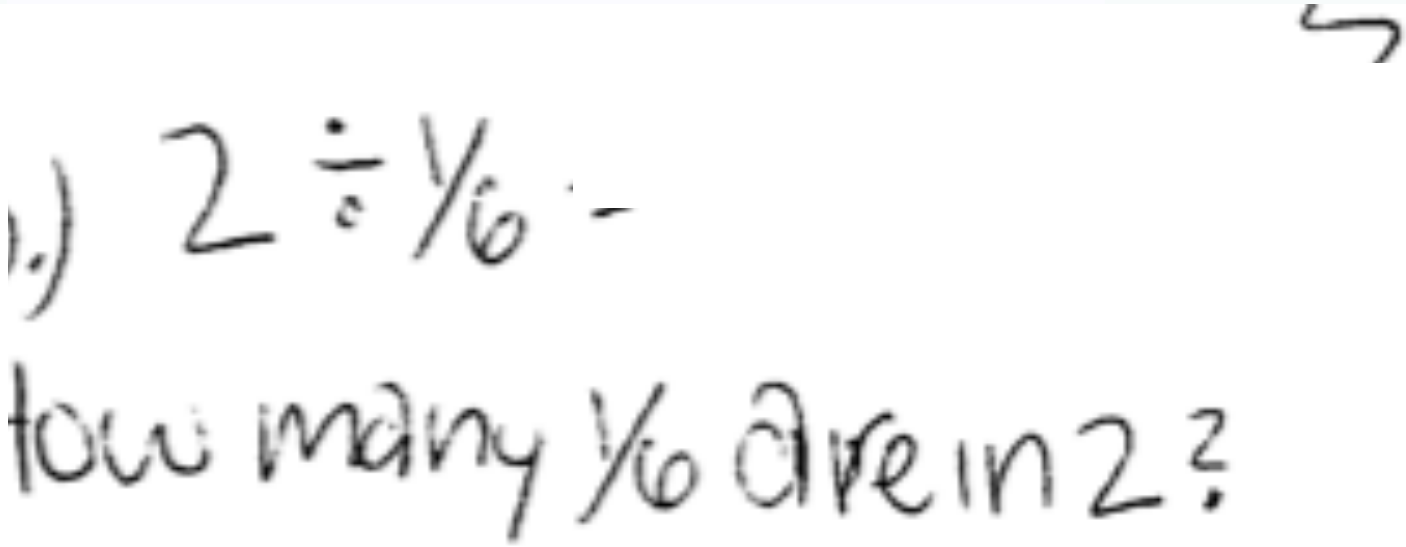
https://mathsolutions.wistia.com/projects/r4bjpdzb31

Beyond Invert & Multiply (forthcoming)

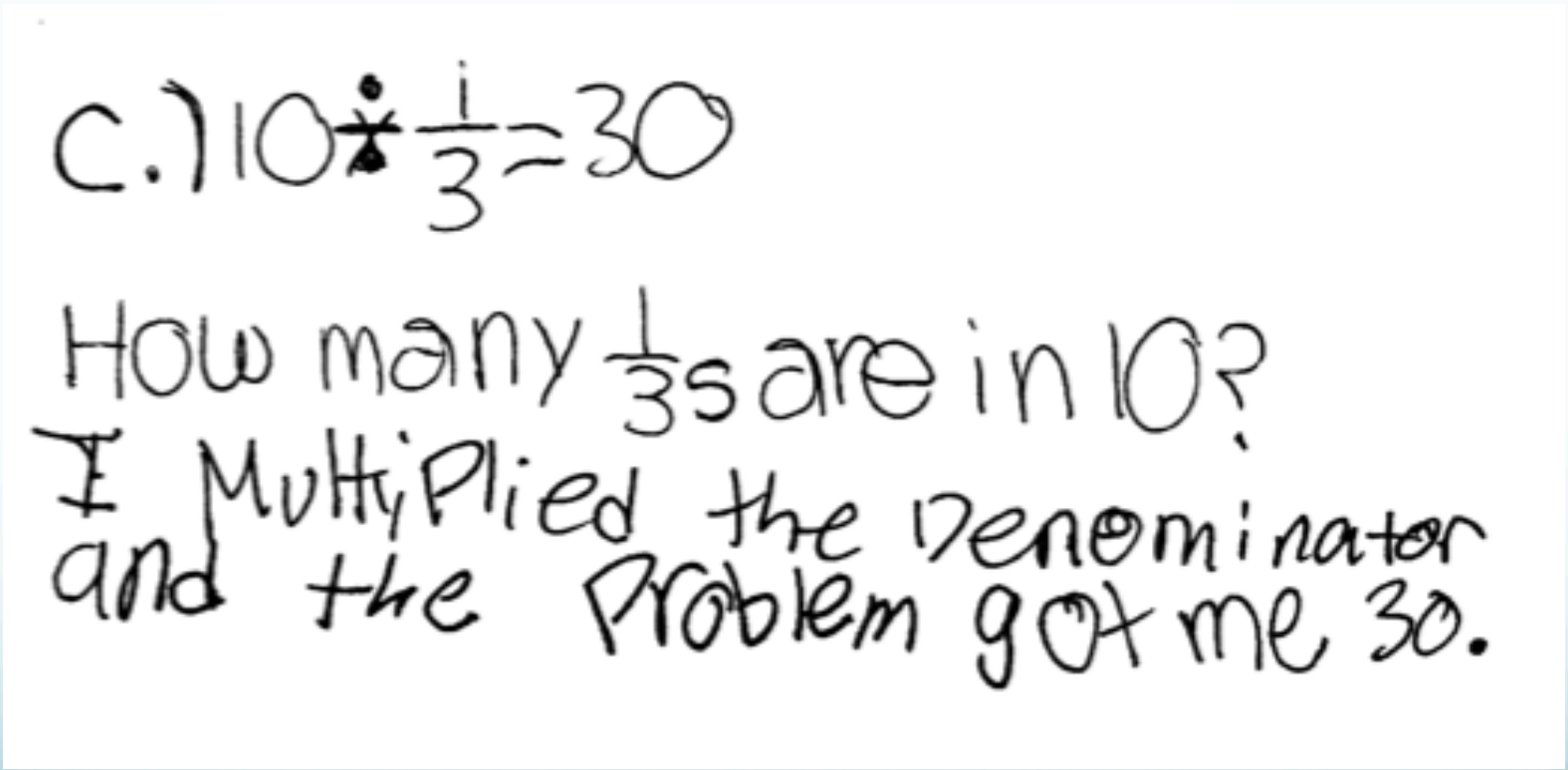
Reasoning about $1 \div \frac{1}{6}$



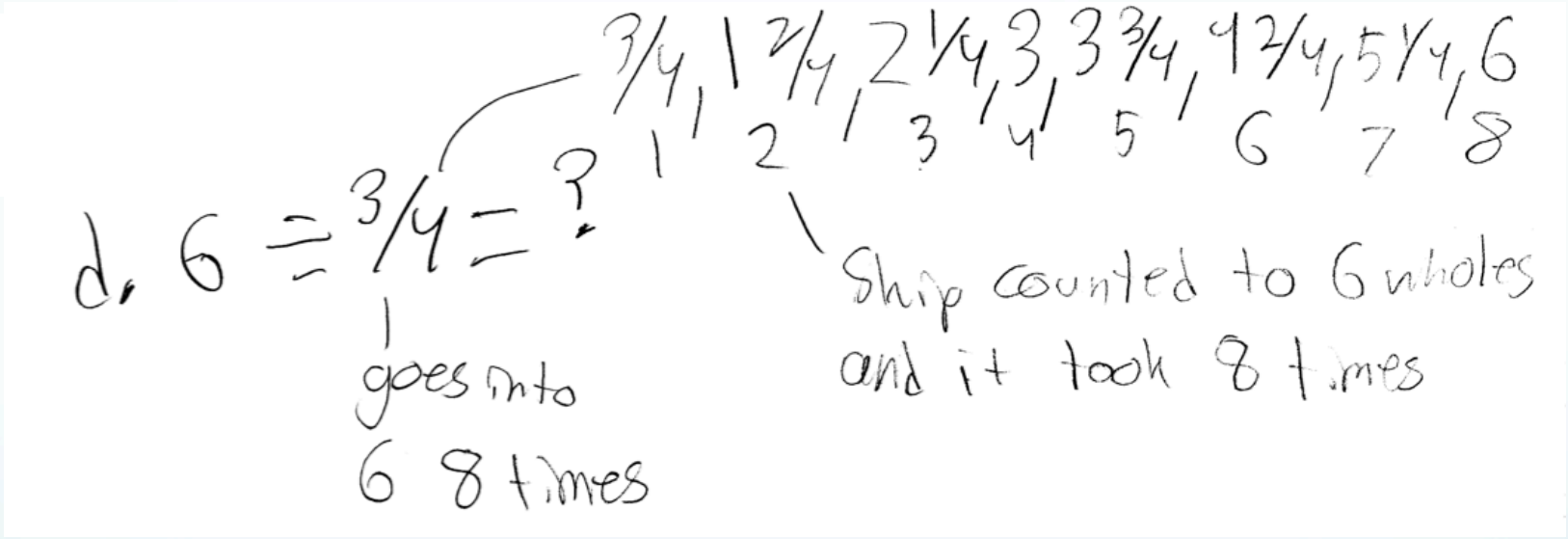
Reasoning about $2 \div \frac{1}{6}$



Reasoning about $10 \div \frac{1}{3}$



Reasoning about $6 \div \frac{3}{4}$



Is $60 \div 2$ the same as $60 \div \frac{1}{2}$?

How Long? How Far? Part 2

Beach Clean-Up (2 people)	
Distance	Each person cleans
8 miles	4 miles
4 miles	2 miles
2 miles	1 mile
1 mile	½ mile
½ mile	?

How Long? How Far? Part 2

$$\frac{1}{2} \div 3$$

$$\frac{1}{6} \div 2$$

$$\frac{3}{4} \div 3$$

Number and Operations - Fractions

3.NF: Develop understanding of fractions as numbers.

4.NF: Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

5.NF: Use equivalent fractions as a strategy to add and subtract fractions.

5.NF: Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Fractions as numbers…

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-Wu, 2002, p. 13

Coming soon!

Beyond Invert and Multiply:

Making Sense of Fraction Computation

(Forthcoming, Math Solutions)

Thank you!

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mathsolutions.com/presentations