It Makes Sense

Using the Rekenrek to Build Number Sense

NCTM Session 624

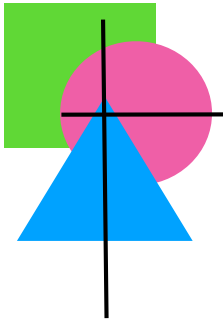
April 6, 2019

Presented by Patty King

Number Sense is

….good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.

Hilde Howden, 1989



How Do Students Develop Number Sense? Do No Harm!

Some students do develop number sense on their own with the right conditions:

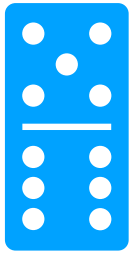
1. The environment supports children's natural curiosity and exploration of numbers

2. Instruction is non-intrusive to their development and does not involve showing procedures to follow

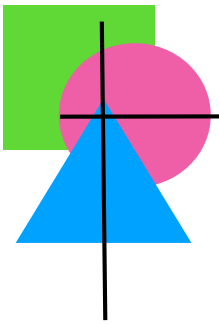
3. Students encouraged to use make sense and use their own strategies, selecting tools as needed

Number Sense

- Spatial relationships or subitizing.
- One More or Less
- Benchmarks 5 & 10; how it relates to 5 and 10
- Part-Whole; how a whole can be broken into parts



Van de Walle, J., A., Lovin, L., H., (2006), **Teaching Student Centered Mathematics K - 3,** Boston Pearson Education



Fluency

Efficient, flexible use of strategies to solve addition and subtraction.

CCSS-M Developing fluency in each grade may involve a mixture of knowing some answers, knowing some answers from patterns and knowing some answers through the use of strategies.

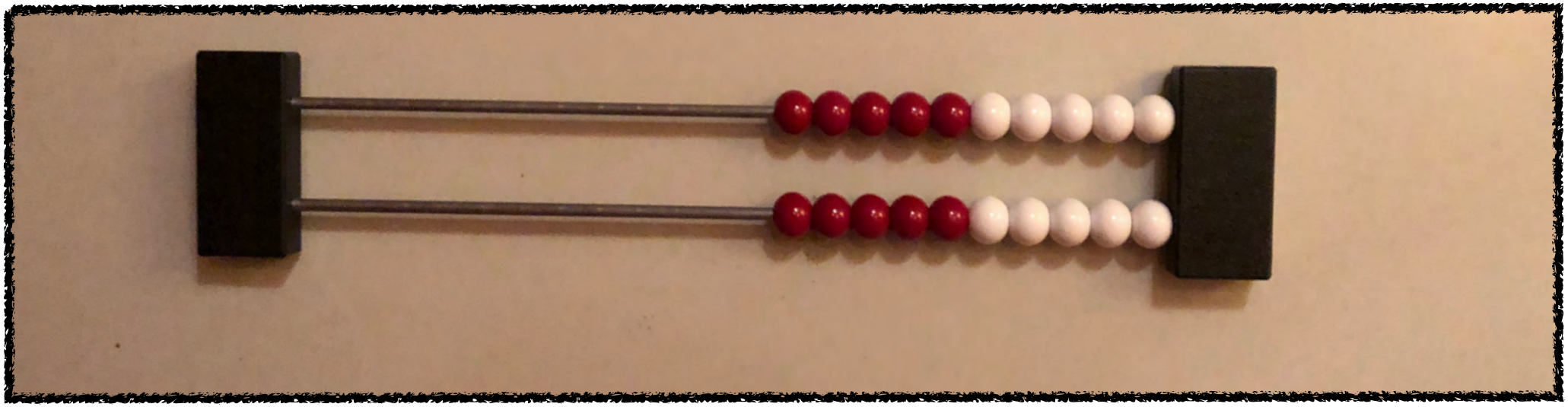
Fluency Standards

Kinder: K.OA.5
Fluently add & subtract within 5

1st grade: 1.OA.6
Demonstrate fluency for addition & subtraction within 10

2nd grade: 2.OA.2
Fluently add & subtract within 20 using mental strategies

The Rekenrek



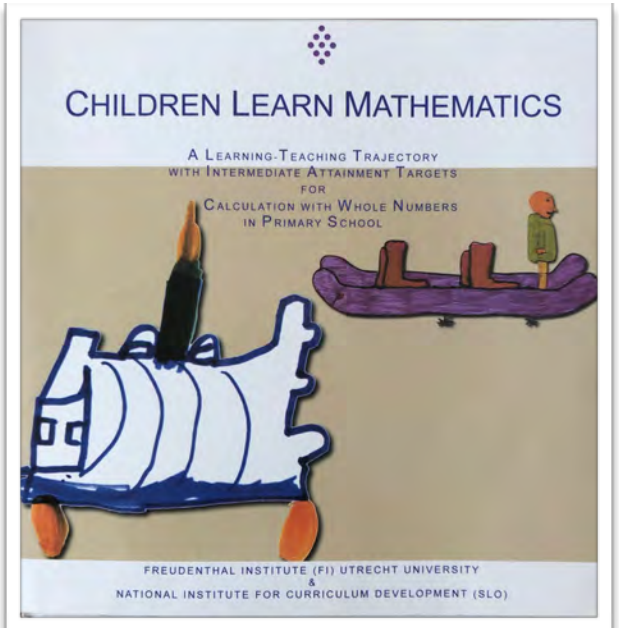
Developed by Adrian Treffers with the Freudenthall Institute

3 phases of instruction

1- counting

2- structuring numbers

3- calculating



Number Rack at Math Learning Center

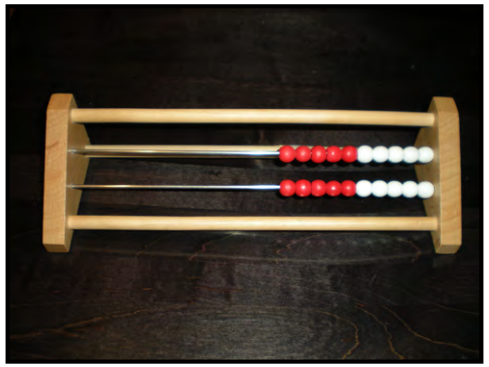
- https://apps.mathlearningcenter.org/number-rack/
- apps: Number Rack

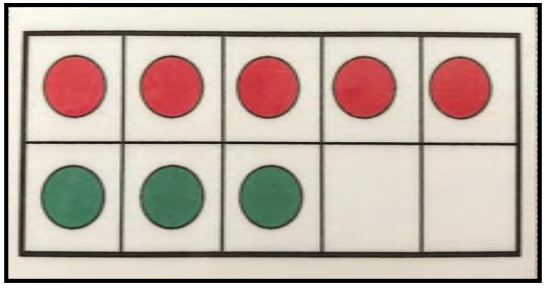
Structuring Numbers

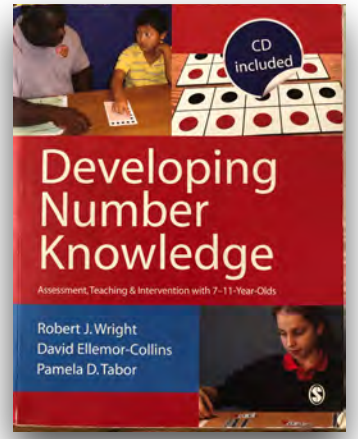
"Through inquiry and discussion, the teacher draws attention to part-whole constructions, working with numbers as parts of other numbers and to relational thinking, making relationships between numbers.

The instructional settings enable students to reason.

As facility develops, the students are weaned off the settings towards solving verbal and written tasks."







Robert Wright, David Ellemor-Collins, Pamela Tabor, 2012

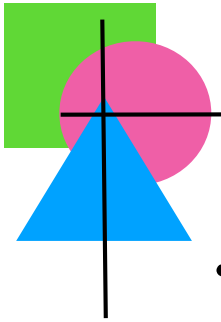
Building Strategies

1. Engage in numerical reasoning to build number knowledge:

    - Students will think, reason and communicate about numbers.

2. Develop fluency based on numerical sense:

    - Students will move away from counting to using strategies for addition and subtraction, which rely on grouping



Teaching Number Sense

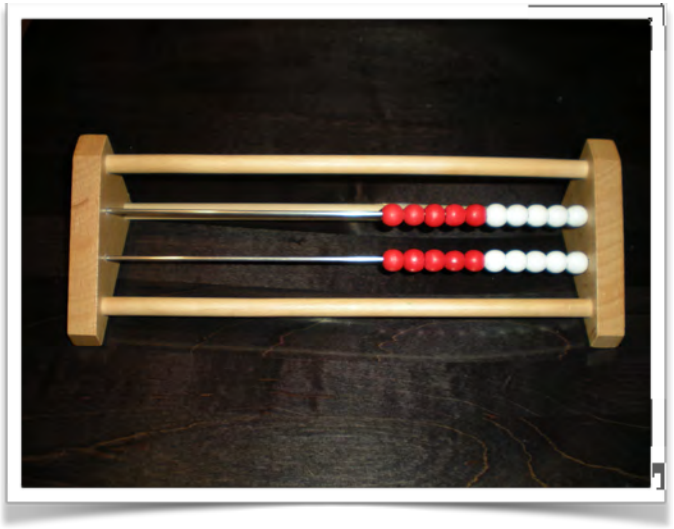
We are not teaching the strategies but providing opportunities in which they make meaning about numbers and relationships.

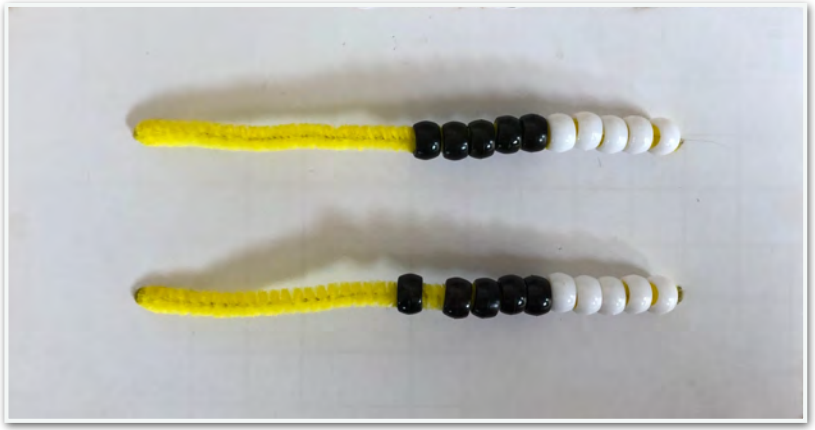
...materials cannot transmit knowledge: the learner must construct the relationships",

1991 Gravemeijer

Rekenreks for Instruction

- Large class size rekenrek
- Student rekenreks





Introduce the Rekenrek

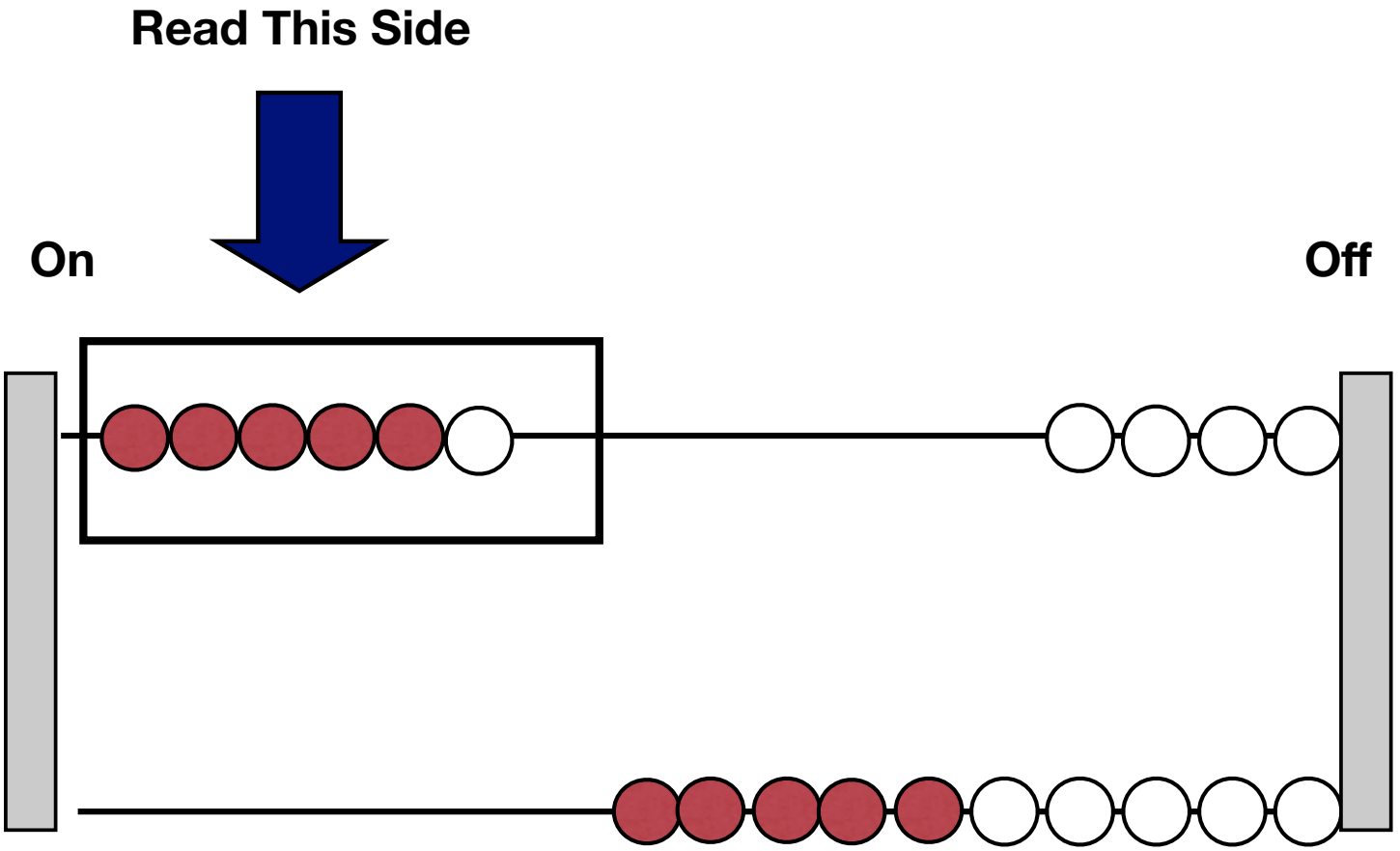
- Allow time for exploration
- Discuss on the structure of the beads:
  What do you notice about the rekenrek?
  - 20 beads in all
  - 10 on top and 10 on bottom
  - 5 red beads on the top and bottom and 5 white beads for a total of 10 red beads and 10 white beads

Provide a Context

Allows children to imagine action for addition and subtraction

Provides support when moving away from the tool

How Many Beads?



Sequence of Instruction

1.Making & Reading Numbers

2.Quick Images: using mental imagery to imagine numbers and then describe number

3.Add and subtract

- part/whole
- Related to 5
- Related to 10

Making Numbers

- Make 7. What does 7 look like?
- Make 9. How do you know you made 9?
- Put 6 on the top and 5 on the bottom. How many? How do you know?
- Make 8...
  ...in one push. ...on two bars. What do you notice?
- Make 16, and this time use 2 pushes. How do you know it is 16?

Making Teen Numbers in Kindergarten

- It is spring.
- Notice how students are describing the numbers.

Number and Operations in Base Ten K-NBT

Work with numbers 11 - 19 to gain foundations for place value.

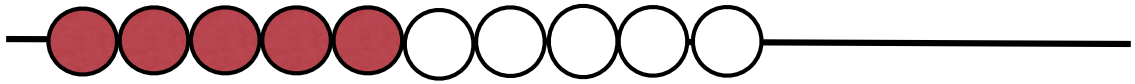
- K-NBT 1. Compose and decompose numbers from 11 - 19 into tens and some further ones: by using objects and record each composition or decomposition by a drawing or equation; understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

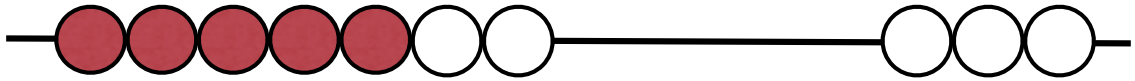
Sequence of Instruction

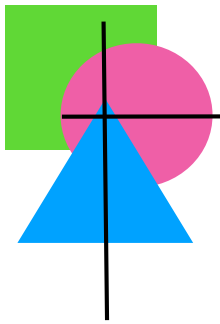
1.Making and reading numbers

2.Quick Images: using mental imagery to imagine numbers and then describe number

3.Add and subtract







Strategy Building

- Watch the student to identify the strategies
- Discuss with a neighbor

Number Sense is not something that adults can easily impose.

- Counting is a important phase
- Develops gradually from the big ideas of part-whole construction of numbers
- Opportunities to share, justify, discuss their strategies

Class Discussion

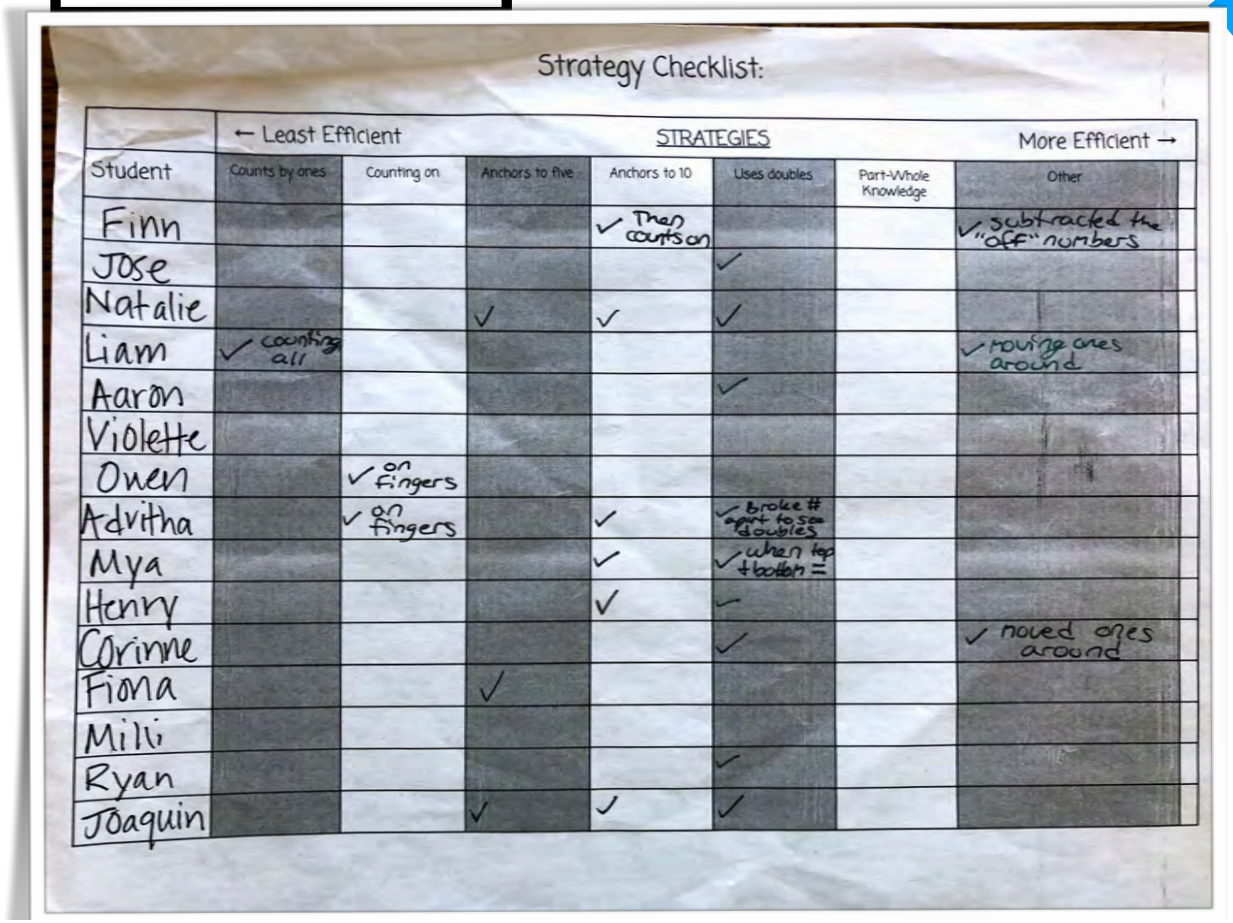
- ☐ Spatial relationships
- ☐ One More or Less
- ☑ Benchmarks 5 & 10; how it relates to 5 and 10
- ☑ Part/whole; how a whole can be broken into parts

Collecting Data on Strategy Development

Student's Current Strategy

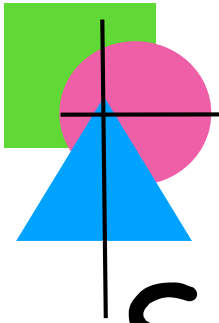
Document Progress

Data Guides Instruction



Strategy Checklist:

Student	← Least Efficient STRATEGIES More Efficient →						
	Counts by ones	Counting on	Anchors to five	Anchors to 10	Uses doubles	Part-Whole Knowledge	Other
Finn				✓ Then counts on			✓ subtracted the "off" numbers
Jose					✓		
Natalie			✓	✓	✓		
Liam	✓ counting all						✓ moving ones around
Aaron					✓		
Violette							
Owen		✓ on fingers					
Adritha		✓ on fingers		✓	✓ broke # apart to see doubles		
Mya				✓	✓ when top + bottom =		
Henry				✓	✓		
Corinne					✓		✓ moved ones around
Fiona			✓				
Milli							
Ryan					✓		
Joaquin			✓	✓	✓		



Sequence of Instruction

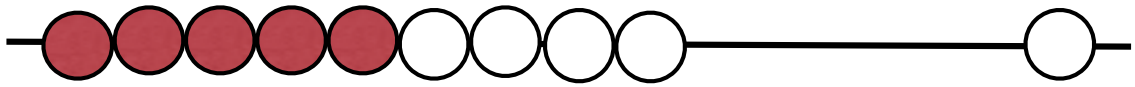
1. Opportunities to Make & Read Numbers.

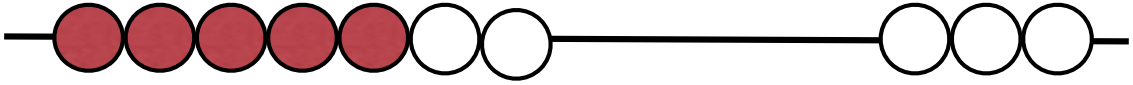
2. Quick Images: using mental imagery to imagine numbers and then describe number.

3. Adding and subtracting: Strategy development

Moving to Adding

- Using quick images students read the top and bottom number and find the total.



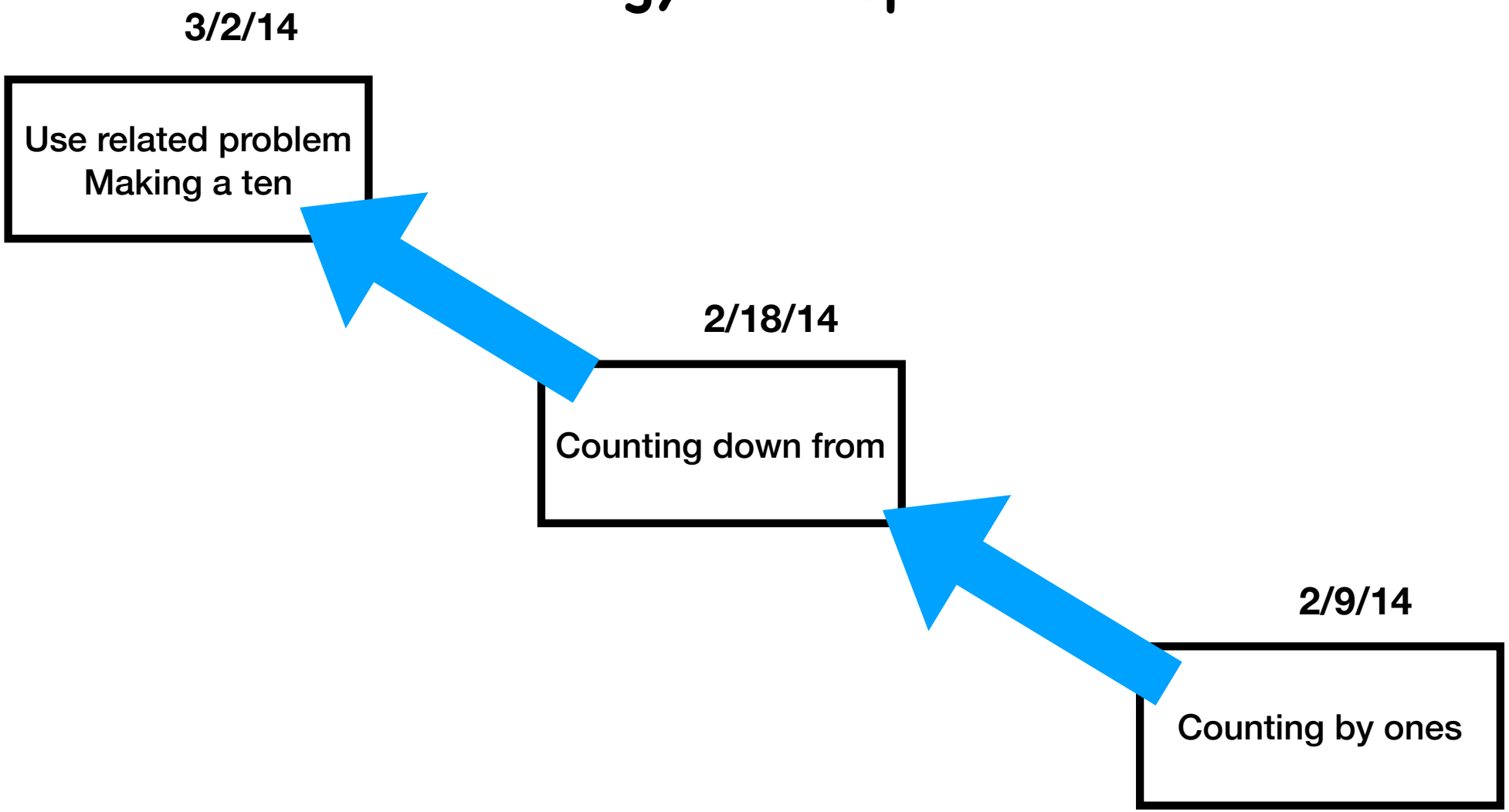


Mental Strategies

- Counting on
- **Making tens 9 + 7 = 9 + 1 + 6 = 10 + 6**
- Decomposing a number leading to a ten 14 - 6 = 14 - 4 - 2 = 10 - 2
- **Related Facts**, using properties such as commutativity or fact families
- Doubles
- Doubles plus one
- Relationship between addition and subtraction, using the inverse operation
- Equivalent but easier or known sums

Maddie's Progress

Strategy Development



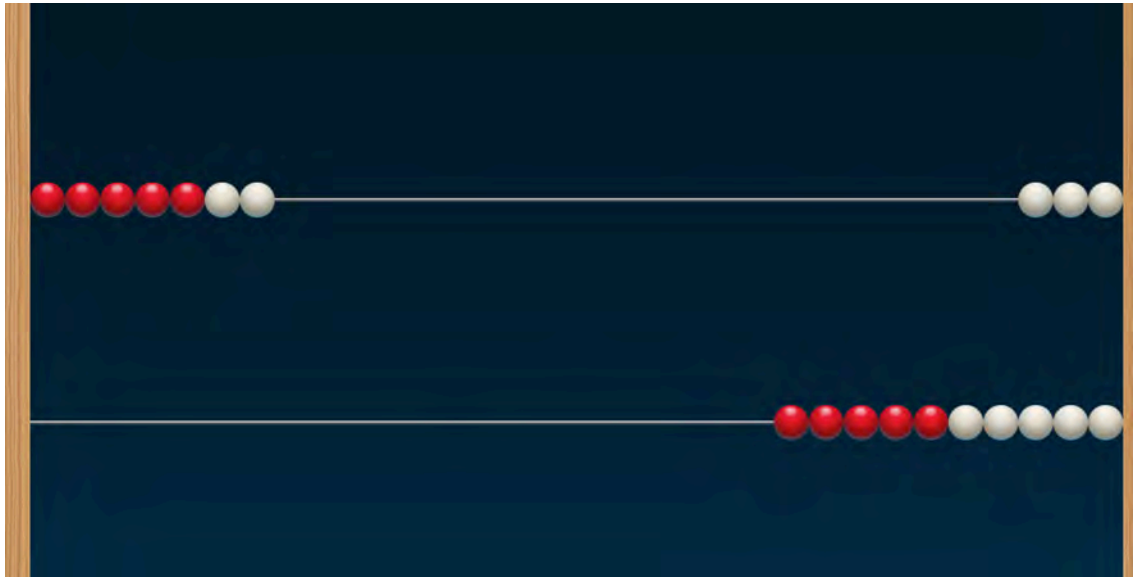
A Gradual Release of the Tool

1. Addition and subtraction story problems; students move the beads on the rekenrek

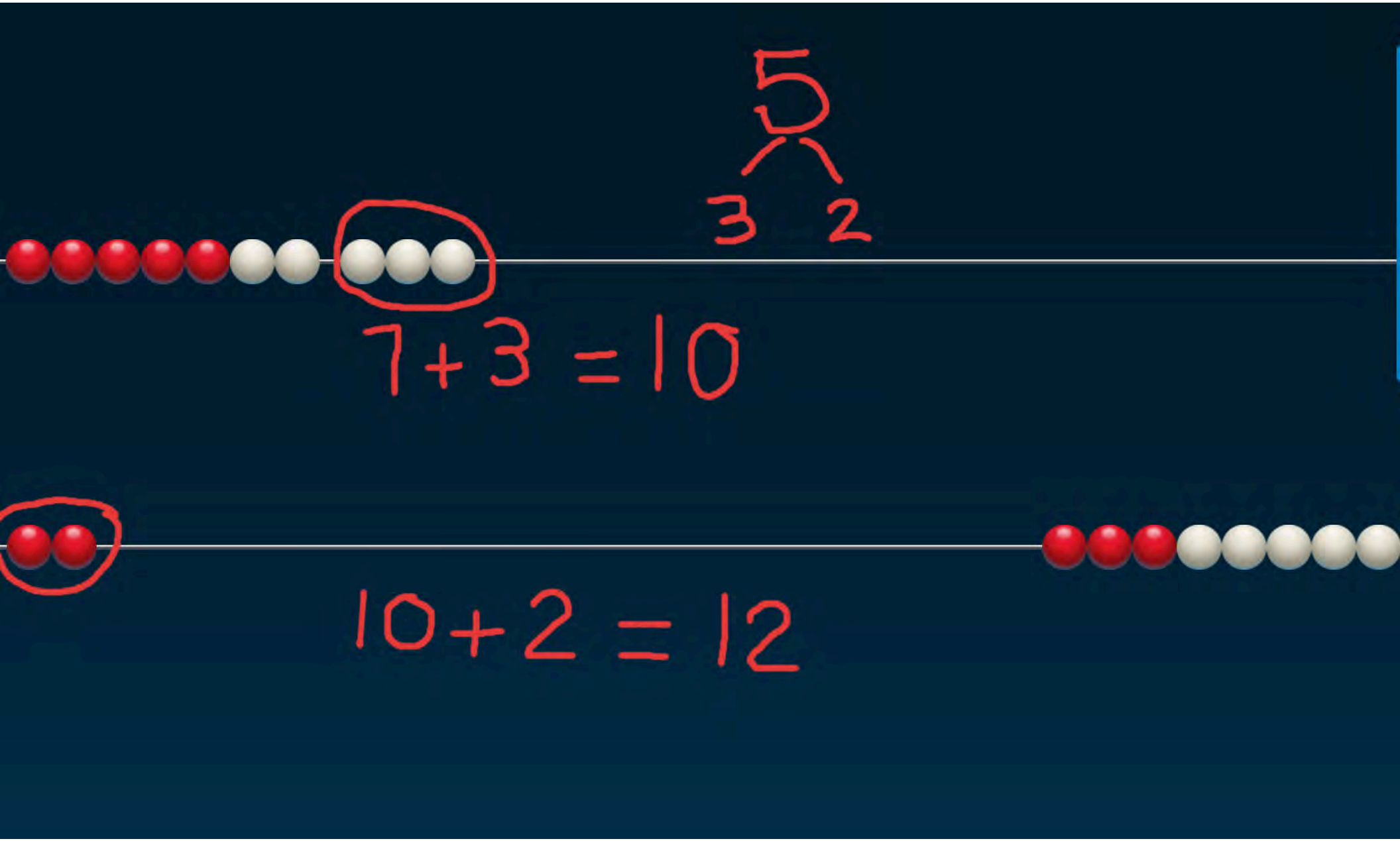
   - Begin with same configuration of beads.
   - Students move the beads.
   - Teacher records how students move

Story Problems
Getting On and Off

1. Seven people were on the train, at the next stop, five more got on. How many are on now? How did you move the beads?

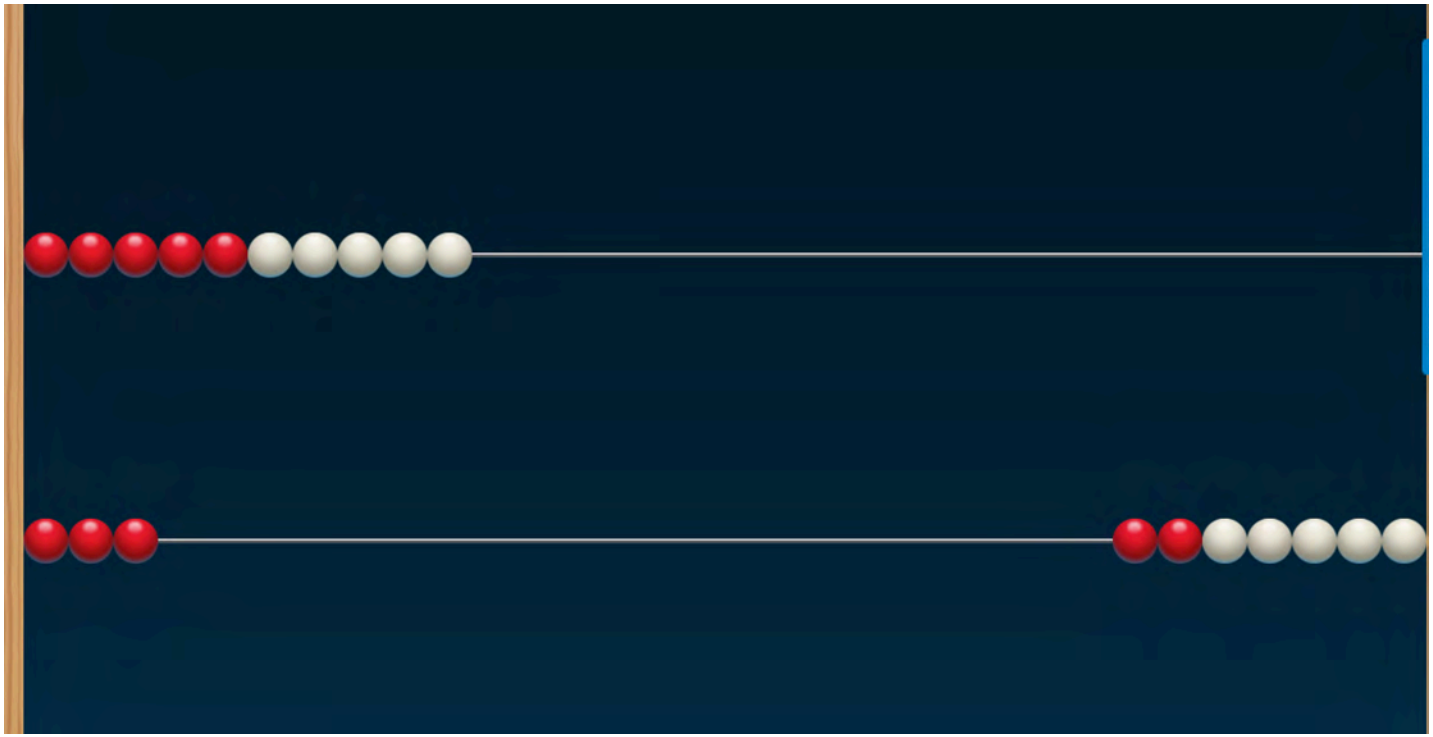


One Solution



Story Problems
Getting On and Off

Thirteen people were on the train. At the next stop six got off. How many are on the train now?



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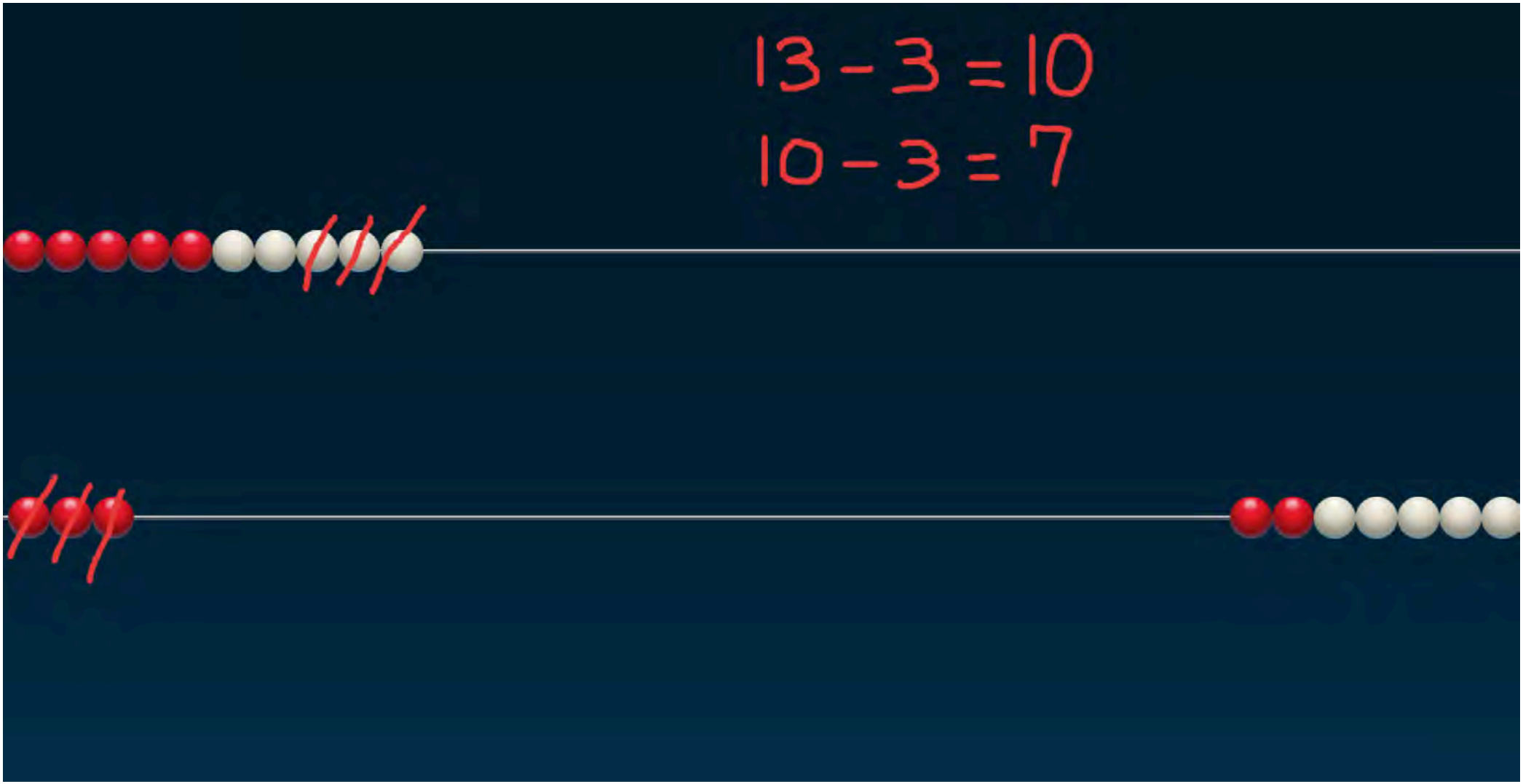


A Gradual Release of the Tool

1. Addition and subtraction story problems; students move the beads on the rekenrek

2. Addition and subtraction story problems with students: seeing the first number but not moving the beads

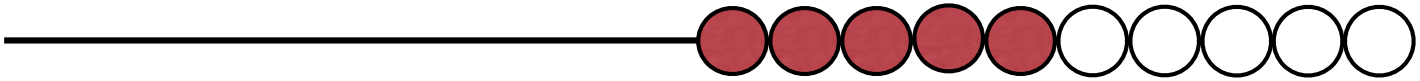
    Show students how the rekenrek looks with the first number

    Students solve without moving the beads

Recording Student Thinking

- Watch the teacher use recording to match student thinking
- What is the purpose of drop down notation in supporting student thinking?

8 + 6



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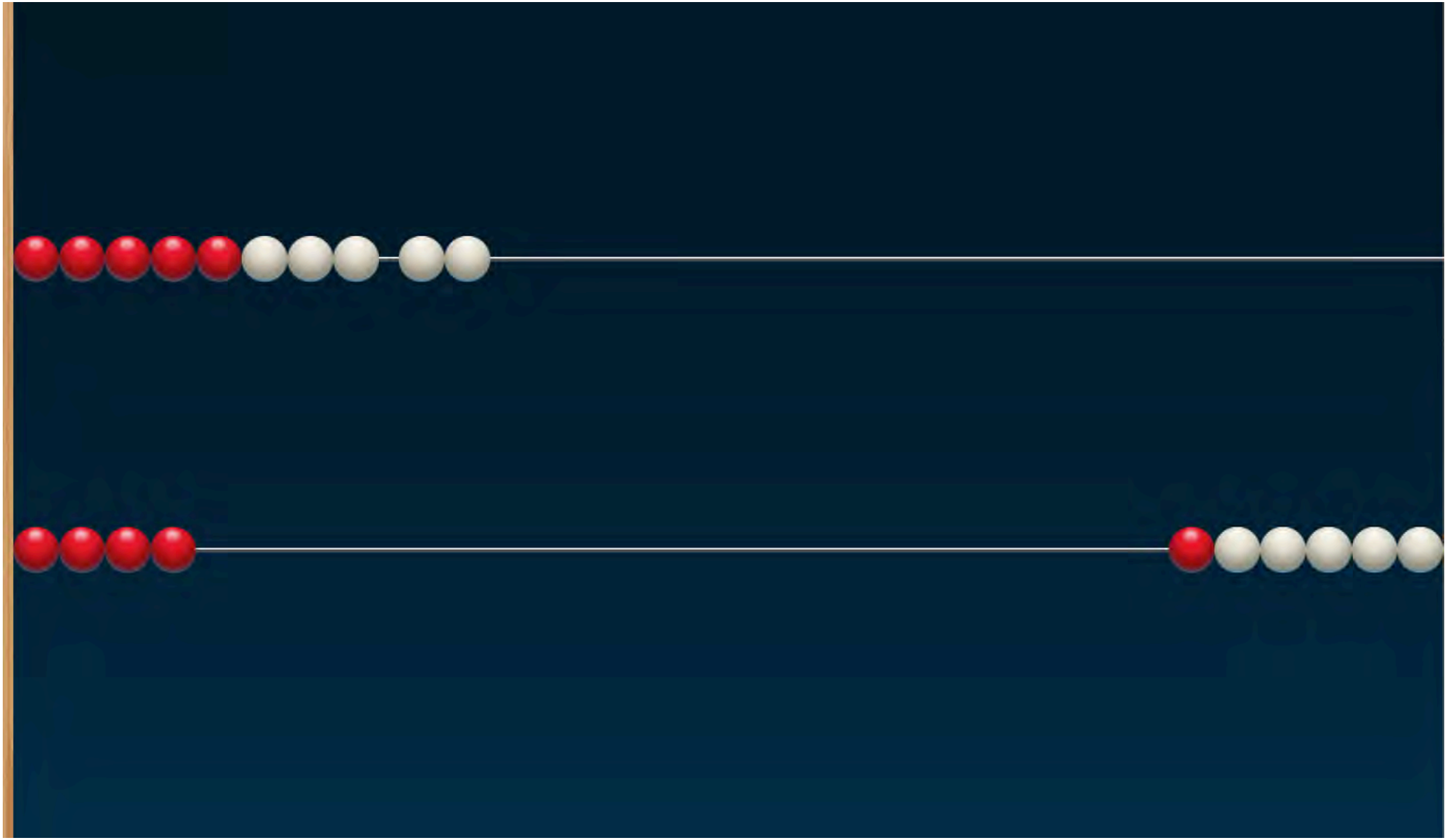
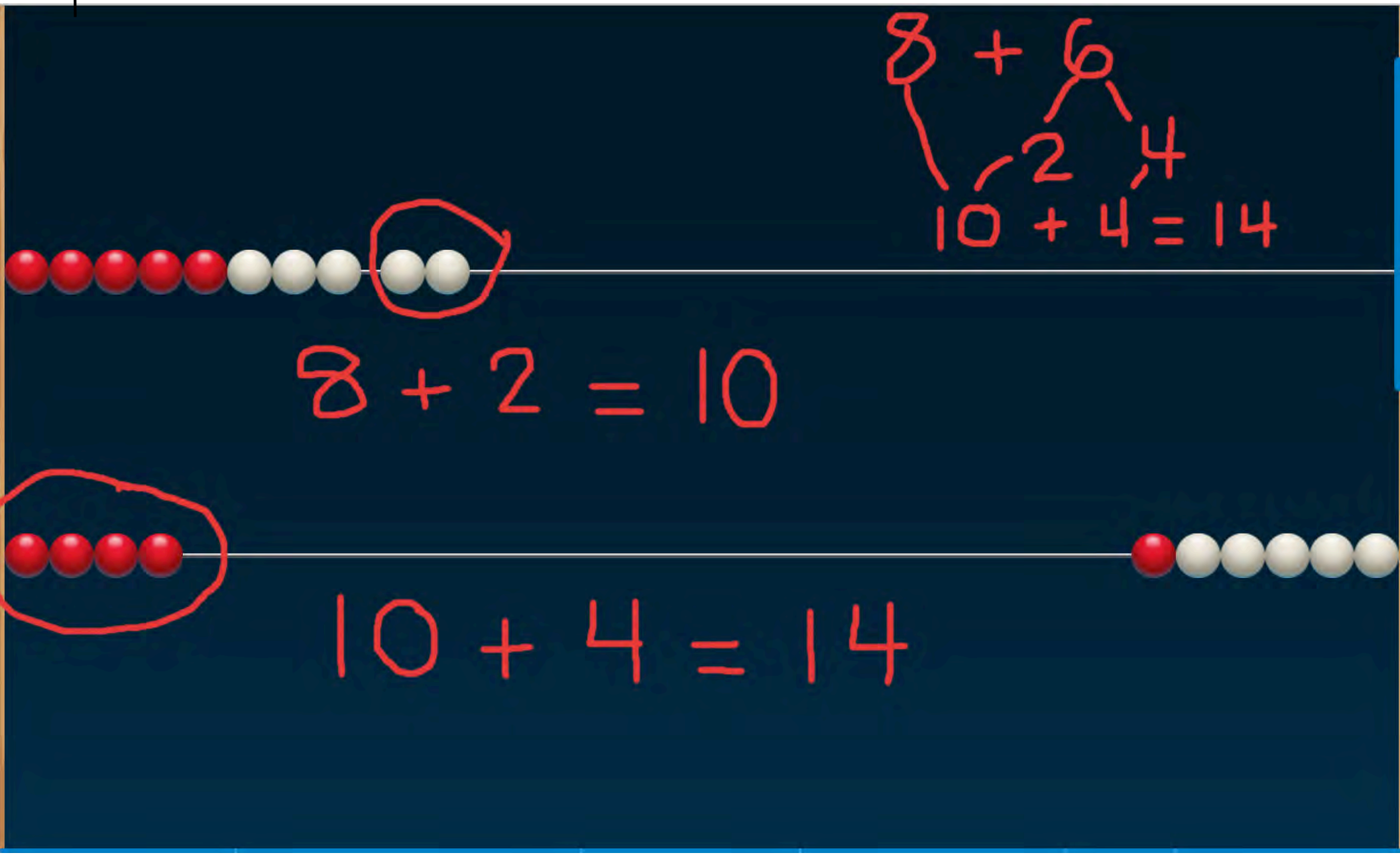
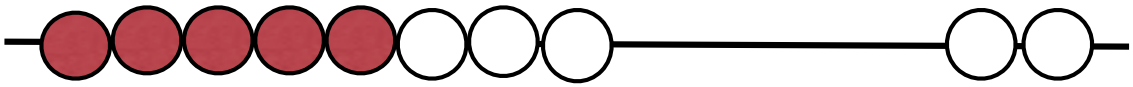


Adding to 8 to Make a 10



10 + 4

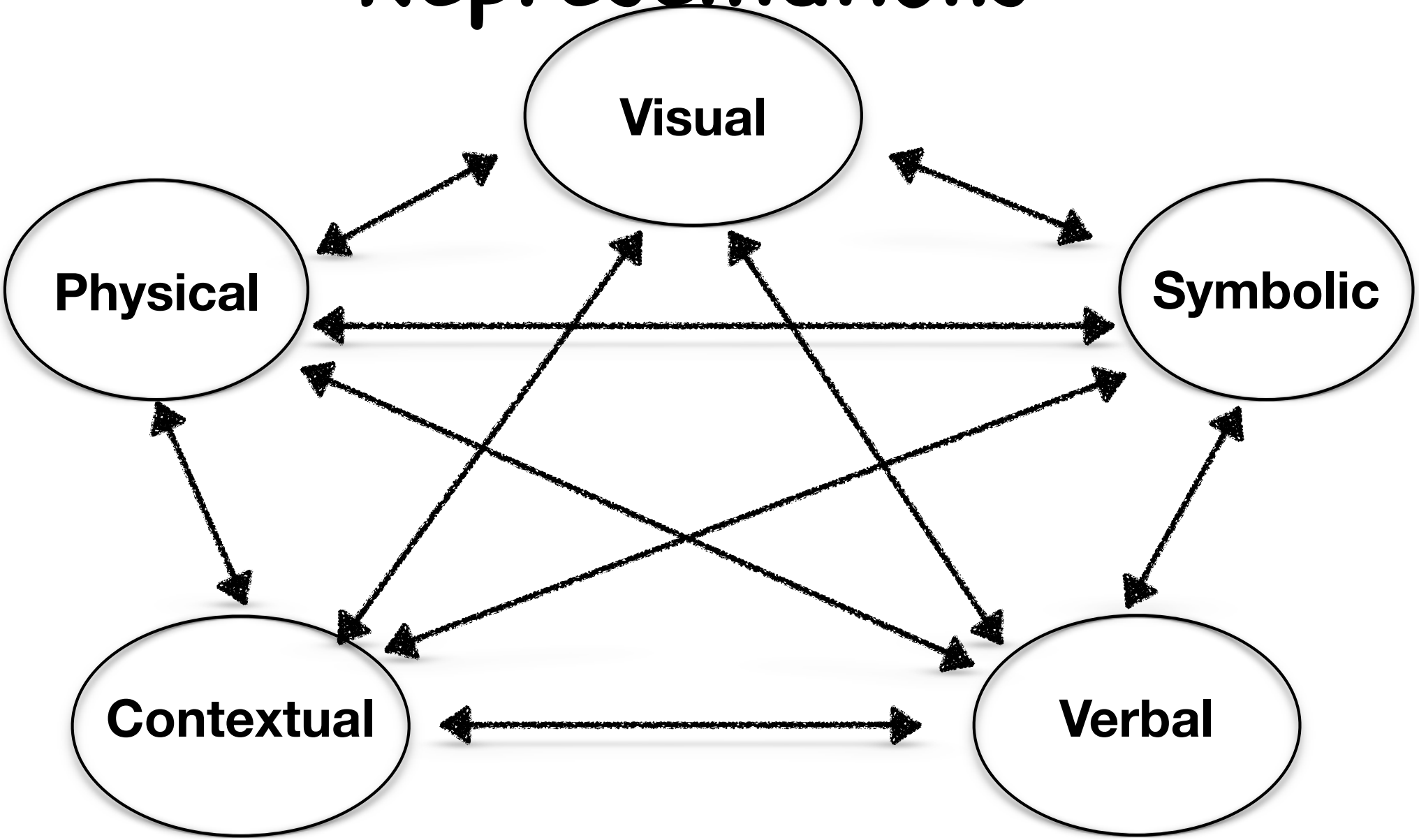


Classroom Discourse

What is the purpose of drop down notation in supporting student thinking?

How does the teachers' recording support students' reasoning?

Connections Among Representations

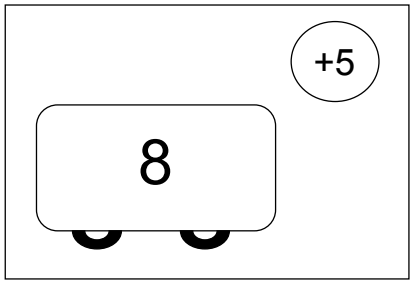


A Gradual Release of the Tool

1. Addition and subtraction story problems; students move the beads on the rekenrek

2. Addition and subtraction story problems with students: seeing the first number and not moving the beads

3. Addition and subtraction bare number problems with a context, students imagine the rekenrek



4. Task posed in pure symbolic notation
   ....Pose written tasks 8 + 5

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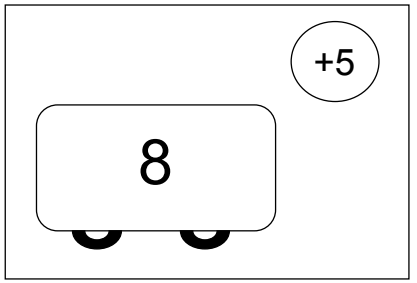


4. Task posed in pure symbolic notation
   ....Pose written tasks 8 + 5

Teacher's Actions

- Thoughtful decisions: Using formative assessment of where are students are
  - Strategically progress lessons
  - Provide support for each student, as needed
  - Record student thinking & connect multiple representations to student thinking
- **Value classroom discussions as a powerful vehicle for learning**

Mathematics Teaching Practices

1.Established mathematics goals to focus learning.

2.Implemented tasks that promote reasoning and problem solving.

3.Use and connect mathematical representations.

4.Facilitate meaningful discourse.

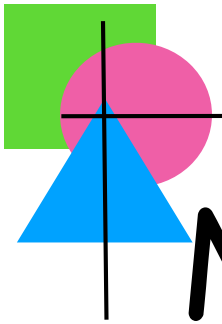
5.Pose purposeful question.

6.Build procedural fluency from conceptual understanding;

7.Support productive struggle.

8.Elicit and use evidence of student thinking.

Principles to Actions, Ensuring Mathematical Success for All, NCTM 2014



Mathematical Practices

1.Make sense of problems and preserve in solving them.

2.Reason abstractly and quantitatively.

3.Construct viable arguments and critique the reasoning of others.

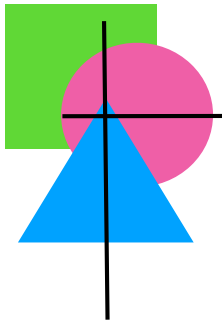
4.Model with mathematics.

5.Use appropriate tools strategically.

6.Attend to precision.

7.Look for and make use of structure.

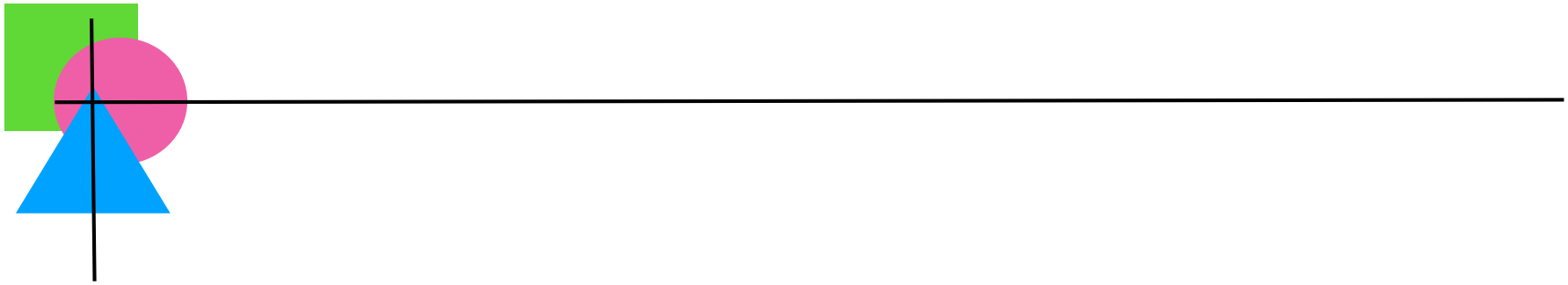
8.Look for and express regularity in repeated reasoning.



Without Focused Instruction

When instruction does not help or encourage children to construct concepts or look for patterns or relationships they will not invent reasoning strategies and thus continue to rely on counting strategies.

Arthur Baroody, 2006



Number sense instruction for fluency lays the foundation for all further learning in mathematics.

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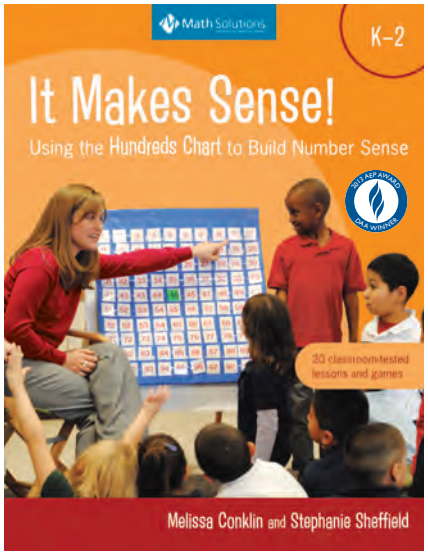
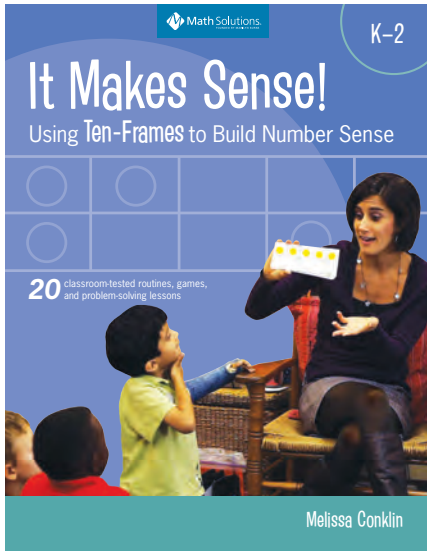
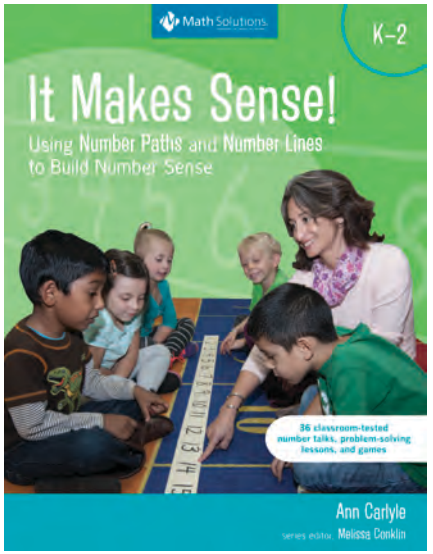
by Patty King & Amy Hurley

Published by Math Solutions in 2021







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