Making Sense of Students’ Numerical Reasoning in Order to Guide Instruction
Math Solutions celebrated its 35 birthday in February 2019.

Figure out in your head the year Math Solutions was founded?
Why Mental Math is Important

• Mental math is an important life skill.

• Learning to compute mentally is closely intertwined with developing number sense.

• Solving problems mentally pushes students to go beyond using algorithmic procedures and encourages them to reason numerically.

• Doing math mentally invites multiple strategies.
Part 1

4 + 9
Explanations from Students: \( 4 + 9 \)

- Added \( 9 + 1 \) and then \( 10 + 3 \)
- Added \( 4 + 6 \) and then \( 10 + 3 \)
- Added \( 4 + 4, 8 + 2, \) and then \( 10 + 3 \)
- Added \( 10 + 4 \), and reasoned that \( 9 + 4 \) is one less than \( 14 \)
- Counted on from \( 9 \)
- Counted on from \( 4 \)
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation
Different Students . . . Different Answers
“So I know that if you add something to a 9, it’s going to be a teen number. Then minus 1 from that number and make it a teen, ‘cause 9 plus 1 equals 10, then you use the extra numbers beyond 10 and it makes, like, it made 13.”
“’Cause . . . because I took the 9 and split it into 1 and into 8, and then I put . . . and then I put 1 plus 4 equals 5 and then I put the 8 and the 5 plus 8 equals 13.”
“Because 9 has a 4, I take the 5 out of the 9 and then 4 plus 4 is 8. And then I take away 2 from the 5 and then I put it with the 8, and that makes 10. And then from the 5 that leaves 3, so it’s 13.”
How Genesis reasoned

9 = 4 + 5     Decomposed 9 into 4 + 5
4 + 4 = 8     Used known fact
2 + 3 = 5     Decomposed 5 into 2 + 3
8 + 2 = 10    Used benchmark of 10
10 + 3 = 13   Added 10 + 3
Part 2
Genesis interviewed by Mallika Scott

A Sampling of an Entire Interview
Addition and Subtraction within 20
“Because I took away 2 from the 6. And then I took away this 2 (pointing at the 2 in the 12) from the 12. And then it left me 4, and 10 take away 4 is 6.”
Explanations:  \[12 - 6\]

- Added 6 + 6
- Counted back from 12
- Subtracted 12 – 2 and then 10 – 4
- Subtracted 6 – 2, \[12 - 2\], and then 10 – 4
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation
How Genesis reasoned

6 – 2 = 4
12 – 2 = 10
10 – 4 = 6
7 + 8
“Because I take away the 2 and the 3 from the 7 and 8, and then it’s 5 plus 5 equals 10, and 3 plus 2 equals 5, and then it’s 15.”
Explanations: 7 + 8

- Made 10 (7 + 3, 8 + 2, or 5 + 5) and then added 5
- Used doubles (7 + 7 + 1 or 8 + 8 – 1)
- Counted on from 7 or 8
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation
Explanations: **11 – 9**

- Counted on from 9 or added 9 + 2
- Explained that 9 is 2 away from 11 or counted back from 11 to 9
- Used 10 + 1 or 10 – 9 to solve 11 – 9
- Counted back 9 from 11
- Changed 11 – 9 to 10 – 8
- Subtracted 11 – 1 and then 10 – 1
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation
How Genesis reasoned

\[
9 - 1 = 8 \\
11 - 1 = 10 \\
10 - 8 = 2
\]
Why does this work?
I baked 20 oatmeal cookies. There were 12 cookies left.

How many cookies were eaten?
Explanations: I baked 20 cookies. There were 12 left.

- Added (e.g., 12 + 3 and then 15 + 5)
- Subtracted without using standard algorithm (e.g., 20 – 10 and then 10 – 2)
- Counted on from 12
- Counted back from 20
- Used standard algorithm to subtract
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation
What did Genesis do?

\[2 + 8 = 10\]

\[10 - 8 = 2\]

So, \[20 - 8 = 12\]

Her go-to strategy is to use the benchmark of 10. She breaks 10 into 2 + 8 and, therefore, knows that 10 − 8 equals 2. Then she reasons that 20 − 8 equals 12.

Why does this work?
Strategies for Mentally Adding & Subtracting
(numbers within 10, 20, 100, 1000)

1. Decomposes numbers within 10
2. Counts on or back
3. Uses known facts (e.g., doubles)
4. Uses benchmark numbers (e.g., 10, 25, multiples of 10 or 25)
5. Uses the inverse relationship between addition and subtraction
6. Decomposes numbers into their place value parts
7. Solves missing addend problems
8. Interprets and solves contextual problems that involve addition or subtraction
Part 3

Another Problem, Six Strategies

7 x 6
Explanations:  7 x 6

- Skip-counted
- Used a known fact (e.g., 6 x 6 = 36, so 7 x 6 = 42)
- Added (e.g., 6 + 6, then 12 + 6, etc.)
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation
7 \times 3 = 21, \ 7 \times 3 = 21, \ 21 + 21 = 42

7 \times 5 = 35, \ 35 + 7 = 42

Skip count by 6 or 7

6 \times 5 = 30, \ 6 \times 2 = 12, \ 30 + 12 = 42

7 \times 7 = 49, \ 49 - 7 = 42

6 \times 6 = 36, \ 36 + 6 = 42
“I’m now convinced that listening to students, one on one, provides access into how they reason in a way that’s unique and invaluable.”

--Marilyn Burns
Our teaching goal is not to cover the Standards . . .

but to *uncover* them.