

**Presenters: Amy Mayfield and Lu Ann Weynand**

# Follow the Standards to Algebra Readiness





<http://facing.org/files/bored> child with blocks.jpg



<http://http://legacy.lincolinteractive.org>

# Solve Mentally

•  $99 + 17 = ?$

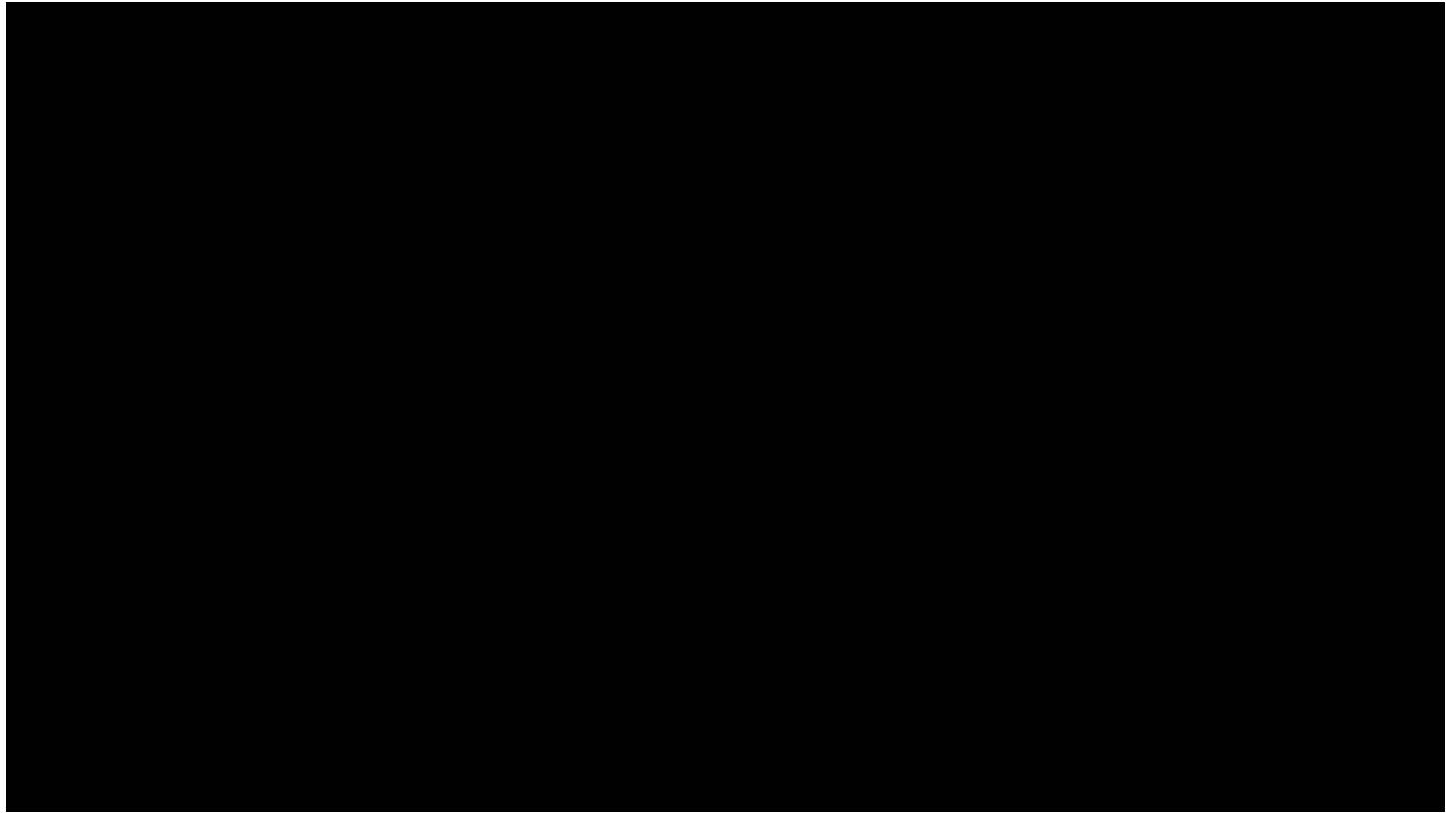
?

How might a student with strong number sense solve this problem?

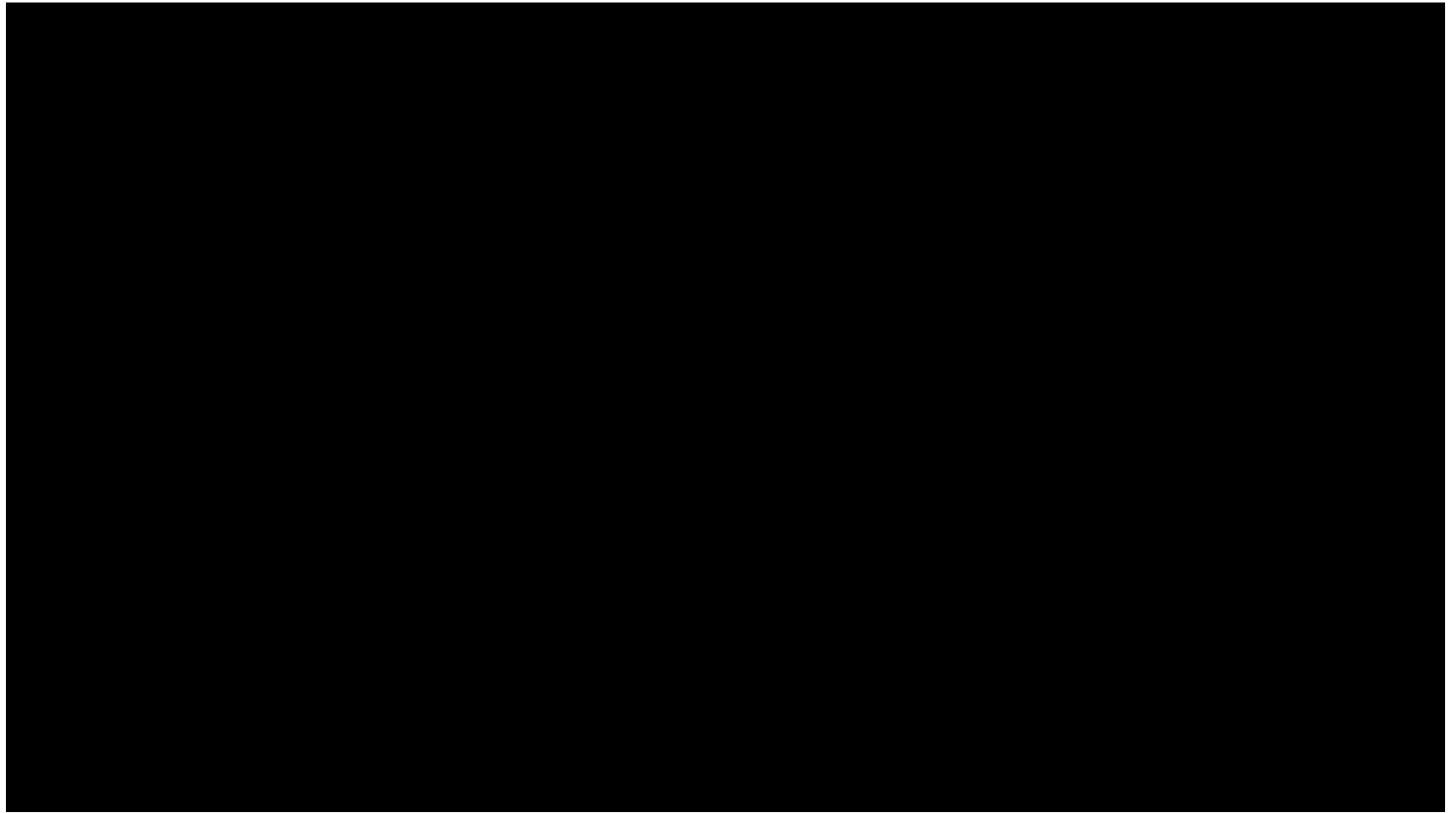
?

How might a student with weak number sense solve this problem?

# Tyrone



# Zakari



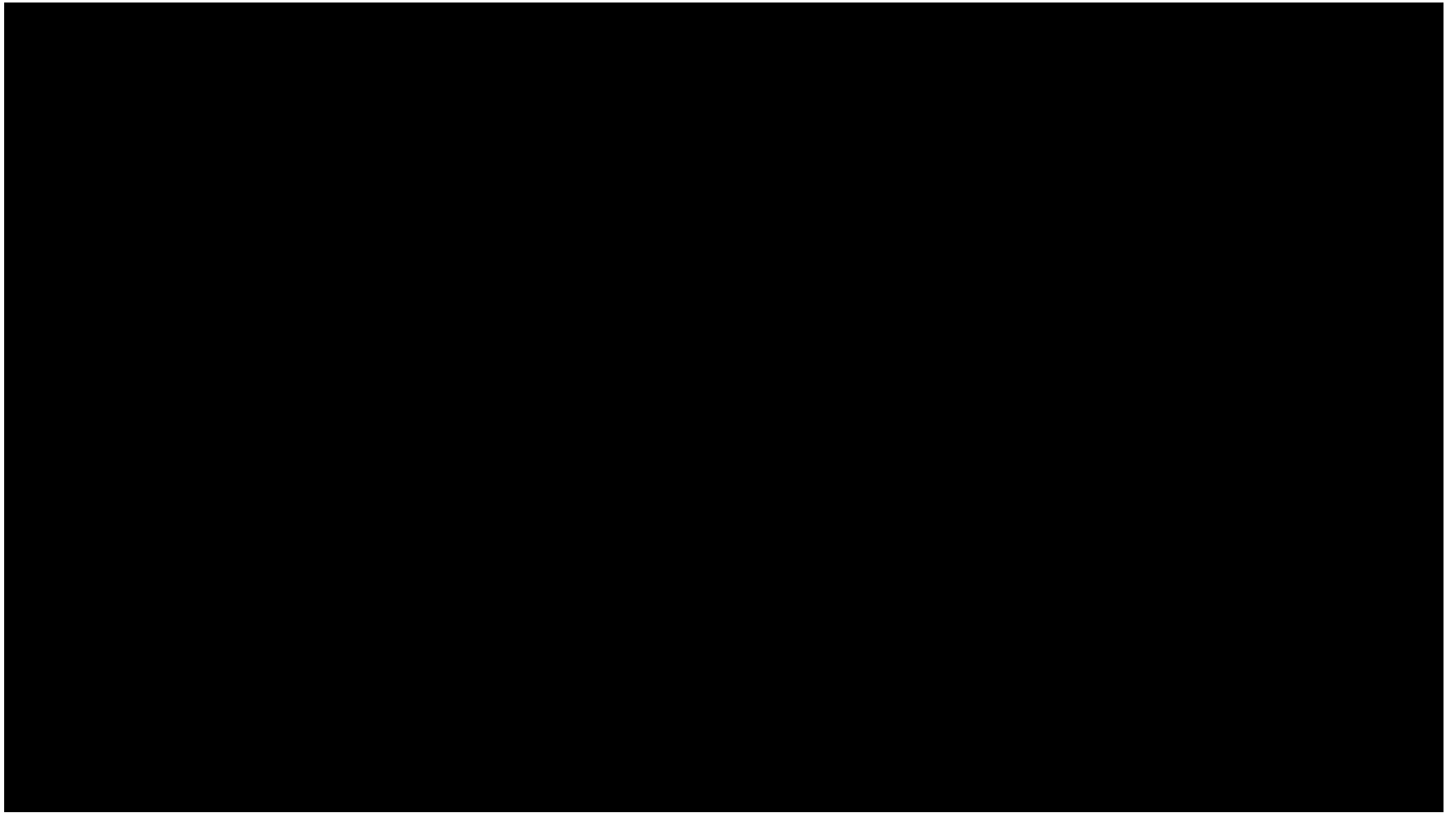
# Turn and Talk....

- What did you observe that will impact these two students as they move forward mathematically?

# Dianna



# Alberto

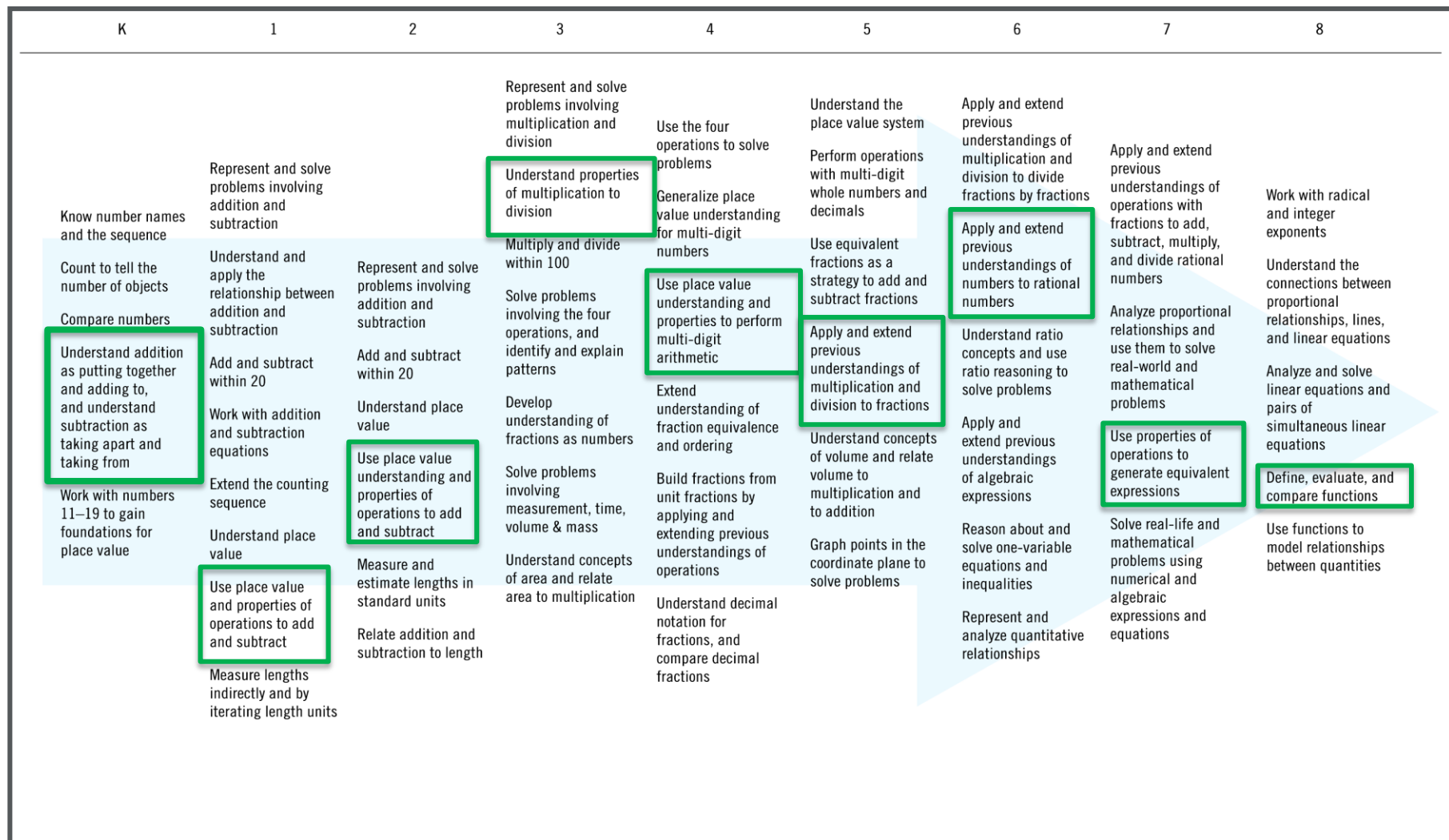




# Turn and Talk....

- What evidence did you see that these students understand numbers and are computationally flexible?

# Pathway to Algebra



# Arithmetic Algebraic Thinking

...using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (multiplication/division).

# Understanding Arithmetic: Three Pillars

- Understanding numbers
- Developing computational fluency
- Examining the behavior of the operations

# Setting the Stage for Algebra Readiness

## Decompose Small Numbers

$$6 = 1 + 5$$

$$6 = 2 + 4$$

$$6 = 3 + 3$$

## Decompose to Find Sums

$$18 + 6 =$$

$$18 + (2 + 4) =$$

$$(18 + 2) + 4 =$$

$$20 + 4 =$$

$$24$$

# Linking to Larger Numbers

$$28 + 14 = \square$$

$$20 + 10 = 30$$

$$8 + 4 = 12$$

$$20 + 12 = 32$$

# Linking to Fractions

$$3 \frac{3}{4} + \frac{1}{2} =$$

$$3 \frac{3}{4} + \left(\frac{1}{4} + \frac{1}{4}\right) =$$

$$\left(3 \frac{3}{4} + \frac{1}{4}\right) + \frac{1}{4} =$$

$$4 \frac{1}{4}$$

$$3 \frac{3}{4} + \frac{1}{2} =$$

$$\left(3 \frac{1}{4} + \frac{1}{2}\right) + \frac{1}{2} =$$

$$3 \frac{1}{4} + \left(\frac{1}{2} + \frac{1}{2}\right) =$$

$$3 \frac{1}{4} + 1 = 4 \frac{1}{4}$$

# Working Towards Algebra Readiness

?

$16 \times 3$

10

6

3

30

18

$30 + 18 = 48$

?

$16 \times 13$

10

6

10

100

60

3

30

18

$100 + 60 + 30 + 18 = 208$



# Linking to Fractions

$$4 \times 3 \frac{1}{3} =$$

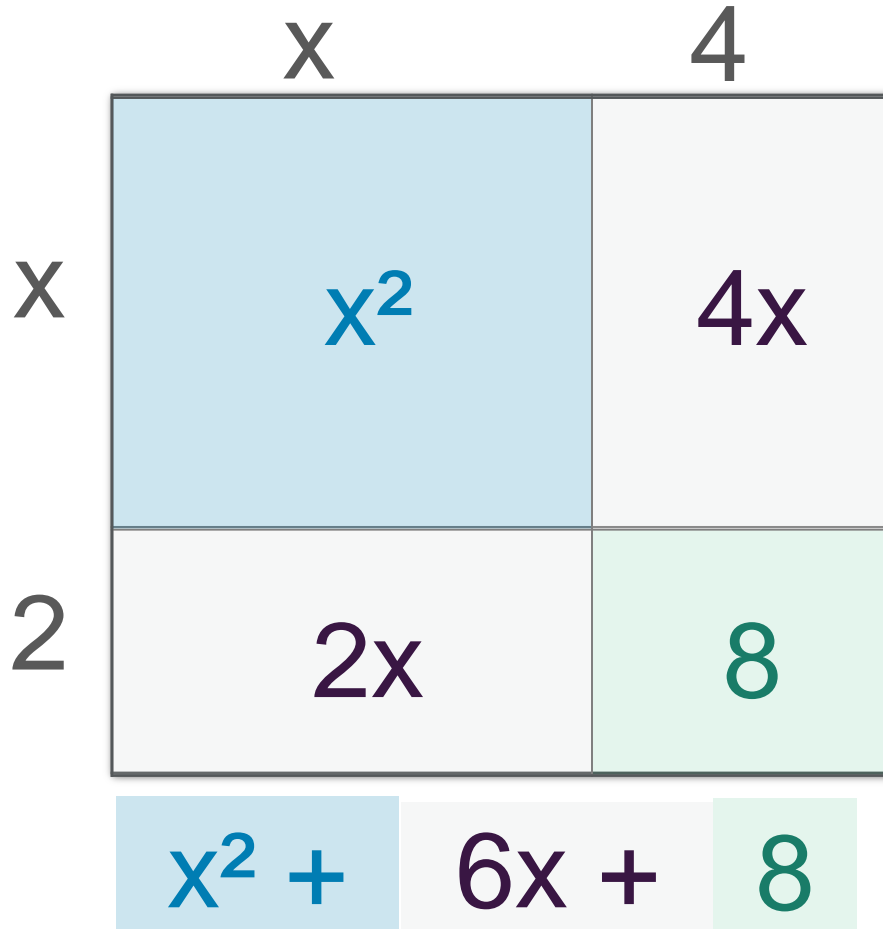
$$(4 \times 3) + (4 \times \frac{1}{3}) =$$

$$12 + \frac{4}{3} =$$

$$13 \frac{1}{3}$$

# Building Critical Algebra Foundations

$$(x + 4) \times (x + 2) = x^2 + 8$$



# Understanding Arithmetic: Three Pillars

- Understanding numbers
- Developing computational fluency
- **Examining the behavior of the operations**

# Interpreting the Equal Sign

“1 plus 7 makes 8”

$$1 + 7 = 8$$

$$3 + 3 = 6 + 5$$

I + means to put together.  
there.

---

$$3 + 3 = \underline{6} + 5$$

That's an equals sign. It means it's a sum.

# Interpreting the Equal Sign

$$8 + 4 = \square + 5$$

# Interpreting the Equal Sign

$$7 = 3 + 4$$

$$8 = 8$$

$$5 + 8 = 8 + 5$$

$$6 - \square = 7 - 4$$

# True or False – How do you know?

$$7 = 3 + 4$$

$$8 = 5 + 13$$

$$6 - 1 = 7$$

$$27 = 7 + 10 + 10$$

$$10 - 3 = 11 - 4$$

Grade 1 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

Grade 1 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

Grade 3 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.



# Relational Thinking

Relational thinking occurs when one observes and uses number relationships between the two sides of the equal sign instead of actually computing amounts.

$$6 + \square = 5 + 9$$

Since  $5 + 9$  is 14, I need to figure out 6 plus what equals 14. It is 8, so the box is 8.

Six is one more than the 5 on the other side. That means the box should be one less than 9, so it must be 8.

# $57 + 22 = 58 + 21$

Circle True or False and explain your choice.

$57 + 22 = 58 + 21$

Because 57 is not equal to 58 and 21 is not equal to 22.

Circle True or False and explain your choice.

$57 + 22 = 58 + 21$

$$\begin{array}{r} 57 \\ + 22 \\ \hline 79 \end{array} = \begin{array}{r} 58 \\ + 21 \\ \hline 79 \end{array}$$

true

Circle True or False and explain your choice.

$57 + 22 = 58 + 21$

58 is 1 more than 57. and 21 is 1 less than 22. So it even's out. 😊

# Encouraging Relational Thinking

$$37 + 54 = 38 + 53$$

$$48 + 63 - 62 = 49$$

$$625 + 450 = 700 + 400$$

$$64 - 38 = 66 - 40$$

# Exploring True, False, and Open Sentences

1. Give each other think time on each number sentence before talking.
2. Share with each other your reasoning.
3. Which examples were interesting to you?

# Look for and make use of structure.



# True or False?

$$6 + 9 = 9 + 6$$

$$4 - 3 = 3 - 4$$

$$90 - 0 = 0 - 90$$

$$7 + 50 = 50 + 7$$

$$6 + \square = 10 + 6$$

$$10 + \square = \square + 10$$

When you add, the order  
does not matter. But it does  
matter when you  
subtract.

Commutative property of addition states that changing the order of the addends does not change the sum.



A child who does not see patterns often does not expect things to make sense and sees all events as discrete, separate, and unrelated.”

A student who expects things to ‘make sense’ *looks* for patterns and generalizations and from these develops understanding.

# Supporting Teachers

$$7 + 3 = \square + 9$$

$$6 + 2 = 1 + 7$$

# Inverse Operations

2. How can knowing  $8 \times 4 = 32$  help you to understand  $32 \div 4$ ?

$$32 \div 4 = 8$$

$$32 \div 8 = 4$$

$$8 \div 32 = \frac{1}{4}$$

$$4 \div 32 = \frac{1}{8}$$

USA:

- **How can I teach my kids to get the answer to this problem?**

Japanese:

- **How can I use this problem to teach the mathematics of this unit?**

~ Phil Daro

# Understanding Arithmetic: Three Pillars

- Understanding numbers
- Developing computational fluency
- Examining the behavior of the operations

“A focus on the operations emphasizes noticing, describing, representing, and explaining consistencies across many problems. Generalizing in this way about the properties and behaviors of the operations is not about solving particular problems but about regularities that are foundational to arithmetic and algebra.”

Connecting Arithmetic to Algebra by Susan Jo Russell, Deborah Schifter, and Virginia Bastable (Portsmouth, NH: Heinemann, 2011).

# Impacting Teacher's Practice

- What two or three things to support algebra readiness do you want to see in your classrooms?
- What ideas do you have for making those happen?

# How to Support Algebra Readiness

- Provide teachers with professional learning that builds:
  - Robust content knowledge
  - Understanding of how student learn
  - Effective instructional strategies
  - Strategies for gathering information about what students do and do not understand
- Ensure teachers and students have quality materials and programs



# Thank you!

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