

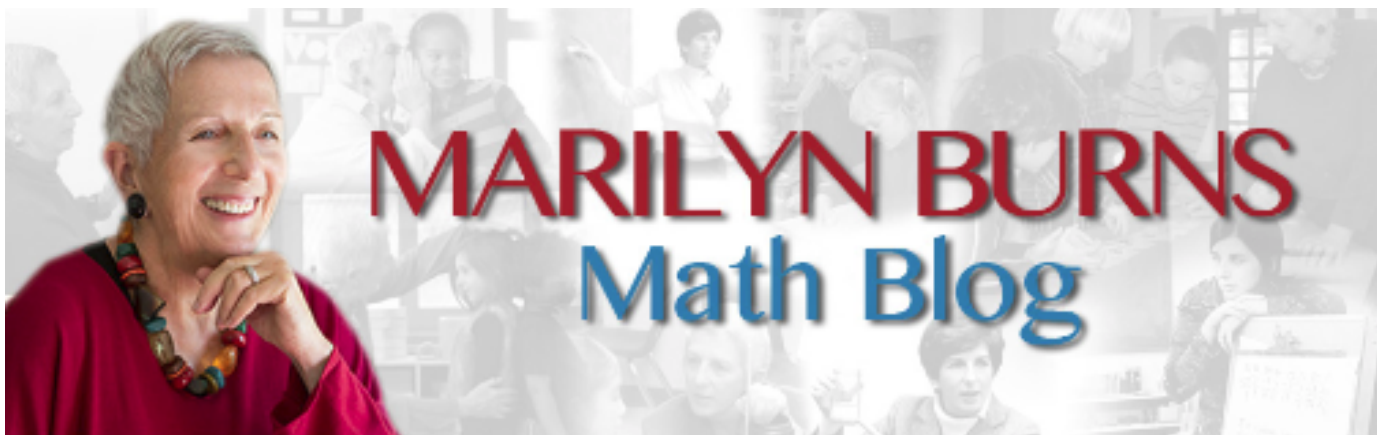
Effective Classroom Teaching for Developing Numerical Understanding and Skills

Marilyn Burns

NCTM Annual Conference

Thursday, April 16, 2015





@mburnsmath



Math Reasoning Inventory™

www.mathsolutions.com



Math Solutions.
FOUNDED BY MARILYN BURNS

15% & 20% TIP TABLE®

Check	15%	20%	Check	15%	20%
\$1.00	\$.15	\$.20	\$26.00	\$3.90	\$5.20
2.00	.30	.40	27.00	4.05	5.40
3.00	.45	.60	28.00	4.20	5.60
4.00	.60	.80	29.00	4.35	5.80
5.00	.75	1.00	30.00	4.50	6.00
6.00	.90	1.20	31.00	4.65	6.20
7.00	1.05	1.40	32.00	4.80	6.40
8.00	1.20	1.60	33.00	4.95	6.60
9.00	1.35	1.80	34.00	5.10	6.80
10.00	1.50	2.00	35.00	5.25	7.00
11.00	1.65	2.20	36.00	5.40	7.20
12.00	1.80	2.40	37.00	5.55	7.40
13.00	1.95	2.60	38.00	5.70	7.60
14.00	2.10	2.80	39.00	5.85	7.80
15.00	2.25	3.00	40.00	6.00	8.00
16.00	2.40	3.20	41.00	6.15	8.20
17.00	2.55	3.40	42.00	6.30	8.40
18.00	2.70	3.60	43.00	6.45	8.60
19.00	2.85	3.80	44.00	6.60	8.80
20.00	3.00	4.00	45.00	6.75	9.00
21.00	3.15	4.20	46.00	6.90	9.20
22.00	3.30	4.40	47.00	7.05	9.40
23.00	3.45	4.60	48.00	7.20	9.60
24.00	3.60	4.80	49.00	7.35	9.80
25.00	3.75	5.00	50.00	7.50	10.00

Tipulator



Check amount

\$16.00 ▾

15% tip

\$2.40 ▾

Total

\$18.40

2 people

\$9.20 ▾

Tip N Split

Tip Calculator

Calculate Tips
& Split the Bill in **5** seconds!



WEAR TIP CALCULATOR

by GABRIEL



Exclusive on Android Wear



IF YOUR BILL WAS
\$42.42
AND



YOUR SERVER WAS



AMAZING!

YOUR SERVER WAS



GREAT

YOUR SERVER WAS



OKAY

YOUR SERVER



SUCKED

THEN YOU SHOULD TIP:

\$8.48

*Swipe left
for total.*

Gratuuity Not Included:

(Tipping Guide)

15%: _____

18%: _____

20%: _____

Total Charge

25.40

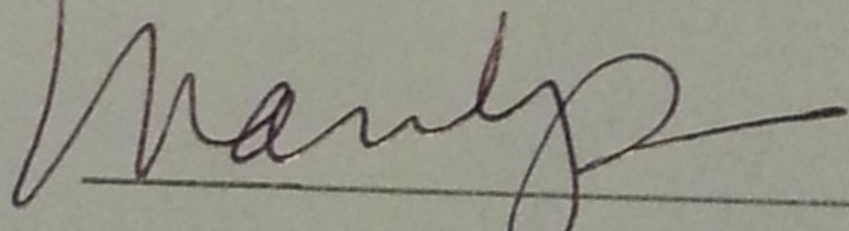
Room Number

1354

Print Name

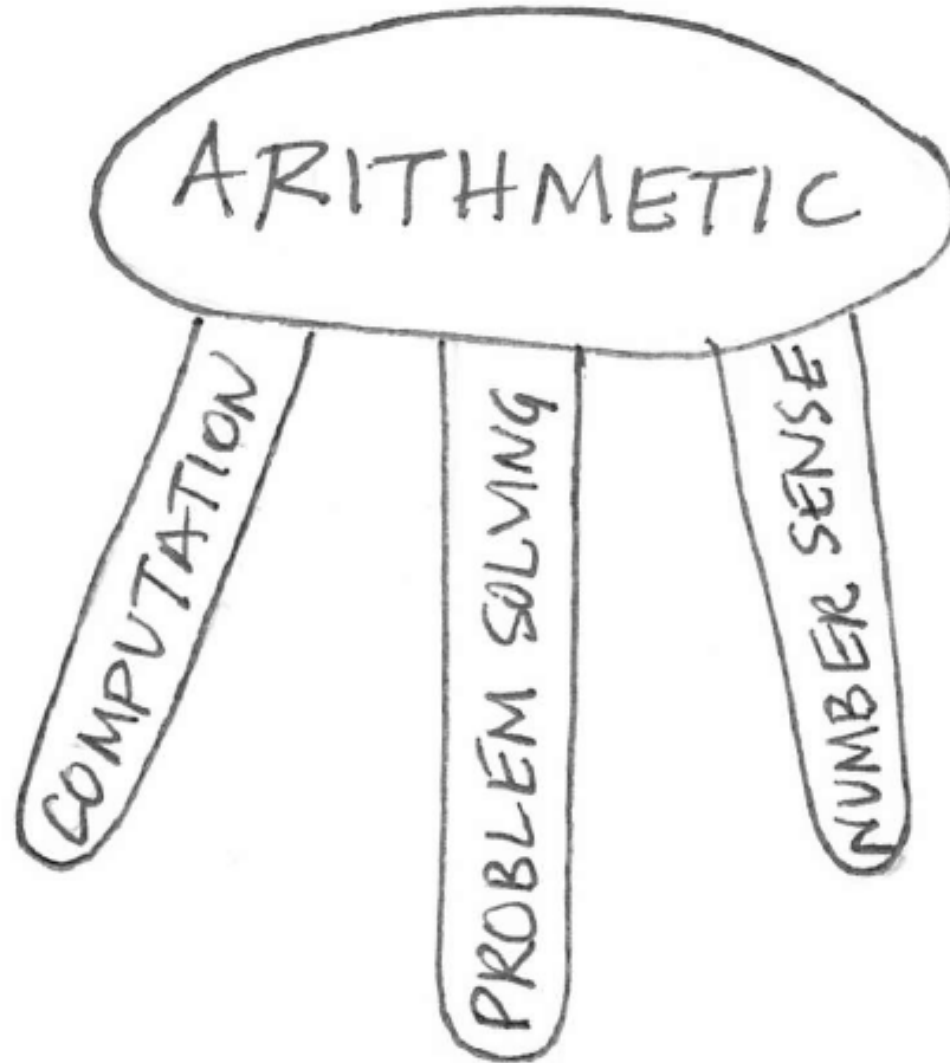
BURNS

SIGNATURE



3 R's

- Reading
- 'Riting
- 'Rithmetic



3 R's

- Reading
- 'Riting
- Reasoning

**Yours is not to reason why,
just invert and multiply.**

**Do only what makes sense to
you. And persist until it does.**

Mathematical Practices

- Numerical Reasoning
- Teachers' Math Knowledge
- Classroom Instruction

Justin: Grade 1

$$10 - 5$$

Justin: Grade 1

$$\begin{array}{r} 010 \\ \times 0 \\ \hline 5 \\ 5 \end{array}$$

Ellen: Grade 3

$$100 - 3$$

Ellen: Grade 3

$$100 - 3$$

$$100 - 98$$

Ellen: Grade 3



A handwritten multiplication problem is shown inside a hand-drawn oval. The problem is written vertically as follows:

$$\begin{array}{r} 700 \\ \times 98 \\ \hline 602 \end{array}$$

School Bus Problem

There are 295 students in the school.

School buses hold 25 students.

How many school buses are needed to
fit all of the students?



Marisa: School Bus Problem

View at <https://mathreasoninginventory.com/>
Click on Resources Tab, then Video Library



Marisa

$$\begin{array}{r} 1+ \quad 1+ \\ 295 \\ + \quad 25 \\ \hline 320 \end{array}$$

$$99 + 17$$



Ana: $99 + 17$

View at <https://mathreasoninginventory.com/>
Click on Resources Tab, then Video Library





99 +

Zakari: $99 + 17$

View at <https://mathreasoninginventory.com/>
Click on Resources Tab, then Video Library



99 + 17: Go to MRI site



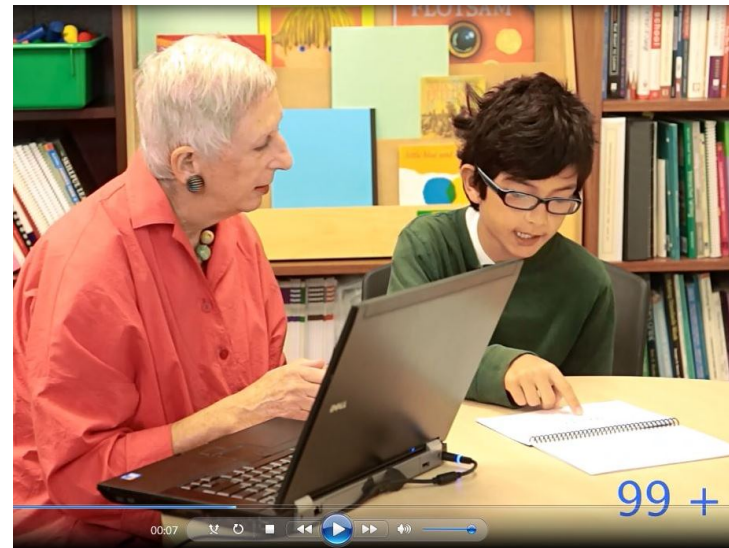
Jada



Manuel



Dina



Amir

Alex

$$\begin{array}{r} \frac{1}{2} \boxed{\frac{3}{3}} + \frac{2}{3} \boxed{\frac{2}{2}} \\ \downarrow \quad \downarrow \\ \frac{3}{6} + \frac{4}{6} = \frac{7}{6} = 1\frac{1}{6} \end{array}$$

Amelia

$$\frac{1}{2} + \frac{2}{3}$$

$$\frac{1}{2} = \frac{1.5}{3} + \frac{2}{3} = \frac{3.5}{3} = 1\frac{.5}{3}$$

“That the quality of mathematics teaching depends on teachers’ knowledge of the subject should not be a surprise.”

What Mathematical Knowledge Is Needed for Teaching Mathematics?

Deborah Loewenberg Ball
University of Michigan

Alexa

$$\begin{array}{r} 49910 \\ \cancel{5000} \\ - 328 \\ \hline 4672 \end{array}$$

Jesús

$$\begin{array}{r} 5000 \\ - 328 \\ \hline 4672 \end{array}$$

$$\begin{array}{r} 4700 \\ 4680 \end{array}$$

Are students using
reasoning strategies
that are appropriate for
the numbers at hand?

Can students be
successful in math class
without having to
reason?

$$503 - 398$$

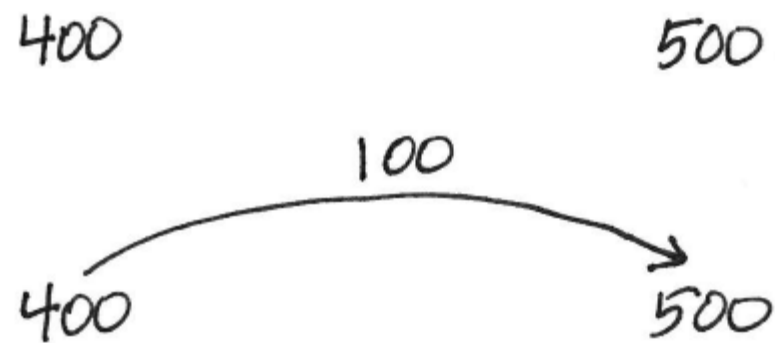
$$503 - 398$$

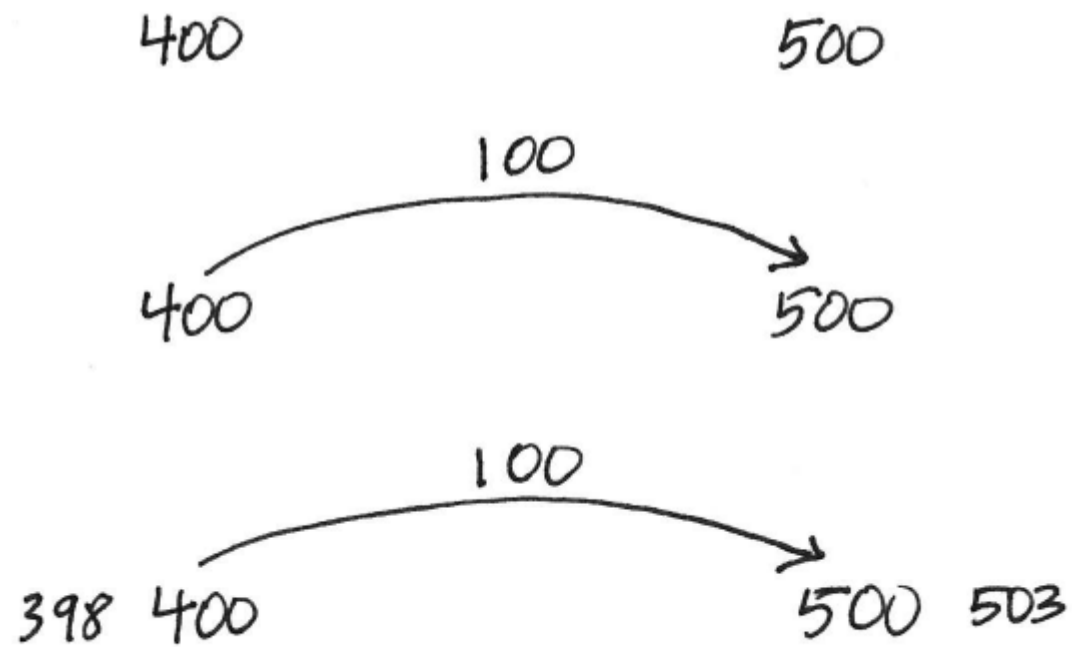
$$500 - 400$$

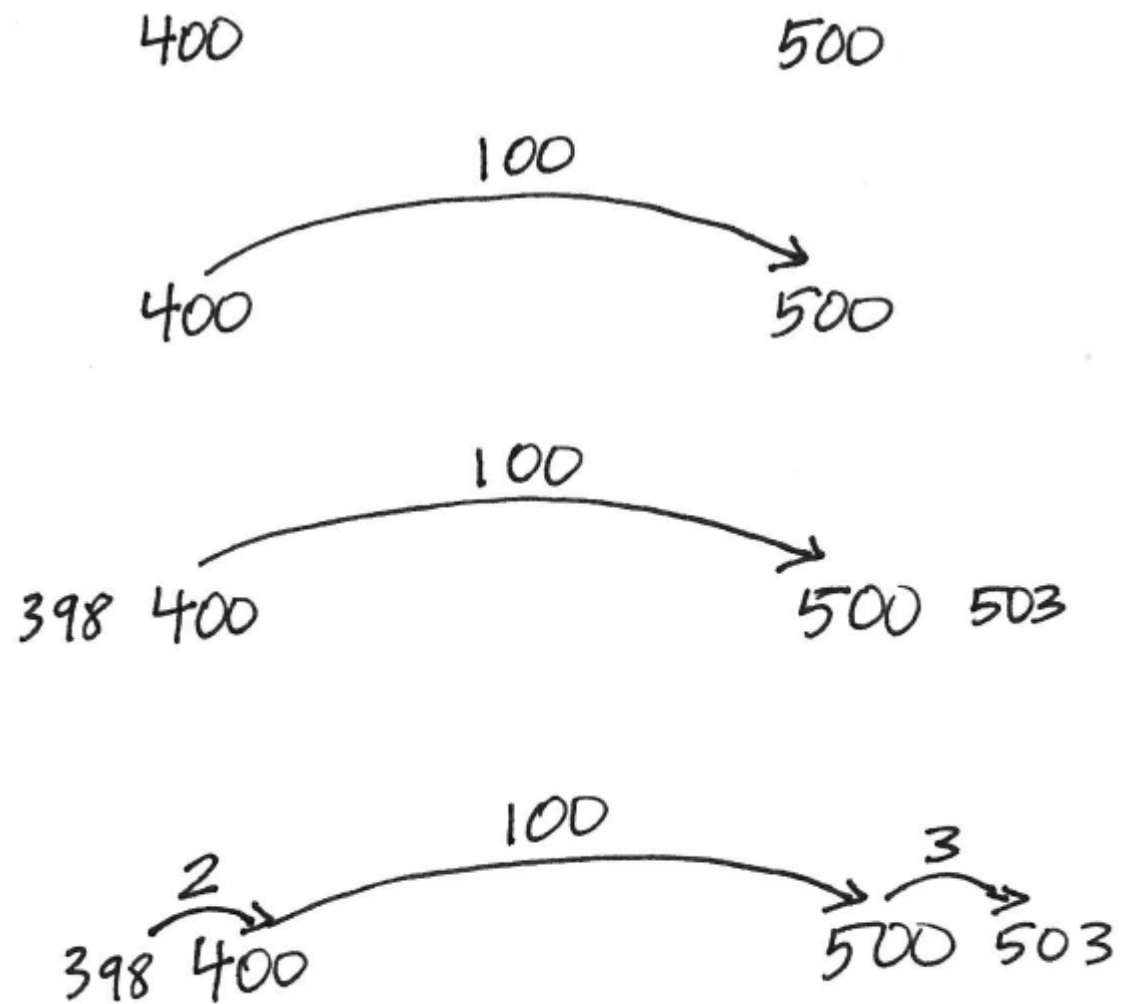
Is the answer greater or
less than 100?

400

500







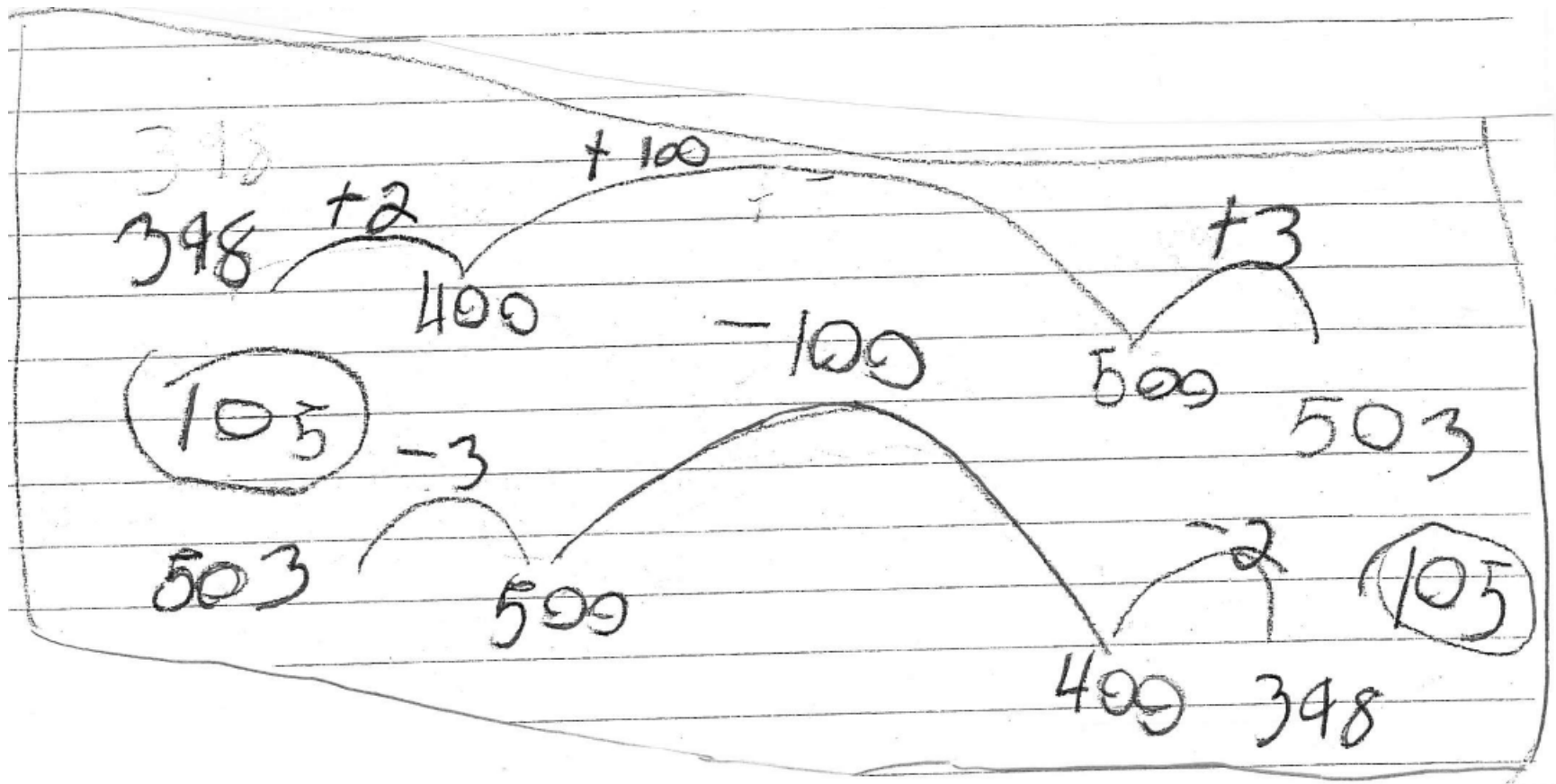
Camille

Handwritten math work on lined paper showing a subtraction problem $503 - 348$ solved using the compensation strategy. The student adds 2 to 503 to get 505, then adds 100 to get 605, and finally subtracts 348 to get 257. There are also some other numbers and a cloud-like shape with the number 105.

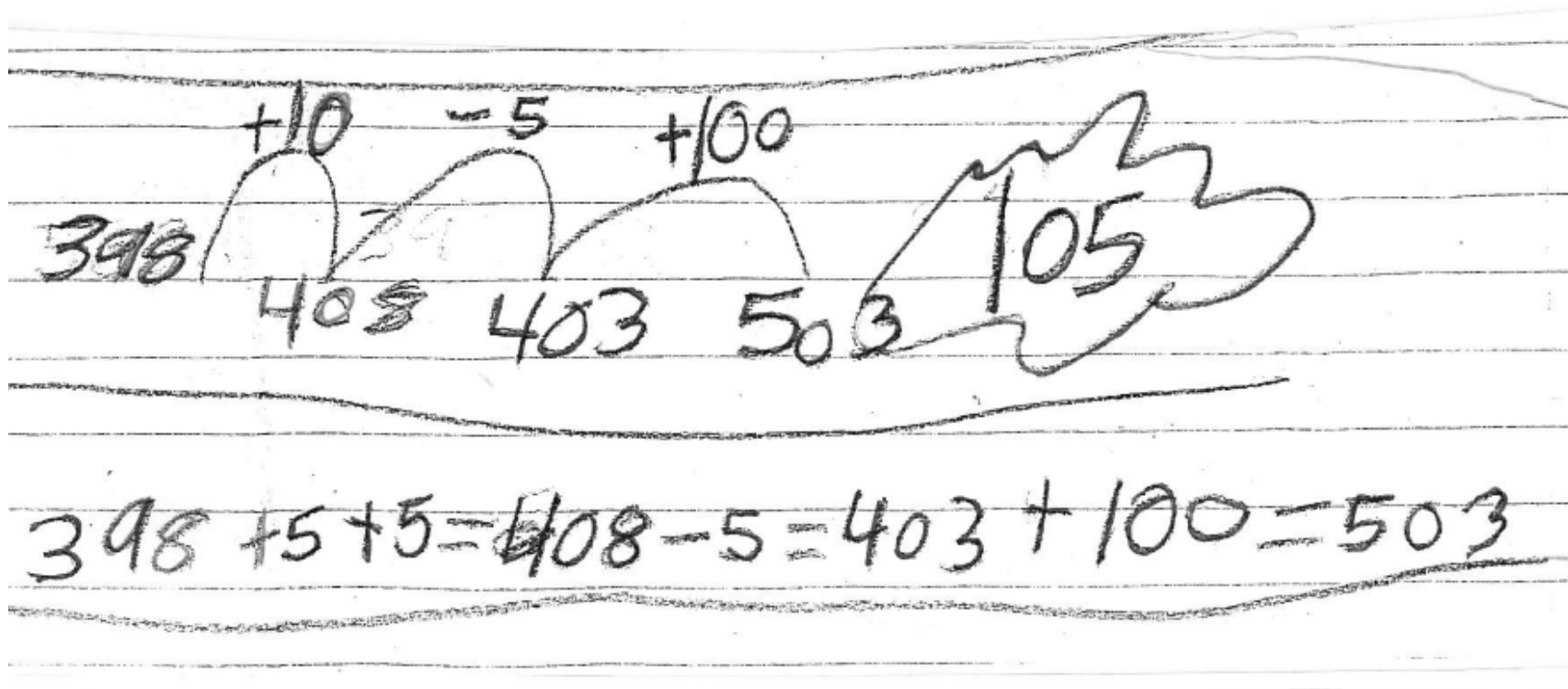
$$\begin{array}{r}
 503 - 348 \\
 +2 \quad +100 \\
 \hline
 505 \\
 +100 \\
 \hline
 605 \\
 -348 \\
 \hline
 257
 \end{array}$$

Other numbers and symbols on the page include: 398, 400, 500, 503, and a cloud-like shape containing the number 105.

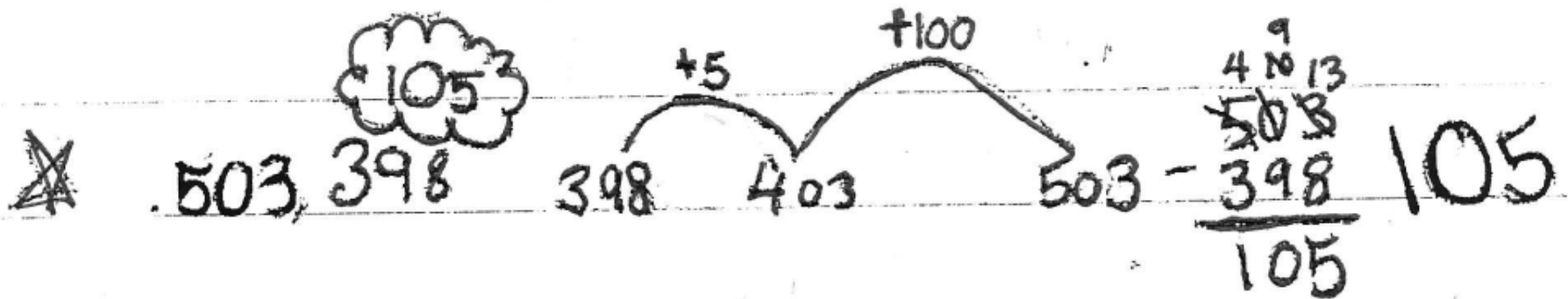
Laura



Fran



Zoe



$$12 \div 0.3$$

$$12 \div 0.3$$

Which is the exact answer?

0.04 0.4 4 40

Traction

$$12 \div 3 = 4$$

Answer has to
be higher
because .03
is smaller.

It has to be 40

Lars

Did it on
mental
chalkboard

$$0.3 \overline{)12}$$

Maverick

$12 \div 0.3$ is the
same as

$$0.3 \times \underline{\quad} = 12$$

$$12 \div 0.3$$

Which is the exact answer?

0.04 0.4 4 40

Trehsom

$$12 \div 3 = 4$$

Answer has to
be higher
because .03
is smaller.

It has to be 40

Lars

Did it on
mental
chalkboard

$$0.3 \overline{)12}$$

Maverick

$12 \div 0.3$ is the
same as

$$0.3 \times \underline{\quad} = 12$$

Quick Write

Explain why 40 is the answer to $12 \div 0.3$.

Well, $12 \div 3 = 4$ then you multiply
.03 by all the possible answers
until you get 40. Also the answer
couldn't be less than 5.

$$\begin{array}{r} .03 \\ 40 \\ \hline 00 \\ 12 \\ \hline \boxed{12} \end{array}$$

Lili

3/24/15

Sarah

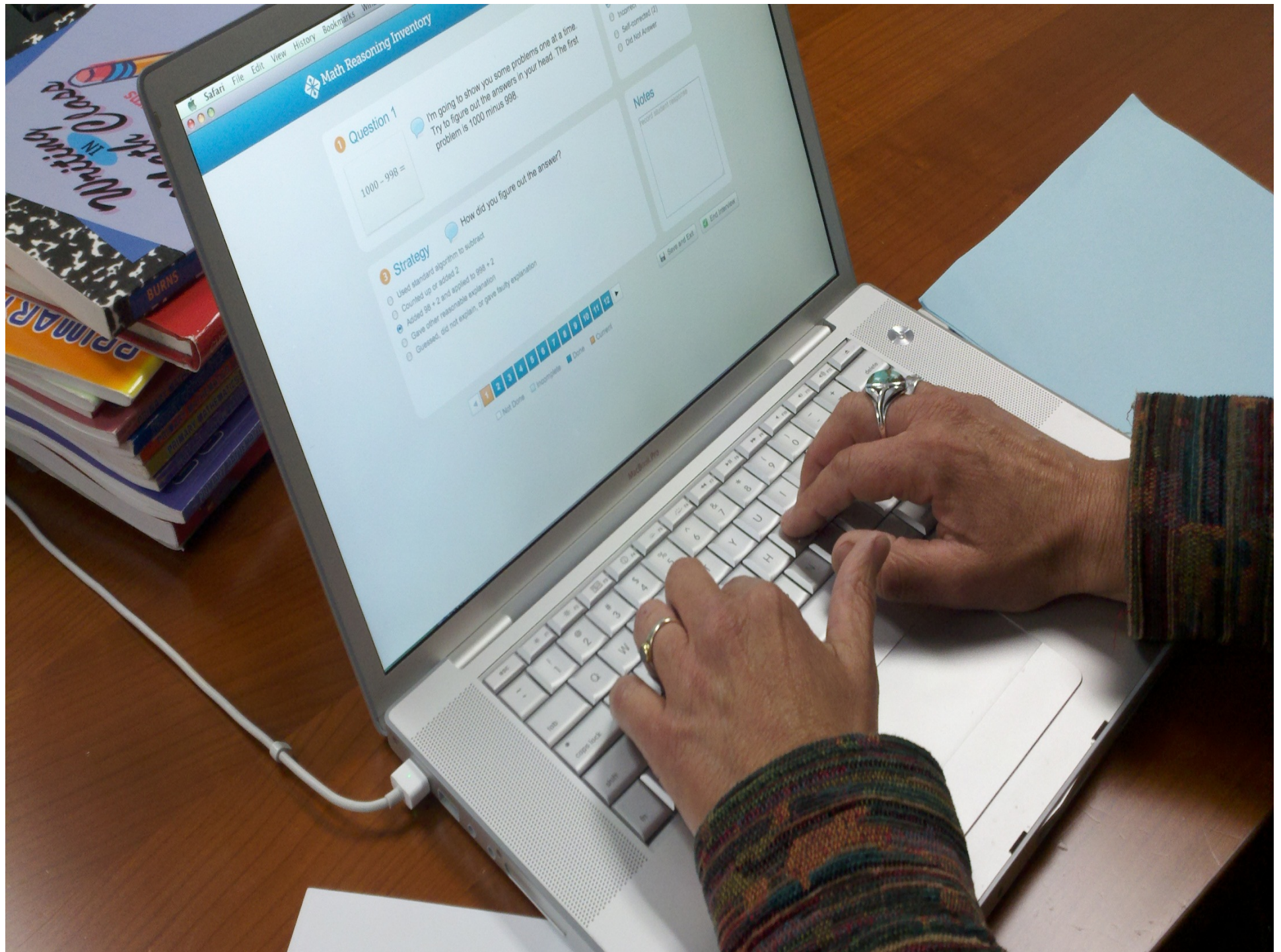
$12 \div 0.3$ is the same thing as $.3 \times \underline{\quad} = 12$
so $\begin{array}{r} 10.3 \\ \times 40 \\ \hline + 00 \\ 120 \end{array}$ and there is one decimal
point So 120 would
turn to $\boxed{12}$ so the
blank space = $\boxed{40}$
 120

· Zac

The way I solved the problem was I knew $12 \div .3 = \frac{12}{.3}$ and I knew that because the numerator was bigger than the denominator, it had to be a whole number. I also thought that $\frac{12}{3} = 4$, so $\frac{12}{.3}$ must be bigger than 4, by 10 times, 10 times 4 is 40 so I knew that must be the answer.

Lars

If done using a mental chackboard you will always get the same answer using paper, unless human error.



**1** Question 9

$$12 \div 0.3$$

0.04 0.4 4 40



For this division problem, look at the choices and decide which is the exact answer.

2 Answer

- ☐ Correct (40)
- ☐ Incorrect
- ☐ Self-corrected (40)
- ☐ Did Not Answer

3 Explanation

How did you figure out the answer?

- ☐ Divided 12 by 3 and then adjusted decimal point
- ☐ Multiplied and then adjusted decimal point
- ☐ Analyzed choices and chose one that seemed most reasonable
- ☐ Moved decimal point one place to the right in both numbers and divided
- ☐ Gave other reasonable explanation
- ☐ Guessed, did not explain, or gave faulty explanation

Notes

record student response

Amir

View at <https://mathreasoninginventory.com/>
Click on Resources Tab, then Video Library



Dina

View at <https://mathreasoninginventory.com/>
Click on Resources Tab, then Video Library



From: *About Teaching Mathematics*, 4th Edition

The Two-Dice Sum Game

You need: counters, 11 per player

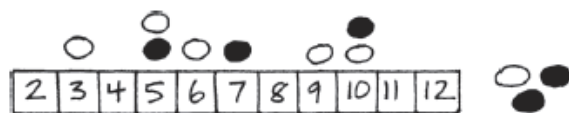
1–6 dice, 2 per pair or group of students

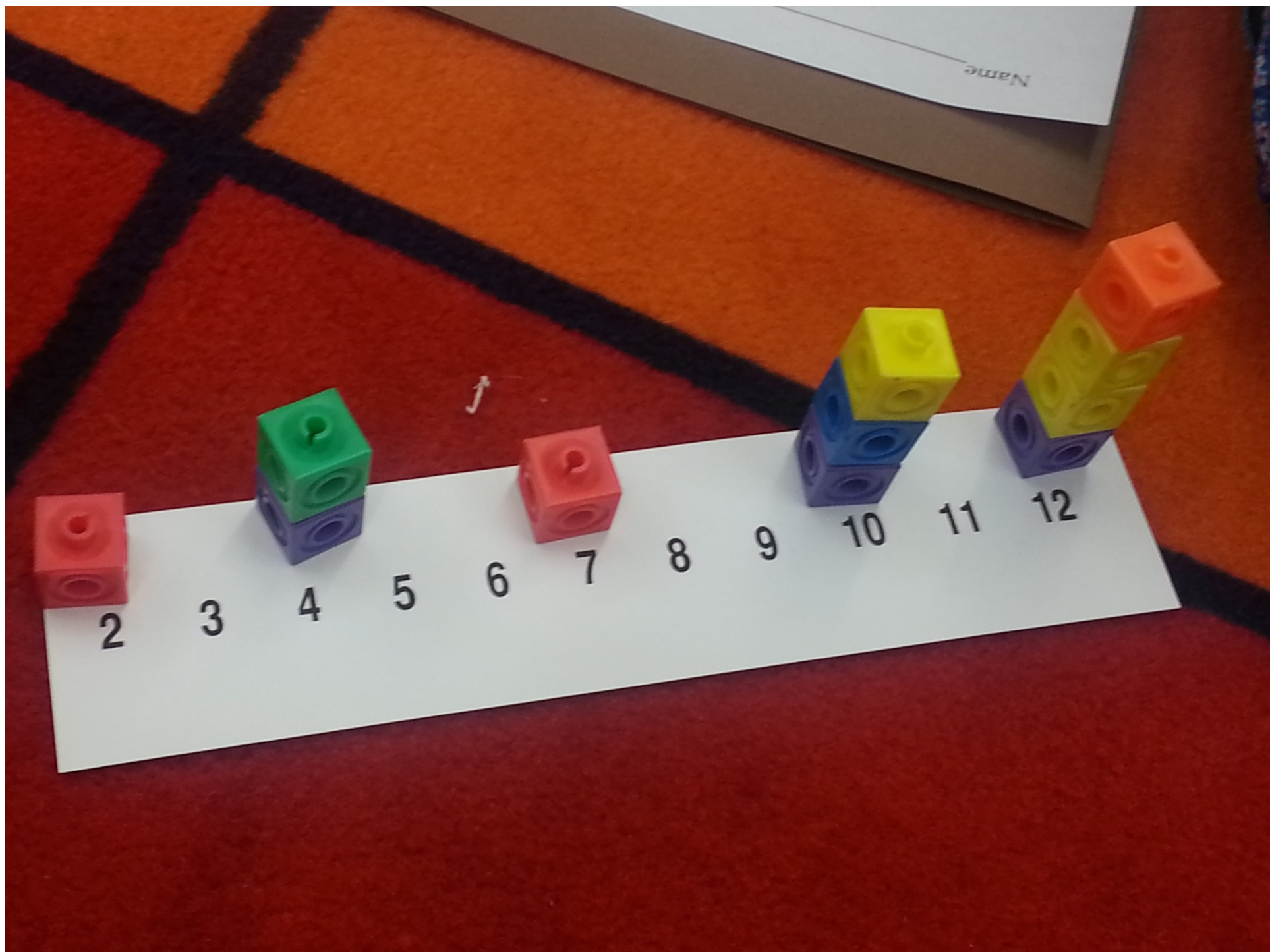
This is a game for two or more players. Each player draws a number line from 2 to 12, with spaces between the numbers large enough so counters fit on the numbers.

Place your eleven counters on your number line in any arrangement. (You may put more than one counter on some numbers and none on others.)

Take turns rolling the dice. On each roll, every player removes one counter that's on the number that matches the sum of numbers that come up on the two dice. (If players have more than one counter on a sum, they may remove only one of them. If players don't have a counter on a sum, they don't do anything.) The winner is the first player to remove all eleven counters.

Decide on the best winning arrangement of counters on the number line. Explain your thinking.





Name _____

Math Menu

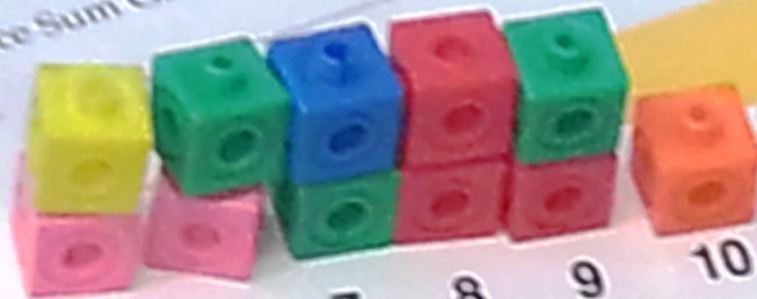
Pattern Block Explorations (I)

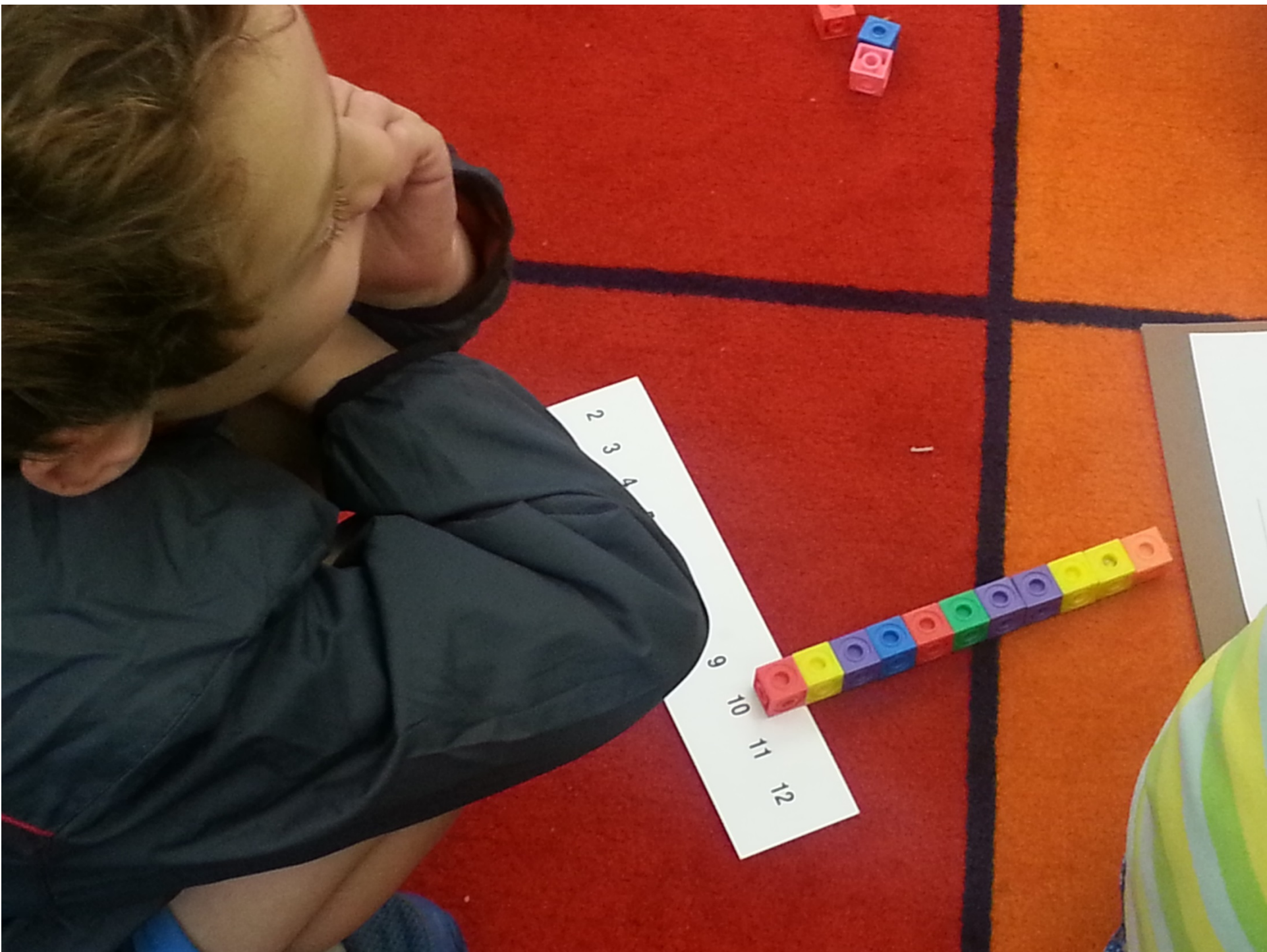
The Two-Dice Sum Game (P)

Partners: _____



2 3 4 5 6 7 8 9 10 11 12





Name _____

Math Menu

Pattern Block Explorations (I) ☐

The Two-Dice Sum Game (P) ☐

Players: _____

2

3

4

5

6

7

8

9

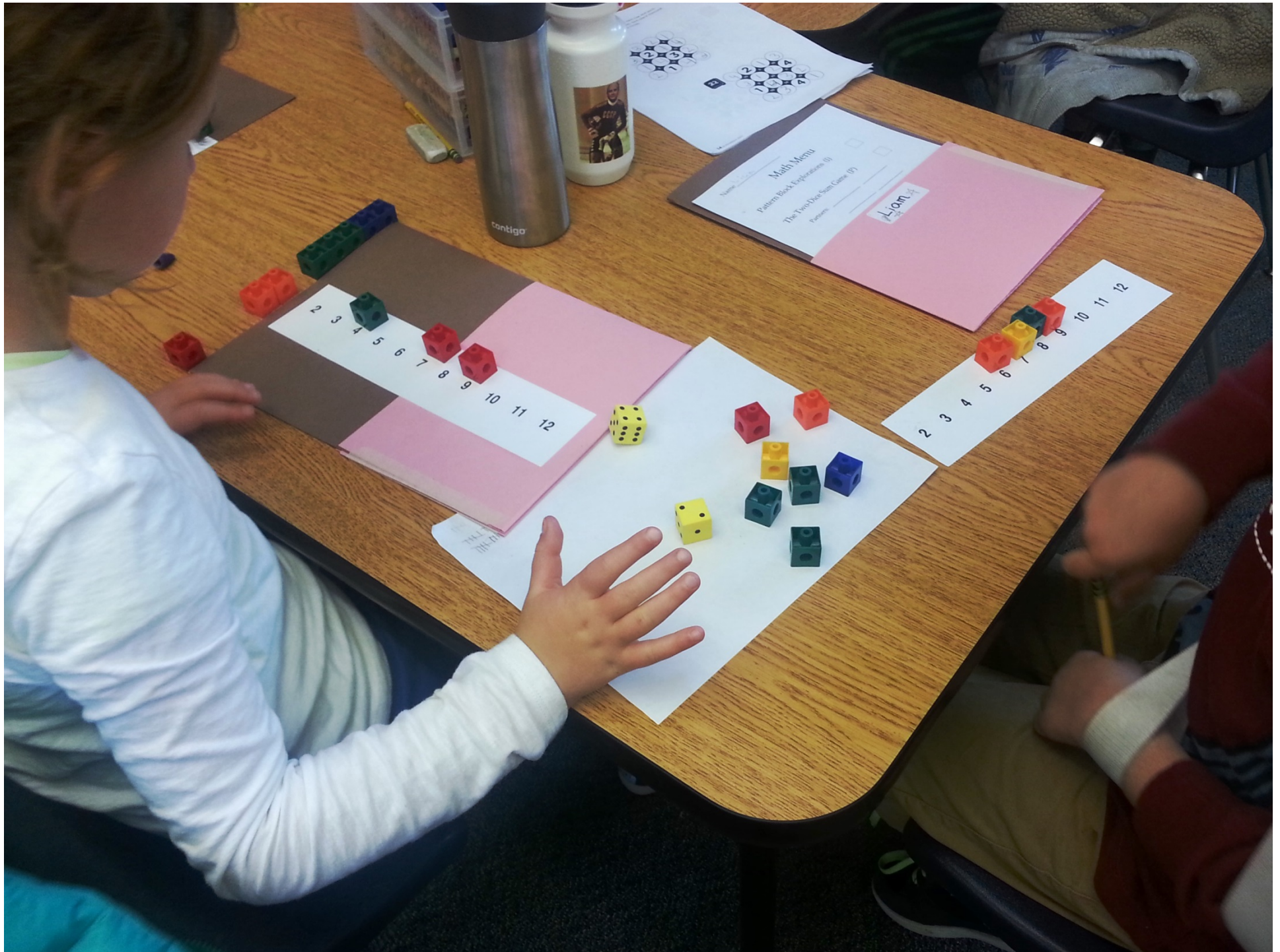
10

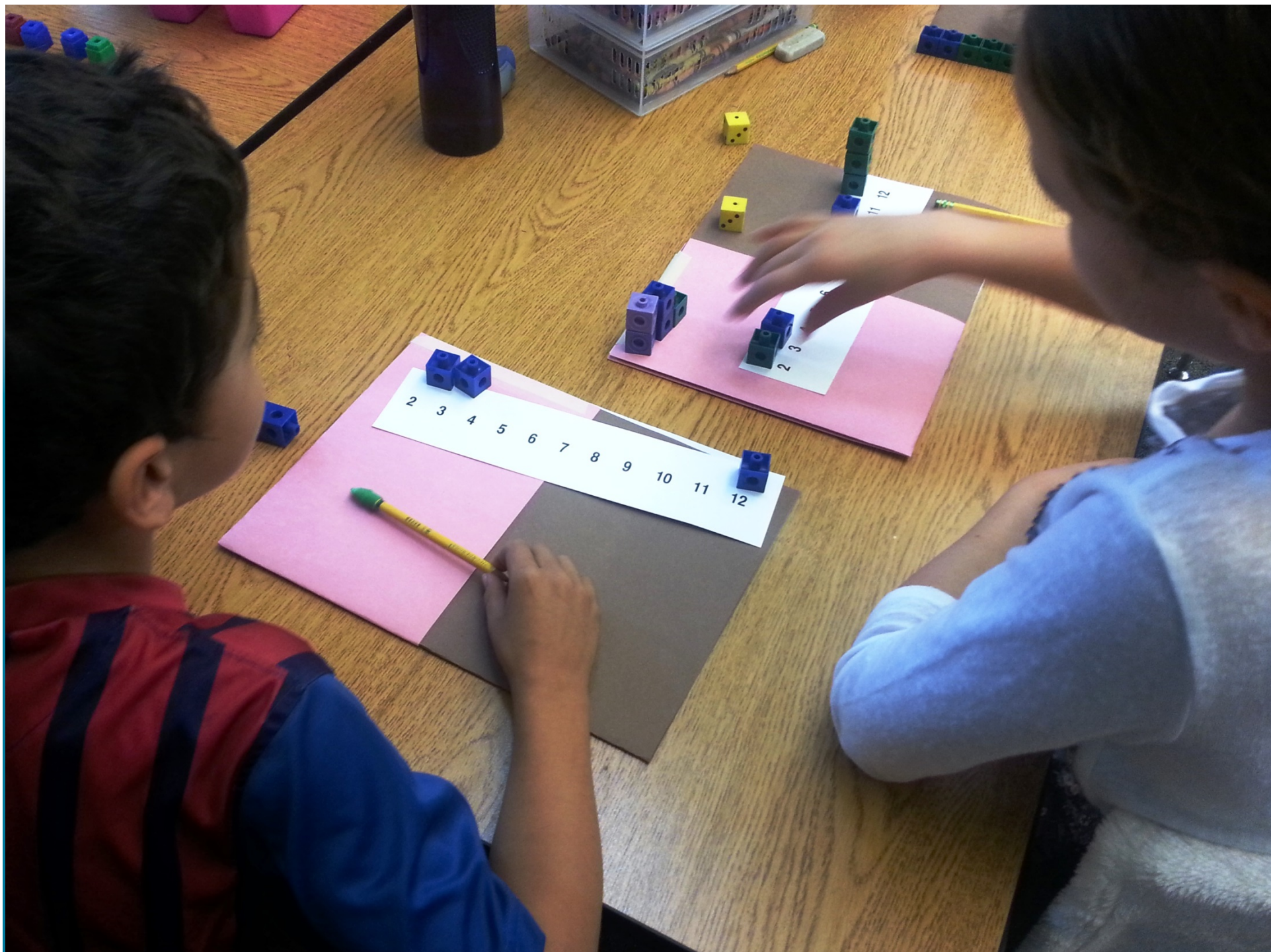
11

12

Comparing Names (I) ☐







Is It Fair? (Version 1)

Who wins?

A

B

Is It Fair? (Version 2)

Who wins?

A

B

Is It Fair? (Version 3)

Who wins?

A

B

Is It Fair? (Version 1)

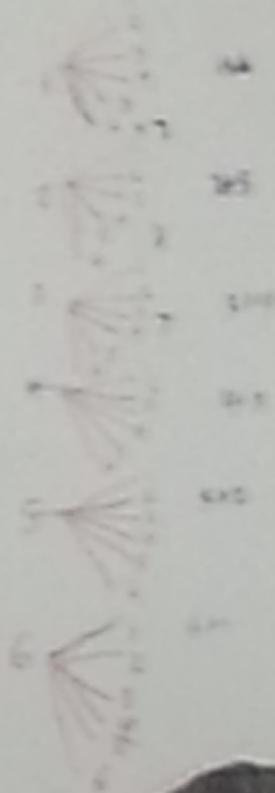
Player A: even

Player B: odd

2 3 4 5 6 7 8 9 10 11 12

Describe
outcomes

Equally
likely
Outcomes



Computer Lab

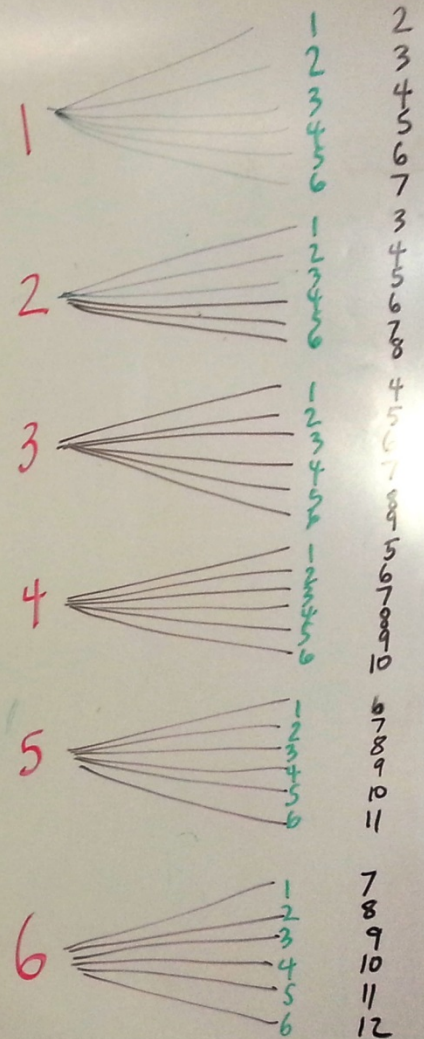
36 Equally Likely Ways to Get 11 Possible Sums

	2	3	4	5	6	7	8	9	10	11	12
1+1		1+2	1+3	1+4	1+5	1+6	2+6	3+6	4+6	5+6	6+6
		2+1	2+2	2+3	2+4	2+5	3+5	4+5	5+5	6+5	
			3+1	3+2	3+3	3+4	4+4	5+4	6+4		
				4+1	4+2	4+3	5+3	6+3			
					5+1	5+2	6+2				
						6+1					

1 2 3 4 5 6 5 4 3 2 1

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

x	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36



Two Dice Sum Game

what your board should look like:

□	□	□	□	□	□	□	□	□	□	□	□
2	3	4	5	6	7	8	9	10	11	12	

Dear Second Graders,

A good Strategy for this game is to put 1 cube on each number.

This is a good strategy because then you have at least one cube for every number. Although, this strategy doesn't always work because once you take off that one cube you can never take off another cube.

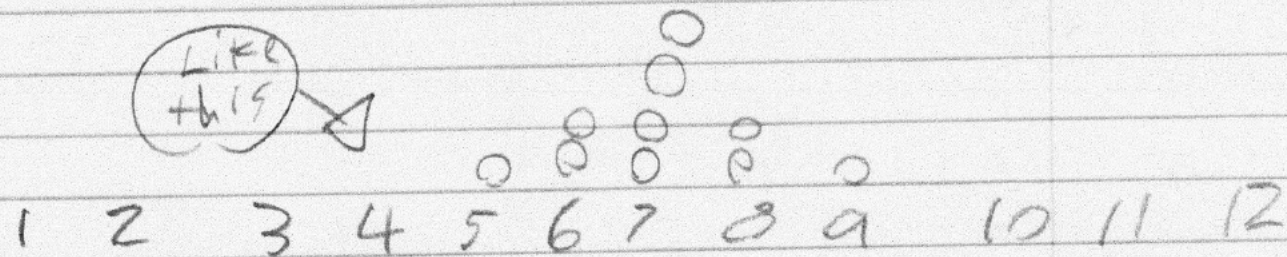
Good Luck!

- Sarah Frascchetti

Dear second graders,

when you play ten or I recommend
making a "mountain around the seven", this
will give you a high probability of winning

sincerely, Jackson



Dear second grade Students From Ms. Marcuses class,

My advice for winning the two dice Sum game is not to pile them all up on one number, but to scatter them out. The odds are higher on 7, 8, and 6. There are the most possible combinations for these, there are 6 possible combos for 7, and 5 possible combos for 6 and 8. There are the most possible combos for 7 and it goes down from there

	2	3	4	5	6	7	8	9	10	11	12
possible Combos	→ 1	2	3	4	5	6	5	4	3	2	1

Sincerely,
Charlie Dyer

From Ms. Adams class.

Dear Second Graders,

Your strategy for the game should be; learn from your past games. know generally which numbers are rolled the most, usually 5-9, and place your cubes accordingly. 8 works for me.

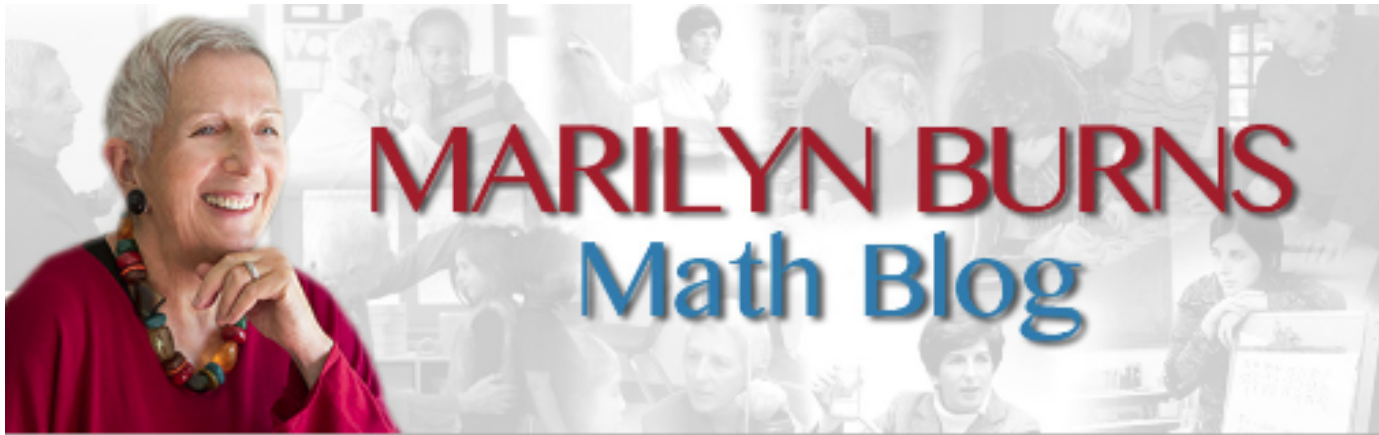
Please reply so I know how this strategy works out for you!

Sincerely,
Anonymous

Our teaching goal is
not to *cover* the
Standards . . .

**Our teaching goal is
not to *cover* the
Standards . . .**

but to *uncover* them.



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ABOUT TEACHING Mathematics

Marilyn Burns

