Houghton Mifflin Harcourt

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# Maximizing Learning or Missing Opportunities?

## Helping Teachers Foster Algebra Readiness

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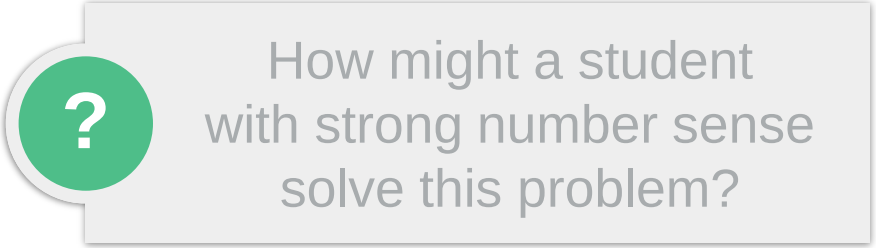
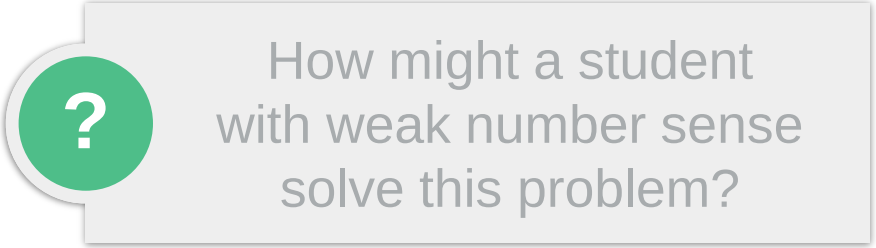
# Solve Mentally

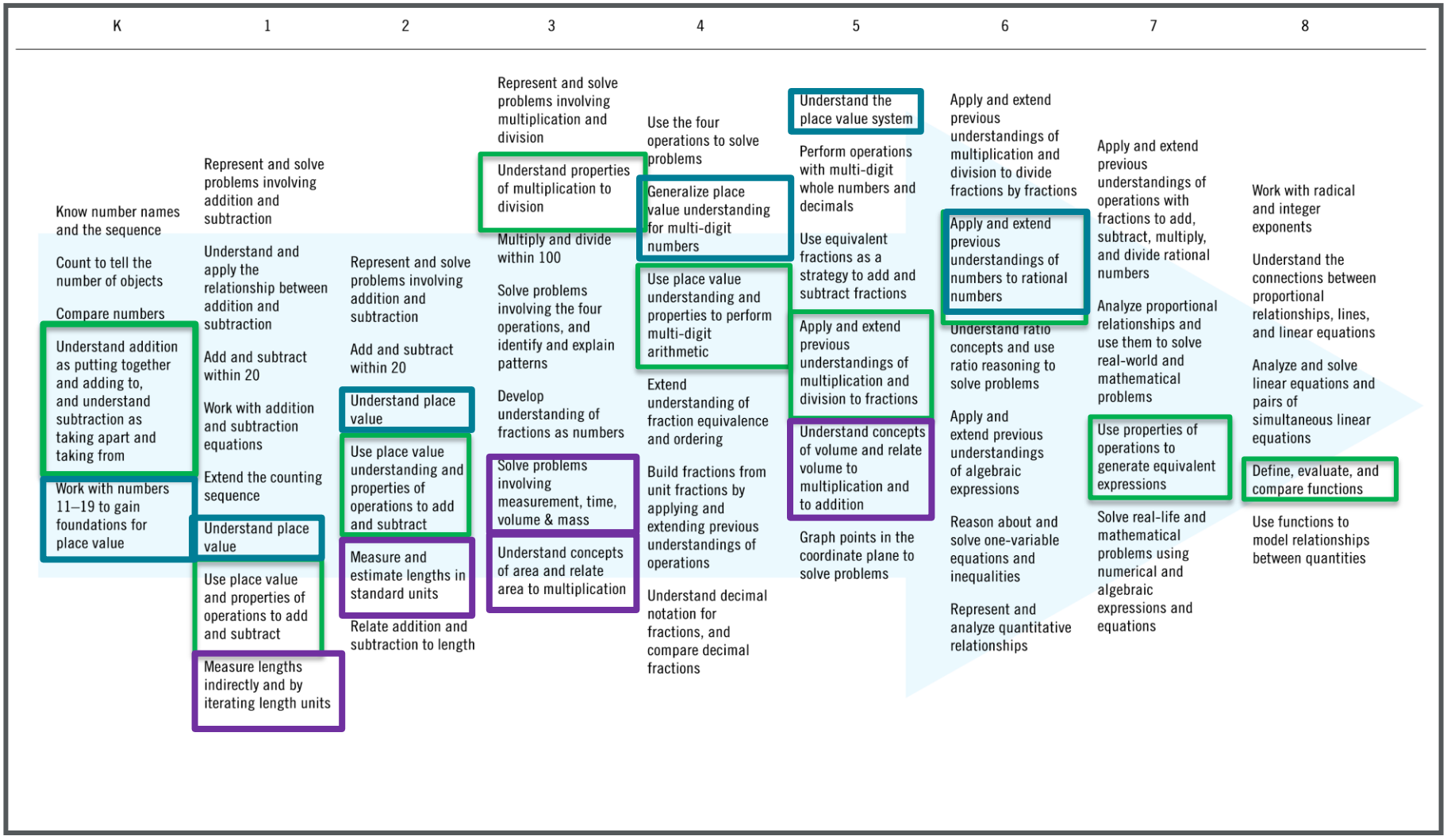
- 1000 – 998 = ?
- 99 + 17 = ?
- 12.6 X 10 = ?



A child who does not see patterns often does not expect things to make sense and sees all events as discrete, separate, and unrelated."
A student who expects things to 'make sense' *looks* for patterns and generalizations and from these develops understanding.

# Pathway to Algebra

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A division of HMH

# Understanding Arithmetic: Three Pillars

- Understanding numbers
- Developing computational fluency
- Examining the behavior of the operations

# Arithmetic ⟷ Algebraic Thinking

…using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction (multiplication/division).

# Setting the Stage for Algebra Readiness

## Decompose Small Numbers

$6 = 1 + 5$

$6 = 2 + 4$

$6 = 3 + 3$

## Decompose to Find Sums

$18 + 6 =$

$18 + (2 + 4) =$

$(18 + 2) + 4 =$

$20 + 4 =$

$24$

# Linking to Larger Numbers

$$28 + 14 = \square$$

$$20 + 10 = 30$$
$$8 + 4 = 12$$
$$20 + 12 = 32$$

# Linking to Fractions

$$3\ \frac{3}{4} + \frac{1}{2} =$$

$$3\ \frac{3}{4} + \left(\frac{1}{4} + \frac{1}{4}\right) =$$

$$\left(3\ \frac{3}{4} + \frac{1}{4}\right) + \frac{1}{4} =$$

$$4\frac{1}{4}$$

# Working Towards Algebra Readiness

? 16 × 3

| | 10 | 6 |
|---|---|---|
| 3 | 30 | 18 |

30 + 18 = 48

? 16 × 13

| | 10 | 6 |
|---|---|---|
| 10 | 100 | 60 |
| 3 | 30 | 18 |

100 + 60 + 30 +18 = 208

# Linking to Fractions

$$4 \times 3\ \frac{1}{3} =$$

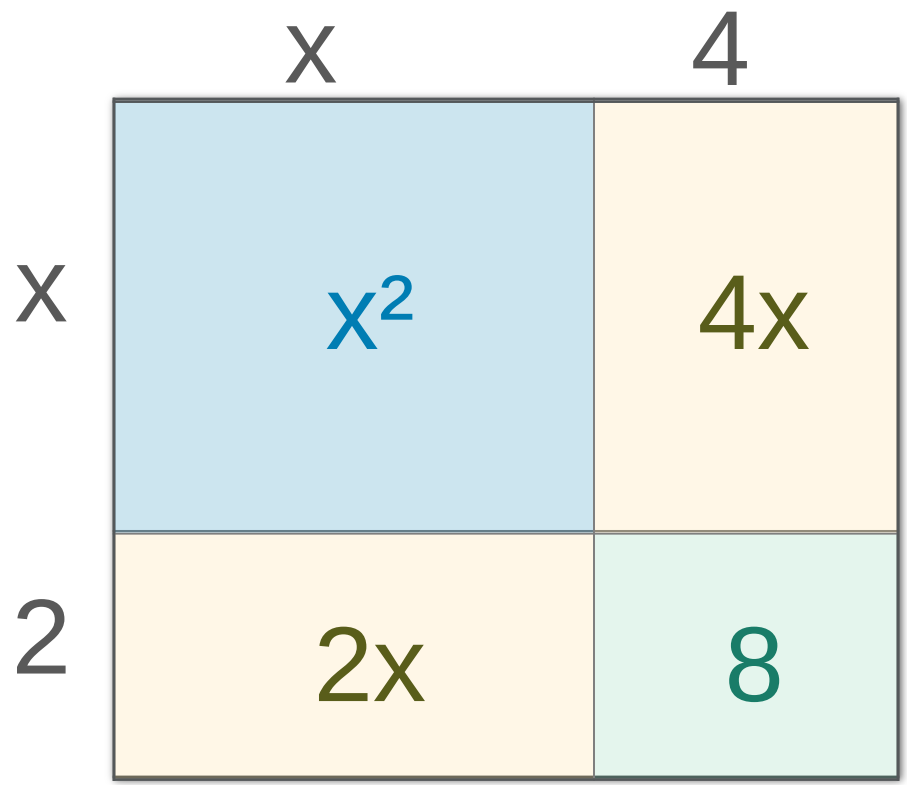
$$(4 \times 3) + (4 \times \frac{1}{3}) =$$

$$12 \quad + \quad \frac{4}{3} \quad =$$

$$13\frac{1}{3}$$

# Building Critical Algebra Foundations

$$(x + 4) \times (x + 2) = x^2 + 8$$




$x^2 + 6x + 8$

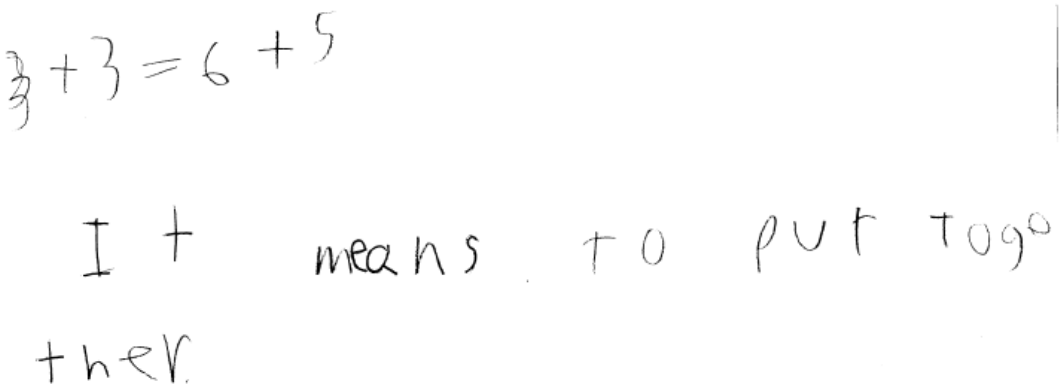
# Understanding Arithmetic: Three Pillars

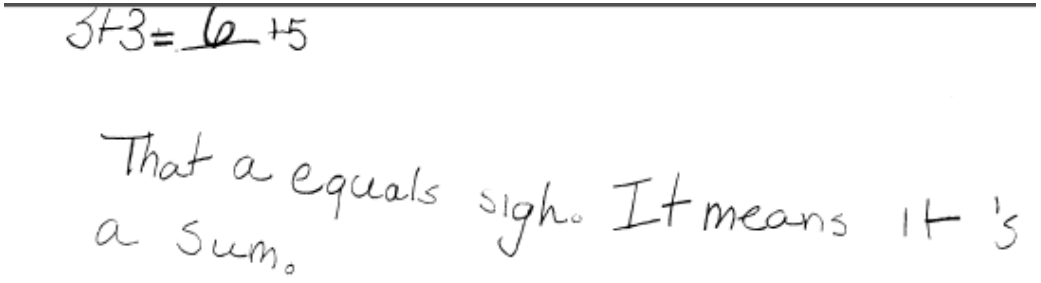
- Understanding numbers
- Developing computational fluency
- **Examining the behavior of the operations**

# Interpreting the Equal Sign

"1 plus 7 makes 8"

1 + 7 = 8



# Interpreting the Equal Sign

$$8 + 4 = \square + 5$$

# Interpreting the Equal Sign

$7 = 3 + 4$

$8 = 8$

$5 + 8 = 8 + 5$

$6 - \square = 7 - 4$

# True or False – How do you know?

$7 = 3 + 4$

$8 = 5 + 13$

$6 – 1 = 7$

$27 = 7 + 10 + 10$

$10 – 3 = 11 - 4$

1.OA.D.7 Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false.

1.OA.D.8 Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

3.OA.A.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers.

# Relational Thinking

Relational thinking occurs when one observes and uses number relationships between the two sides of the equal sign instead of actually computing amounts.

# 6 + □ = 5 + 9

Since 5 + 9 is 14, I need to figure out 6 plus what equals 14. It is 8, so the box is 8.

Six is one more than the 5 on the other side. That means the box should be one less than 9, so it must be 8.

# Encouraging Relational Thinking

37 + 54 = 38 + 53

48 + 63 – 62 = 49

625 + 450 = 700 + 400

126 – 37 = ☐ - 40

# Exploring True and False Sentences

1. Give each other think time on each number sentence before talking.

2. Share with each other your reasoning. How did you decide if it was true or false?

3. Which examples were interesting to you?

# Exploring Open Sentences

1. Pause for think time. Push yourself to use relational thinking.

2. Share with each other your reasoning.

3. Will that strategy work for other numbers?

# Look for and make use of structure.

# True or False?

6 + 9 = 9 + 6

4 - 3 = 3 – 4

90 – 0 = 0 - 90

7 + 50 = 50 + 7

6 + ☐ = 10 + 6

10 + ☐ = ☐ + 10

Commutative property of addition states that changing the order of the addends does not change the sum.

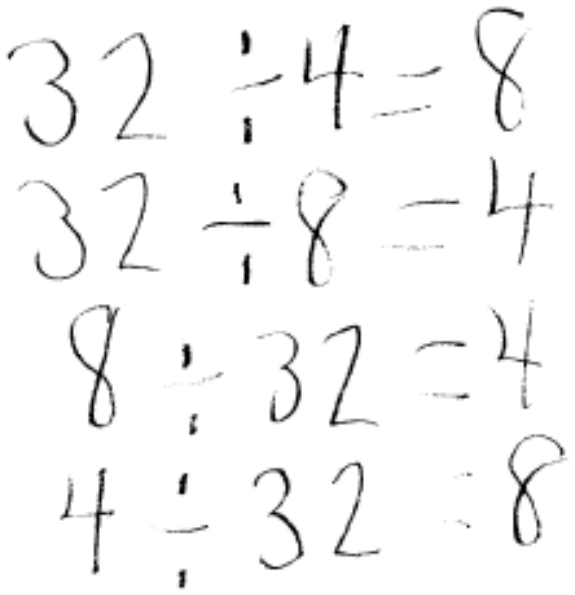
# Supporting Teachers

$7 + 3 = \square + 9$

$6 + 2 = 1 + 7$

# Inverse Operations

2. How can knowing 8 x 4 = 32 help you to understand 32 ÷ 4?



# Understanding Arithmetic: Three Pillars

- Understanding numbers
- Developing computational fluency
- Examining the behavior of the operations

"A focus on the operations emphasizes noticing, describing, representing, and explaining consistencies across many problems. Generalizing in this way about the properties and behaviors of the operations is not about solving particular problems but about regularities that are foundational to arithmetic and algebra."

Connecting Arithmetic to Algebra by Susan Jo Russell, Deborah Schifter, and Virginia Bastable (Portsmouth, NH: Heinemann, 2011).

# Impacting Teacher's Practice

- What two or three things to support algebra readiness do you want to see in your classrooms?
- What ideas do you have for making those happen?

# How to Support Algebra Readiness

- Provide teachers with professional development that builds:
  - –Robust content knowledge
  - –Understanding of how student learn
  - –Effective instructional strategies
  - – Strategies for gathering information about what students do and do not understand
- Ensure teachers and students have quality materials and programs

# Thank you!

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# Progress to Algebra in Grades K-8

| K | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Know number names and the count sequence<br><br>Count to tell the number of objects<br><br>Compare numbers<br><br>Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from<br><br>Work with numbers 11-19 to gain foundations for place value | Represent and solve problems involving addition and subtraction<br><br>Understand and apply properties of operations and the relationship between addition and subtraction<br><br>Add and subtract within 20<br><br>Work with addition and subtraction equations<br><br>Extend the counting sequence<br><br>Understand place value<br><br>Use place value understanding and properties of operations to add and subtract<br><br>Measure lengths indirectly and by iterating length units | Represent and solve problems involving addition and subtraction<br><br>Add and subtract within 20<br><br>Understand place value<br><br>Use place value understanding and properties of operations to add and subtract<br><br>Measure and estimate lengths in standard units<br><br>Relate addition and subtraction to length | Represent & solve problems involving multiplication and division<br><br>Understand properties of multiplication and the relationship between multiplication and division<br><br>Multiply & divide within 100<br><br>Solve problems involving the four operations, and identify & explain patterns in arithmetic<br><br>Develop understanding of fractions as numbers<br><br>Solve problems involving measurement and estimation of intervals of time, liquid volumes, & masses of objects<br><br>Geometric measurement: understand concepts of area and relate area to multiplication and to addition | Use the four operations with whole numbers to solve problems<br><br>Generalize place value understanding for multi-digit whole numbers<br><br>Use place value understanding and properties of operations to perform multi-digit arithmetic<br><br>Extend understanding of fraction equivalence and ordering<br><br>Build fractions from unit fractions by applying and extending previous understandings of operations<br><br>Understand decimal notation for fractions, and compare decimal fractions | Understand the place value system<br><br>Perform operations with multi-digit whole numbers and decimals to hundredths<br><br>Use equivalent fractions as a strategy to add and subtract fractions<br><br>Apply and extend previous understandings of multiplication and division to multiply and divide fractions<br><br>Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition<br><br>Graph points in the coordinate plane to solve real-world and mathematical problems* | Apply and extend previous understandings of multiplication and division to divide fractions by fractions<br><br>Apply and extend previous understandings of numbers to the system of rational numbers<br><br>Understand ratio concepts and use ratio reasoning to solve problems<br><br>Apply and extend previous understandings of arithmetic to algebraic expressions<br><br>Reason about and solve one-variable equations and inequalities<br><br>Represent and analyze quantitative relationships between dependent and independent variables | Apply and extend previous understanding of operations with fractions to add, subtract, multiply, and divide rational numbers<br><br>Analyze proportional relationships and use them to solve real-world and mathematical problems<br><br>Use properties of operations to generate equivalent expressions<br><br>Solve real-life and mathematical problems using numerical and algebraic expressions and equations | Work with radical and integer exponents<br><br>Understand the connections between proportional relationships, lines, and linear equations<br><br>Analyze and solve linear equations and pairs of simultaneous linear equations<br><br>Define, evaluate, and compare functions<br><br>Use functions to model relationships between quantities* |

*Indicates a cluster that is well thought of as part of a student's progress to algebra, but that is currently not designated as Major by one or both of the assessment consortia in their draft materials. Apart from the two asterisked exceptions, the clusters listed here are a subset of those designated as Major in both of the assessment consortia's draft documents.

# True or False?

**Sort these into two groups – those that are true and those that are false. Try to use relational reasoning rather than performing the calculations on both sides of the equal sign.**

| | |
|---|---|
| $345 + 71 = 70 + 344$ | $\frac{6}{7} + \frac{3}{7} = 1 + \frac{3}{7} - \frac{1}{7}$ |
| $37 + 56 = 39 + 54$ | $33 - 27 = 34 - 26$ |
| $9 \times 7 = 10 \times 7 - 7$ | $4 \times 6 = 6 + 6 + 6 + 6$ |
| $0.3 \times 7 = 0.7 \times 3$ | $7 \times 8 = (2 \times 8) + (5 \times 8)$ |
| $6 \times \frac{1}{3} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ | $7 \times \frac{4}{5} = \frac{4}{5} \times 7$ |


 

# Open Sentences

**Determine the number that goes in the box without performing calculations.**

| | |
|---|---|
| $\frac{\square}{3} = \frac{7}{7}$ | $18 + 22 = 32 + \square$ |
| $2.4 \div \square = 4.8 \div 6$ | $\frac{7}{8} = \square + \frac{4}{8}$ |
| $10 + 8 + \square = 12 + 9$ | $126 - 37 = \square - 40$ |
| $\square \times \frac{3}{4} = 12 \times 3 \times \frac{1}{4}$ | $8 \times 7 = \square \times 7 + 7$ |
| $(4 \times 3) + (4 \times 0.25) = 4 \times \square$ | $12.8 - 1.9 = \square - 2$ |


 