AGENDA
Geometry
Grades 6-8

OVERVIEW
This course focuses on strategies and tools that build on students’ thinking and spatial reasoning skills developed in elementary school. Teachers gain an understanding of levels of geometric thinking and the types of learning experiences that promote rigorous thinking. Specific attention is paid to area, surface area, volume, congruence, the Pythagorean Theorem, coordinate geometry and transformations.

OUTCOMES
• Articulate the progression of content in the strand of geometry, specifically around deductive reasoning.
• Analyze problem-solving activities that deepen understanding and develop participants’ skills in geometry, geometric measurement, and informal proof.
• Incorporate effective strategies for teaching mathematics vocabulary into lessons.
• Apply an understanding of the Van Hiele levels of Geometric thought to lesson design choices.
• Challenge participants with rigorous math problems that require the habits of mind called for in current state standards and assessments.

Day One

Opening
This introduction includes the course goals, an overview of current state geometry standards in Grades 6–8, and pertinent logistical information.

Making Sense of Formulas
Current state standards call for deeper understanding of area formulas that goes beyond use and applications. The area of quadrilaterals can be composed or decomposed using triangles and rectangles. Furthermore, the formula for the area of a circle can be derived from decomposition into sectors that can then be composed into a parallelogram. This derivation of the area formula also indirectly introduces the idea of limits, a fundamental concept in calculus.

BREAK

Classifying Quadrilaterals
Classification is a fundamental concept at the core of geometric thinking. Furthermore, this is an area of mathematics that, for the first time for many students, shifts students from thinking in more algebraic terms to thinking in more subjective terms where different yet equivalent outcomes are possible. Shapes being classified differently can create much disequilibrium for geometry learners.

*LUNCH*

**Examining Levels of Geometric Thought**
The levels of geometric thought, as defined by Pierre van Hiele and Dina van Hiele-Geldof, are at the center of teachers’ instructional decisions, particularly in teaching geometry. These five levels move students sequentially from concrete to abstract. Instructional choices must be appropriate for students’ places in this sequence.

*BREAK*

**Investigating Volumes of Prisms**
The volume formula for right rectangular prisms, \( V = l \times w \times h \), is derived from multiplying the area of the base times the height of the prism. Using cubes to represent a unit layer of the shape allows learners to see a model relate back to this formula. Understanding the definition of a prism is important to the understanding of why this works.

**Closing**
Participants take time to reflect on the experiences of the day and ways that these experiences will positively impact their classroom instruction. Participants fill out an exit ticket before leaving to provide feedback for day 2.

**Day Two**

**Opening**
This introduction includes the course goals, the course learning outcomes, an overview of habits of mathematical thinkers, and pertinent logistical information. Participants play a short game with partners identifying missing angles in a diagram and justifying their reasoning.

**Exploring Congruence**
Students make connections and build understanding of corresponding parts and shape congruence through exploration and discourse. In this session, participants consider how to support students as they move from using measurement as a justification for congruence to using more abstract thinking about congruence within shape. Participants explore triangle congruence with an activity designed to encourage students to justify and generalize their thinking, both important actions in the development of geometrical maturity.
BREAK

Exploring Coordinate Geometry and Transformations
The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformations. In this session, participants investigate rigid transformations that preserve distance and angles and use these transformations to more deeply understand congruence.

LUNCH

Experiencing the Pythagorean Theorem
The Pythagorean Theorem is an important topic in middle school geometry and it becomes increasingly important in the high school years. Oftentimes, this theorem is reduced to a formula that merely calculates a missing side length of a right triangle. Students need to see the geometric connections as well as apply the formula algebraically. This can be done through meaningful experiences using dot paper, geoboards, and other manipulatives.

BREAK

Exploring Surface Area Using Nets
Students develop deeper understanding of the relationship between surface area and volume of shapes when they explore using cubes and creating the nets of corresponding models. As participants engage in a simulation in which a cheese factory is seeking advice about the most economical way to package blocks of cheese, they see how students use the models and nets to gather information and write recommendations to the company.

Closing
This session connects the various sessions back to the course goals and learning outcomes so participants are prepared to move forward as they go back into classrooms and implement both the instructional strategies and content modeled throughout the course.

Math Solutions Guiding Principles
Drawing upon academic work and our own classroom-grounded research and experience, Math Solutions has identified the following four instructional needs as absolutely essential to improving instruction and student outcomes:

• Robust Content Knowledge
• Understanding of How Students Learn
• Insight into Individual Learners Through Formative Assessment
• Effective Instructional Strategies
These four instructional needs drive the design of all Math Solutions courses, consulting, and coaching. We consider them our guiding principles and strive to ensure that all educators:

- Know the math they need to teach—know it deeply and flexibly enough to understand various solution paths and students’ reasoning
- Understand the conditions necessary for learning, what they need to provide, and what students must make sense of for themselves
- Recognize each student’s strengths and weaknesses, content knowledge, reasoning strategies, and misconceptions
- Have the expertise to make math accessible for all students, to ask questions that reveal and build understanding, and to help students make sense of and solve problems