Presenters: Amy Mayfield and Lu Ann Weynand

Houghton
Mifflin
Harcourt.

Set Your Sights High: Teaching Arithmetic with an Algebra Lens

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FOUNDED BY MARILYN BURNS

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Solve Mentally

- 99 + 17 = ?

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Tyrone
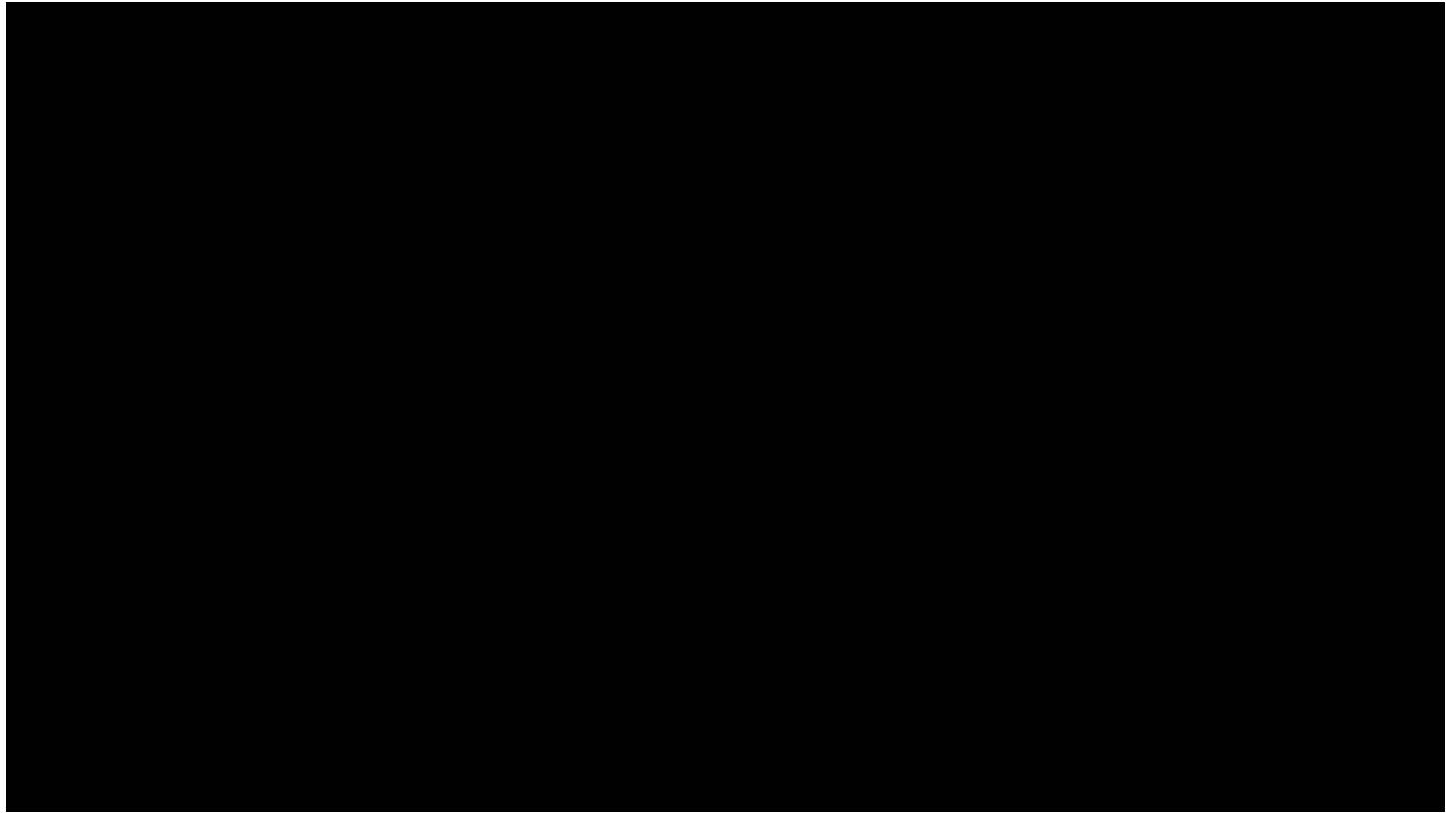
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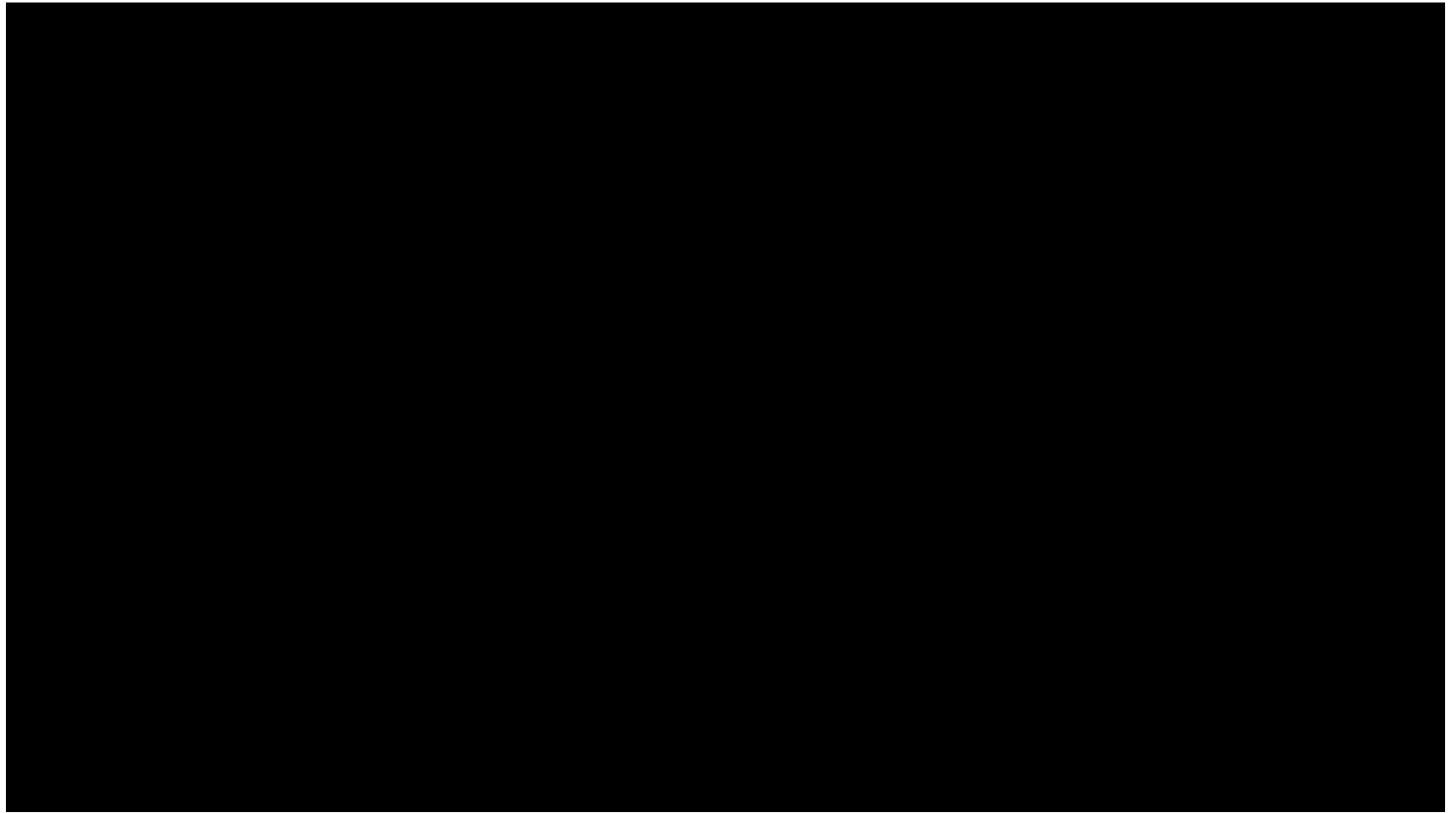
Turn and Talk….

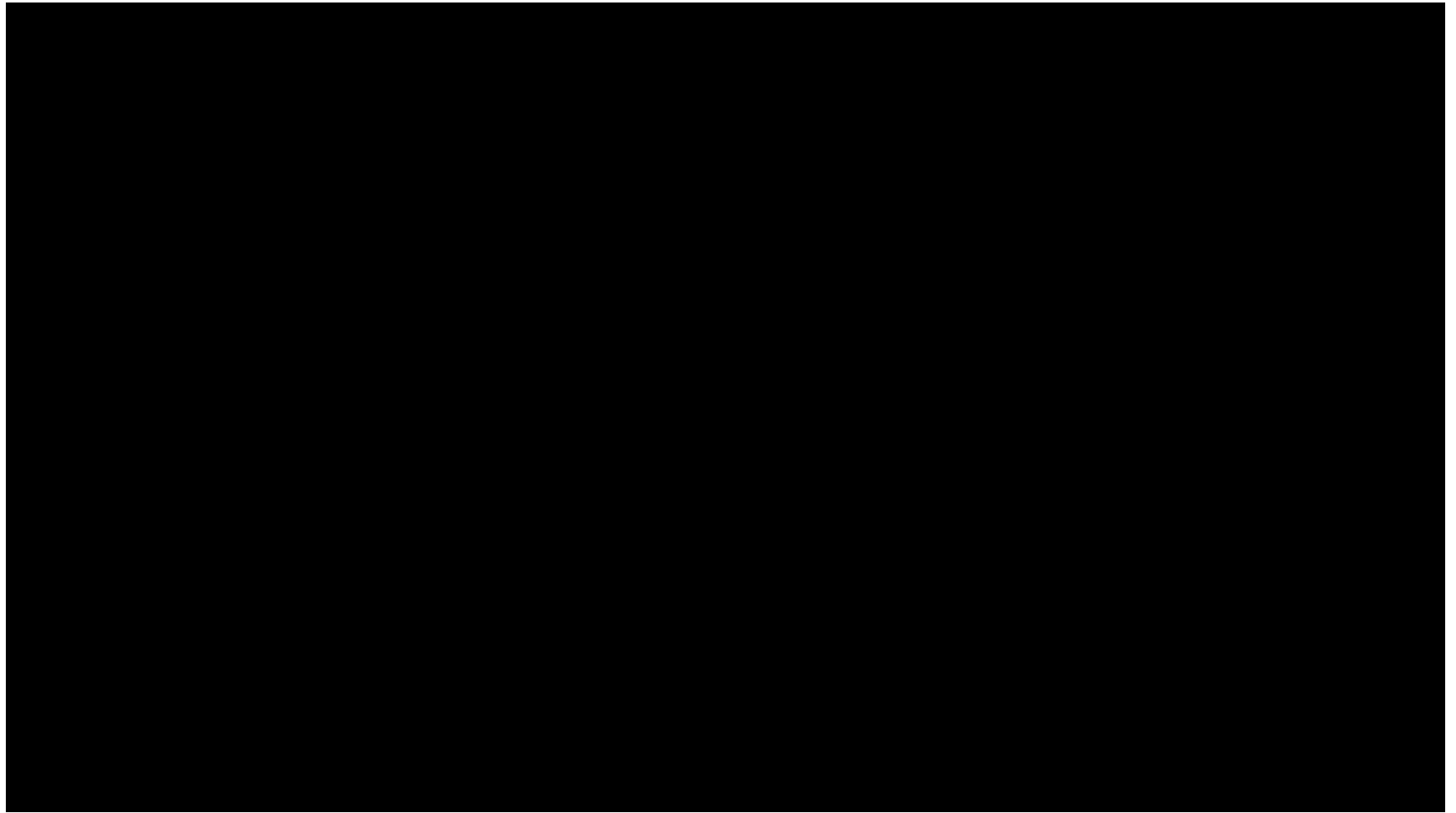- What did you observe that will impact these two students as they move forward mathematically?

Dianna

Alberto

Turn and Talk….

- What did you observe that will impact these two students as they move forward mathematically?
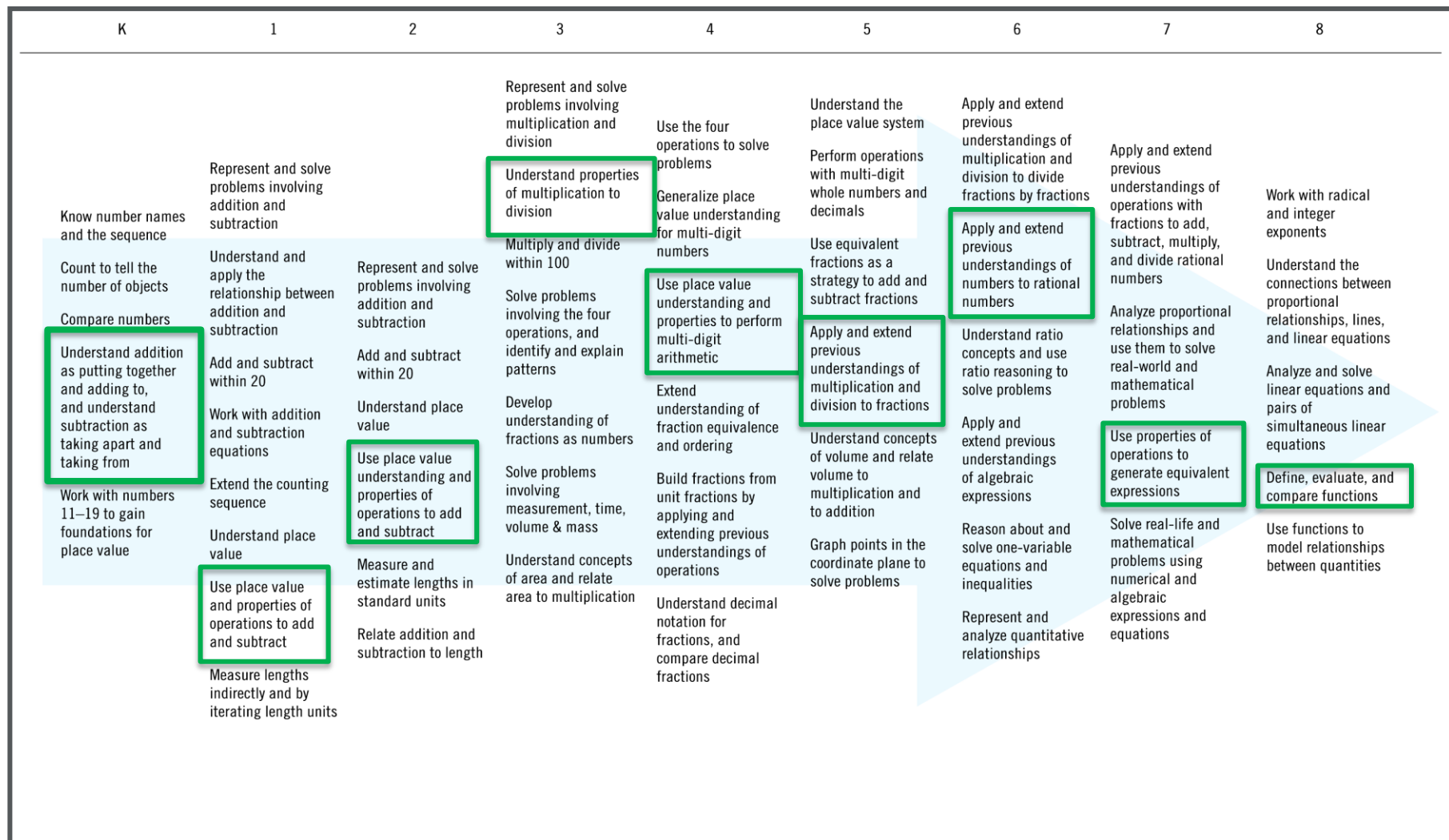
USA:

- **How can I teach my kids to get the answer to this problem?**

Japanese:

- **How can I use this problem to teach the mathematics of this unit?**

~ Phil Daro

Pathway to Algebra

K	1	2	3	4	5	6	7	8

K
- Know number names and the sequence
- Count to tell the number of objects
- Compare numbers
- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from
- Work with numbers 11–19 to gain foundations for place value

1
- Represent and solve problems involving addition and subtraction
- Understand and apply the relationship between addition and subtraction
- Add and subtract within 20
- Work with addition and subtraction equations
- Extend the counting sequence
- Understand place value
- Use place value and properties of operations to add and subtract
- Measure lengths indirectly and by iterating length units

2
- Represent and solve problems involving addition and subtraction
- Add and subtract within 20
- Understand place value
- Use place value understanding and properties of operations to add and subtract
- Measure and estimate lengths in standard units
- Relate addition and subtraction to length

3
- Represent and solve problems involving multiplication and division
- Understand properties of multiplication to division
- Multiply and divide within 100
- Solve problems involving the four operations, and identify and explain patterns
- Develop understanding of fractions as numbers
- Solve problems involving measurement, time, volume & mass
- Understand concepts of area and relate area to multiplication

4
- Use the four operations to solve problems
- Generalize place value understanding for multi-digit numbers
- Use place value understanding and properties to perform multi-digit arithmetic
- Extend understanding of fraction equivalence and ordering
- Build fractions from unit fractions by applying and extending previous understandings of operations
- Understand decimal notation for fractions, and compare decimal fractions

5
- Understand the place value system
- Perform operations with multi-digit whole numbers and decimals
- Use equivalent fractions as a strategy to add and subtract fractions
- Apply and extend previous understandings of multiplication and division to fractions
- Understand concepts of volume and relate volume to multiplication and to addition
- Graph points in the coordinate plane to solve problems

6
- Apply and extend previous understandings of multiplication and division to divide fractions by fractions
- Apply and extend previous understandings of numbers to rational numbers
- Understand ratio concepts and use ratio reasoning to solve problems
- Apply and extend previous understandings of algebraic expressions
- Reason about and solve one-variable equations and inequalities
- Represent and analyze quantitative relationships

7
- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers
- Analyze proportional relationships and use them to solve real-world and mathematical problems
- Use properties of operations to generate equivalent expressions
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations

8
- Work with radical and integer exponents
- Understand the connections between proportional relationships, lines, and linear equations
- Analyze and solve linear equations and pairs of simultaneous linear equations
- Define, evaluate, and compare functions
- Use functions to model relationships between quantities

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Arithmetic ⟷ Algebraic Thinking

…using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (multiplication/division).

Understanding Arithmetic: Three Pillars

- Understanding numbers

- Developing computational fluency

- Examining the behavior of the operations

Setting the Stage for Algebra Readiness

Decompose Small Numbers

$$6 = 1 + 5$$
$$6 = 2 + 4$$
$$6 = 3 + 3$$

Decompose to Find Sums

$$18 + 6 =$$
$$18 + (2 + 4) =$$
$$(18 + 2) + 4 =$$
$$20 + 4 =$$
$$24$$

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Linking to Larger Numbers

28 + 14 = ☐

20 + 10 = 30
8 + 4 = 12
20 + 12 = 32

Linking to Fractions

$$3 \frac{3}{4} + \frac{1}{2} =$$

$$3 \frac{3}{4} + \left(\frac{1}{4} + \frac{1}{4}\right) =$$

$$\left(3 \frac{3}{4} + \frac{1}{4}\right) + \frac{1}{4} =$$

$$4\frac{1}{4}$$

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Working Towards Algebra Readiness

? 16×3

$$
\begin{array}{c|c|c}
 & 10 & 6 \\
\hline
3 & 30 & 18 \\
\end{array}
$$

30 + 18 = 48

? 16×13

$$
\begin{array}{c|c|c}
 & 10 & 6 \\
\hline
10 & 100 & 60 \\
\hline
3 & 30 & 18 \\
\end{array}
$$

100 + 60 + 30 + 18 = 208

Linking to Fractions

$$4 \times 3\frac{1}{3} =$$

$$(4 \times 3) + (4 \times \frac{1}{3}) =$$

$$12 \quad + \quad \frac{4}{3} \quad =$$

$$13\frac{1}{3}$$

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Building Critical Algebra Foundations

$$(x + 4) \times (x + 2) = x^2 + 8$$

	x	4
x	x^2	$4x$
2	$2x$	8

$$x^2 + 6x + 8$$

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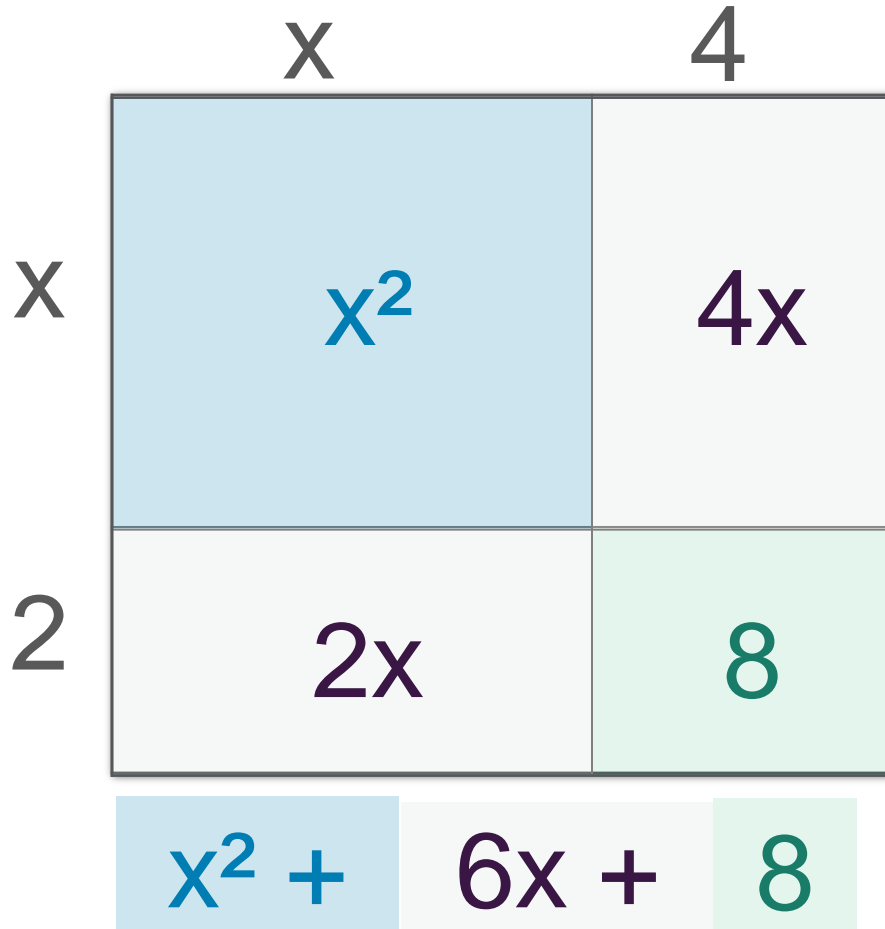
Reflect….

- What instructional implications are you thinking about now?

Understanding Arithmetic: Three Pillars

- Understanding numbers

- Developing computational fluency

- **Examining the behavior of the operations**

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Interpreting the Equal Sign

"1 plus 7 makes 8"

1 + 7 = 8

$$3 + 3 = 6 + 5$$

It means to put toge
ther

$$3+3 = \underline{6} + 5$$

That a equals sign. It means it's
a sum.

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Interpreting the Equal Sign

$$8 + 4 = \square + 5$$

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Interpreting the Equal Sign

$7 = 3 + 4$

$8 = 8$

$5 + 8 = 8 + 5$

$6 - \boxed{} = 7 - 4$

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True or False – How do you know?

$7 = 3 + 4$

$8 = 5 + 13$

$6 - 1 = 7$

$27 = 7 + 10 + 10$

$10 - 3 = 11 - 4$

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1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

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Relational Thinking

Relational thinking occurs when one observes and uses number relationships between the two sides of the equal sign instead of actually computing amounts.

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6 + □ = 5 + 9

Since 5 + 9 is 14, I need to figure out 6 plus what equals 14. It is 8, so the box is 8.

Six is one more than the 5 on the other side. That means the box should be one less than 9, so it must be 8.

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57 + 22 = 58 + 21

Circle **True or False** and explain your choice.

57 + 22 = 58 + 21

Because 57 is not equal to 58 and 21 is not equal to 28

Circle **True or False** and explain your choice.

Circle **True or False** and explain your choice.

57 + 22 = 58 + 21

$$\frac{57 + 22}{79} = \frac{58 + 21}{79}$$

true

Circle **True or False** and explain your choice.

57 + 22 = 58 + 21

58 is 1 more then 57. and 21 is 1 Less than 22. So it evens out.

Circle True or False and explain your choice.

Encouraging Relational Thinking

$37 + 54 = 38 + 53$

$48 + 63 - 62 = 49$

$625 + 450 = 700 + 400$

$64 - 38 = 66 - 40$

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Exploring True, False, and Open Sentences

1. Give each other think time on each number sentence before talking.

2. Share with each other your reasoning.

3. Which examples were interesting to you?

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Look for and make use of structure.

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True or False?

$6 + 9 = 9 + 6$

$4 - 3 = 3 - 4$

$90 - 0 = 0 - 90$

$7 + 50 = 50 + 7$

$6 + \square = 10 + 6$

$10 + \square = \square + 10$

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When you add, the order does not matter. But it does matter when you subtract.

Commutative property of addition states that changing the order of the addends does not change the sum.

Reflecting on Your Thinking

- How do tasks like this promote thinking about operations rather than just trying to get an answer?

Thinking about Instructional Choices

$$7 + 3 = \square + 9$$

$$6 + 2 = 1 + 7$$

Inverse Operations

2. How can knowing 8 x 4 = 32 help you to understand 32 ÷ 4?

$$32 \div 4 = 8$$
$$32 \div 8 = 4$$
$$8 \div 32 = 4$$
$$4 \div 32 = 8$$

Understanding Arithmetic: Three Pillars

- Understanding numbers

- Developing computational fluency

- Examining the behavior of the operations

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"A focus on the operations emphasizes noticing, describing, representing, and explaining consistencies across many problems. Generalizing in this way about the properties and behaviors of the operations is not about solving particular problems but about regularities that are foundational to arithmetic and algebra."

Connecting Arithmetic to Algebra by Susan Jo Russell, Deborah Schifter, and Virginia Bastable (Portsmouth, NH: Heinemann, 2011).

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How to Support Algebra Readiness

- Seek out professional development that builds:

    – Robust content knowledge

    – Understanding of how student learn

    – Effective instructional strategies

    – Strategies for gathering information about what students do and do not understand

- Work with colleagues to make good decisions about how to use your available materials and programs

Thank you!

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Presentation at:

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