Digging into Fraction Division

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Have you ever heard…..

Yours is not to reason why,
Just invert and multiply!

As opposed to....

"Children who are successful at making sense of mathematics are those who believe that mathematics makes sense."

-Lauren Resnick

During our time together...

- Explore fraction division
- Engage in some activities to help students connect fraction division to whole number division
- Share some useful tools
- Make sense of the invert and multiply algorithm

Do you always have to invert and multiply?

Your friend tells you she doesn't understand why your teacher makes you invert and multiply to divide fractions. She says you can just divide across the numerators and denominators to get your answer. She shows you the two examples below to prove her point:

$$\frac{4}{9} \div \frac{2}{3} = \frac{2}{3} \qquad \frac{15}{8} \div \frac{3}{4} = \frac{5}{2}$$

What do you think of her idea?
- Is she right?
- If so, why? If not, why not?

Write a word problem that can be solved by the expression below:

$$\frac{3}{4} \div \frac{1}{2}$$

Two types of division situations:

- **Quotative** (also called measurement division):
  - Size of group is known; number of groups is unknown
  - 6 ÷ 2:   How many 2 's are in 6?
- **Partitive**:
  - Number of groups is known; how many in each group is unknown
  - 6 ÷ 2:   Split 6 into 2 groups ➔ 6 is 2 groups of what?

Quotative Division

- **Quotative** (also called measurement division):

  - $6 \div 2$: How many 2's are in 6?

    $\dfrac{3}{4} \div \dfrac{1}{2}$: How many $\dfrac{1}{2}$'s are in $\dfrac{3}{4}$?

    $1 \div \dfrac{1}{4}$: How many $\dfrac{1}{4}$'s are in 1?

How Long? How Far? Part 1

How many $\frac{1}{4}$ minutes are in 1 minute?

How many $\frac{1}{4}$s are in 1?

- Use your Cuisenaire rods and number line to show how many $\frac{1}{4}$s are in 1.

- Use your Cuisenaire rods and number line to show how many $\frac{1}{4}$s are in 2.

- Use your Cuisenaire rods and number line to show how many $\frac{1}{3}$s are in 2.

How many $\frac{1}{4}$s are in 1?

View video: "How many 1⁄4s are in 1?"

https://mathsolutions.wistia.com/medias/mbae8pjeny

Reasoning about $1 \div \frac{1}{6}$

$1 \div \frac{1}{6} = 6$

6 becaues ~~the red fod~~ 6 of the reel rod fit perfictly in 1.

Reasoning about $2 \div \frac{1}{6}$

$) \ 2 \div \frac{1}{6} =$

How many $\frac{1}{6}$ are in 2?

Reasoning about $10 \div \frac{1}{3}$

c.) $10 \div \frac{1}{3} = 30$

How many $\frac{1}{3}$s are in 10?
I Multiplied the Denominator and the Problem got me 30.

Reasoning about $6 \div \dfrac{2}{3}$

d. $6 \div 3/4 = ?$

goes into
6 8 times

¾, 1¾, 2¼, 3, 3¾, 4¾, 5¼, 6
1   2   3   4   5   6   7   8

Ship counted to 6 wholes
and it took 8 times

Partitive Division

- **Partitive division:**
  - $6 \div 2$: Split 6 into 2 groups ➔ 6 is 2 groups of what?
  - $\frac{3}{4} \div 4$: Split $\frac{3}{4}$ into 4 groups ➔ $\frac{3}{4}$ is 4 groups of what?

How Long? How Far? Part II

Distance	Number of people	Each person cleans
8 miles	2	4 miles
4 miles	2	2 miles
2 miles	2	1 mile
1 mile	2	½ mile
½ mile	2	? mile

How Long? How Far? Part II

1. $\frac{1}{4} \div 3$

2. $\frac{1}{6} \div 2$

3. $\frac{1}{2} \div 3$

4. $\frac{3}{4} \div 3$
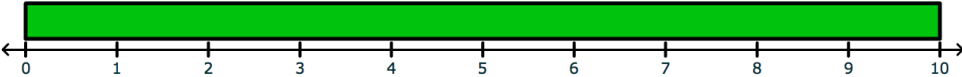
5. $\frac{2}{3} \div 6$

Free Tools from Conceptua Math
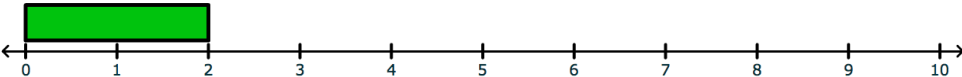
- Measurement (quotative)
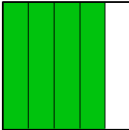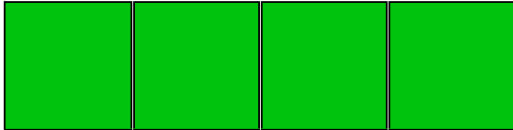
conceptuamath.com

Link to Dividing with Fractions

Ted has a part-time job cleaning the rooms at a hotel after the guests check out. He works 4 hours a day. It takes him 1/2 hour to clean each room. How many rooms can he clean each day?
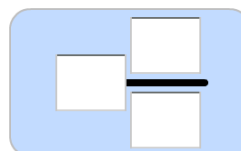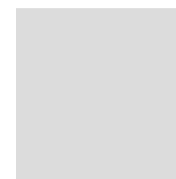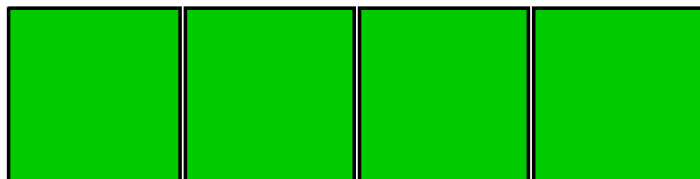
How many groups of $\dfrac{1}{2}$ in 4 ?

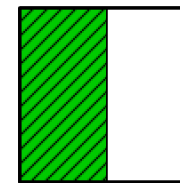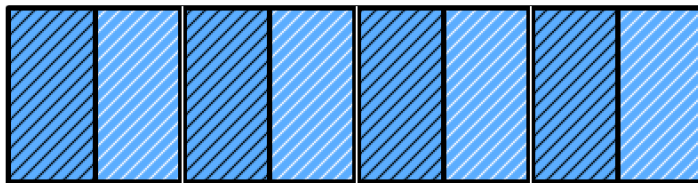When the model to the left shows the dividend, create the divisor in the model below.

units

How many Contained in the Starting Value?

🔊 Ted has a part-time job cleaning the rooms at a hotel after the guests check out. He works 4 hours a day. It takes him 1/2 hour to clean each room. How many rooms can he clean each day?

🔊 How many groups of $\dfrac{1}{2}$ in 4 ?

8 — rooms

Partitive Division

- Equal Shares (partitive)
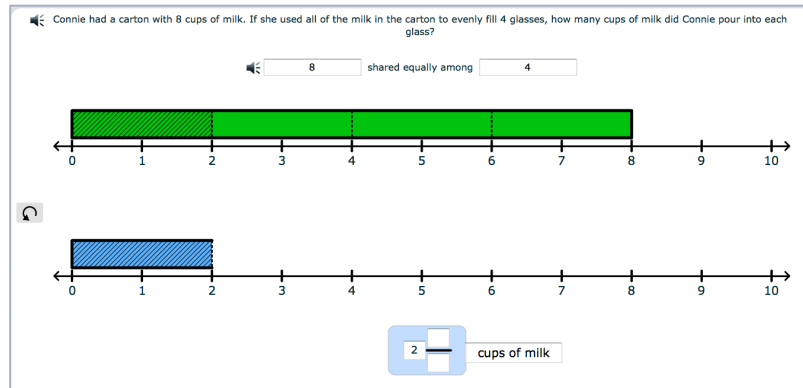
conceptuamath.com
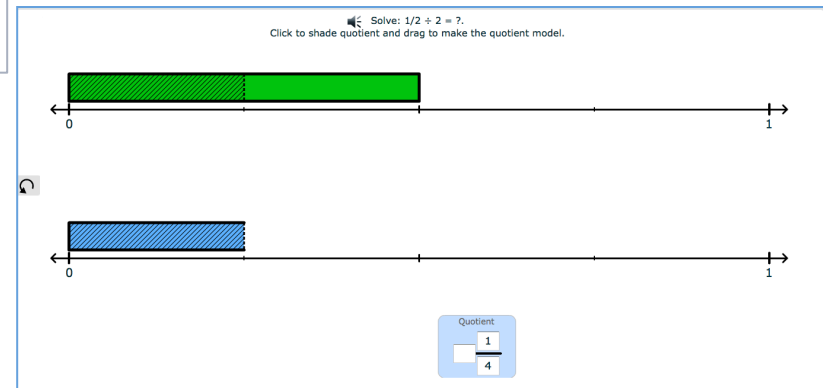
Dividing with Fractions

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Janice purchased 3/4 of a pound of mixed nuts. The container listed that there were an equal amount of each of 4 different types of nuts. How much of a pound of each type of nut does it contain?

$$\frac{3}{4}$$ shared equally among 4

When the model below shows the dividend, divide it into equal shares.

Quotient

units

Divide Proper Fraction by Whole Number: Sharing 9-1-2

Divide Starting Value into Equal Parts

Janice purchased 3/4 of a pound of mixed nuts. The container listed that there were an equal amount of each of 4 different types of nuts. How much of a pound of each type of nut does it contain?

$\dfrac{3}{4}$ shared equally among 4

The quotient is the size of one equal share; when the model to the left is divided into the correct equal shares, shade the quotient and drag it to the model below to measure it.
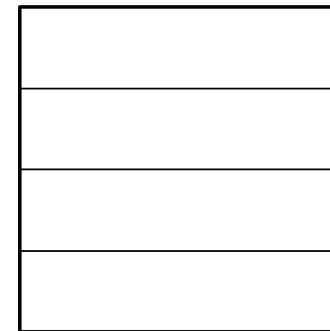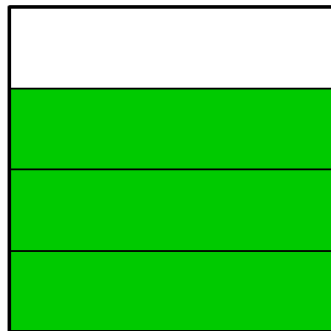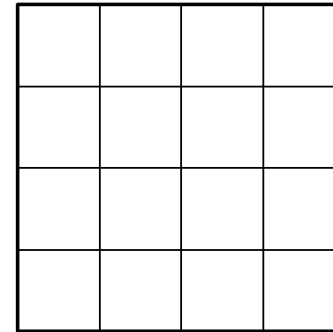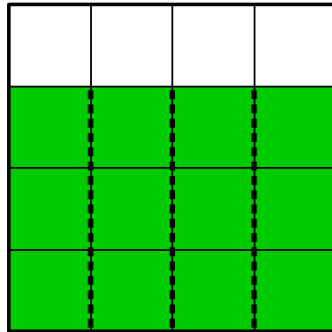
Quotient

units

Value of 1 Share: Quotient

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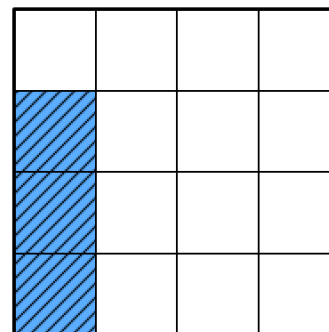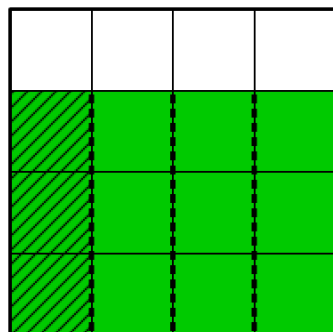
Quotient

$$\frac{3}{16}$$ of a pound

Ok, so why do we invert and multiply?

You have 28 students and you want to split them into 2 equal groups. How many students are in each group?

$$28 \div \underline{} = 14$$
$$28 \times \underline{} = 14$$

Ok, so why do we invert and multiply?

You have 28 students and you want to split them into 2 equal groups. How many students are in each group?

$$28 \div \underline{\ \ 2\ \ } = 14$$
$$28 \times \underline{\ \ \tfrac{1}{2}\ \ } = 14$$

Let's try some more…

$$24 \div \underline{} = 8$$
$$24 \times \underline{} = 8$$

$$15 \div \underline{} = 3$$
$$15 \times \underline{} = 3$$

$$10 \div \underline{} = 40$$
$$10 \times \underline{} = 40$$

What do you notice?

$$24 \div 3 = 8$$
$$24 \times \frac{1}{3} = 8$$

$$15 \div 5 = 3$$
$$15 \times \frac{1}{5} = 3$$

$$10 \div \frac{1}{4} = 40$$
$$10 \times 4 = 40$$

Generalization

Multiplying by a number and dividing by its multiplicative inverse are mathematically equivalent.

Let's return to where we began..

- Try the divide across method:

$$\frac{3}{4} \div \frac{2}{3} = \frac{\frac{3}{2}}{\frac{4}{3}}$$

Let's return to where we began

$$\frac{\frac{3}{2}}{\frac{4}{3}} \times \frac{\frac{3}{4}}{\frac{3}{4}}$$

Renaming to fractions with common denominators before dividing

$$\frac{3}{4} \div \frac{2}{3}$$

$$\left(\frac{3}{4} \times \frac{3}{3}\right) \div \left(\frac{2}{3} \times \frac{4}{4}\right) = \frac{9}{12} \div \frac{8}{12}$$

Do the same thing to the dividend and divisor

$$12 \div 6 = 2$$
$$(12 \times 4) \div (6 \times 4) = 2$$
$$(12 \div 2) \div (6 \div 2) = 2$$
$$(12 \times 5) \div (6 \times 5) = 2$$
$$(12 \div 3) \div (6 \div 3) = 2$$

$$\frac{3}{4} \div \frac{2}{3}$$

$$\left(\frac{3}{4} \times \frac{3}{2}\right) \div \left(\frac{2}{3} \times \frac{3}{2}\right) = \frac{3}{4} \times \frac{3}{2} \div 1$$

Borrowing from the classroom...

*Now **I can** reason why,*
About invert and multiply!

For more information…

- About *Beyond Invert and Multiply: Making Sense of Fraction Computation*
  - Go to mathsolutions.com
- About *Conceptua*
  - Go to conceptuamath.com
  - Email lsusi@conceptuamath.com

Thank you!!

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