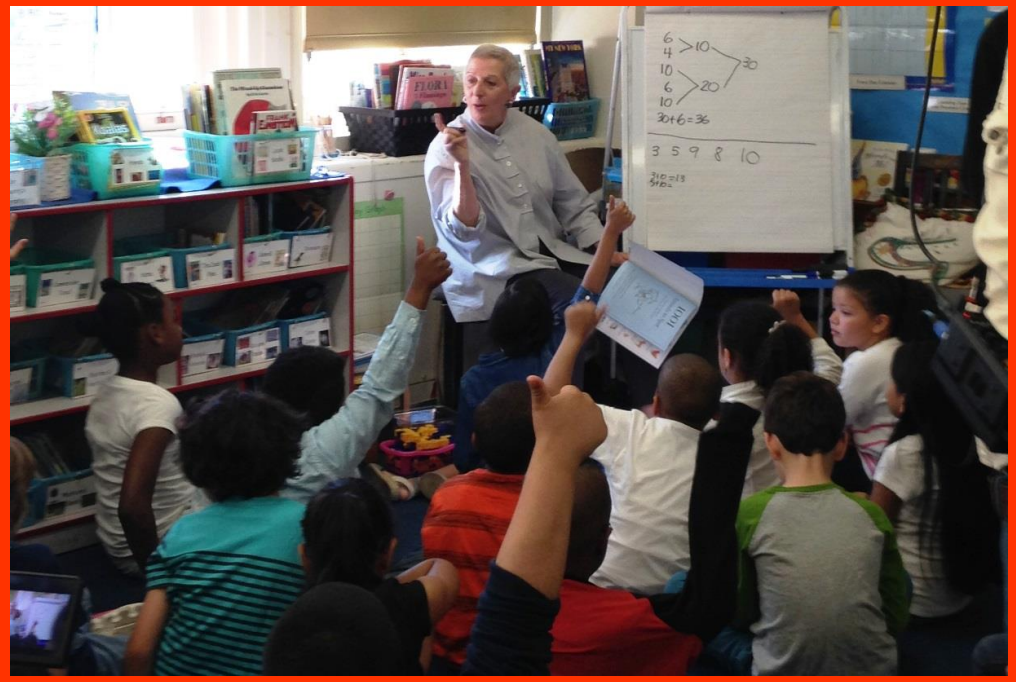
Planning and Preparing for Lessons: Rarely Easy, Always Important

Marilyn Burns

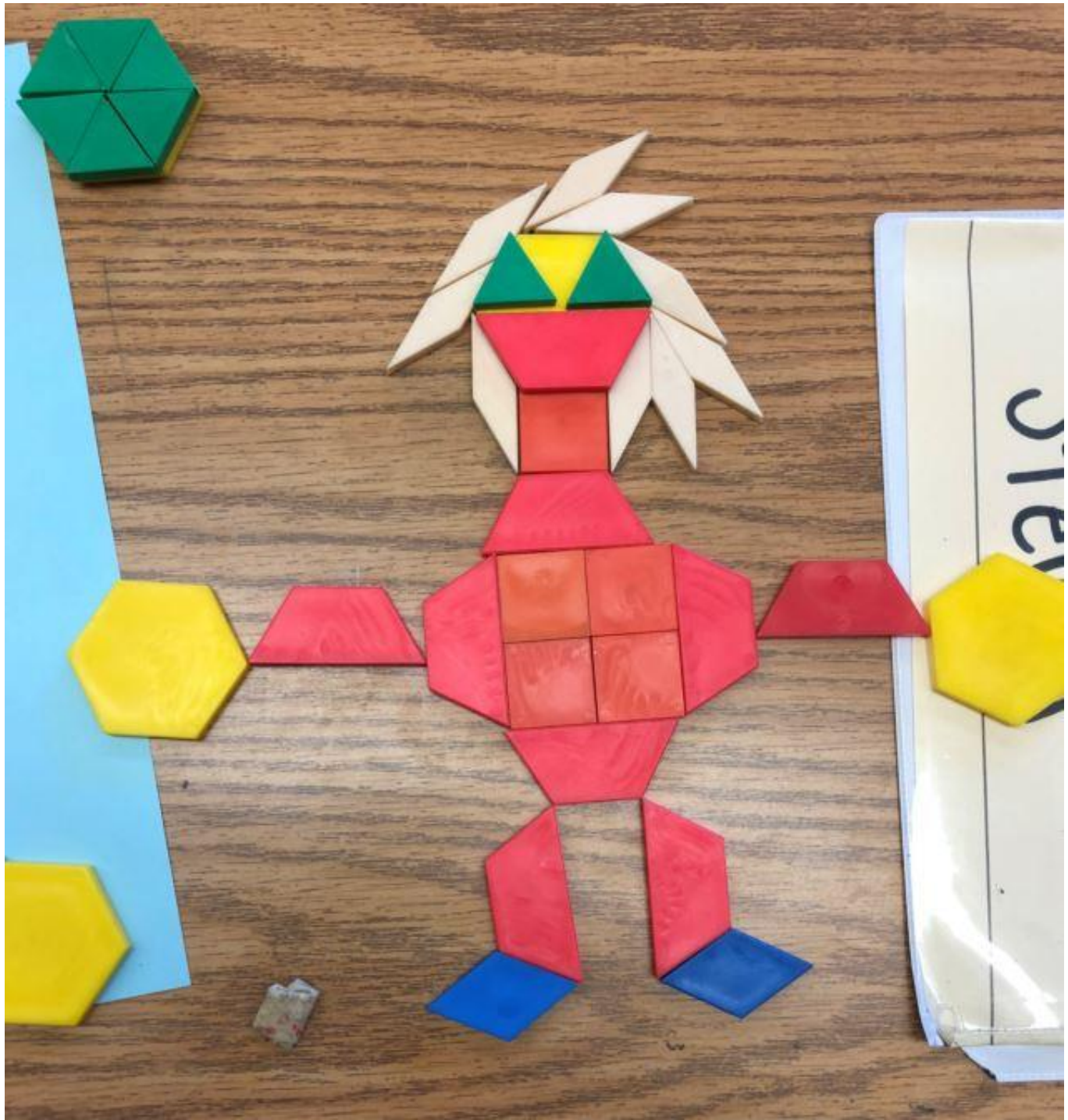


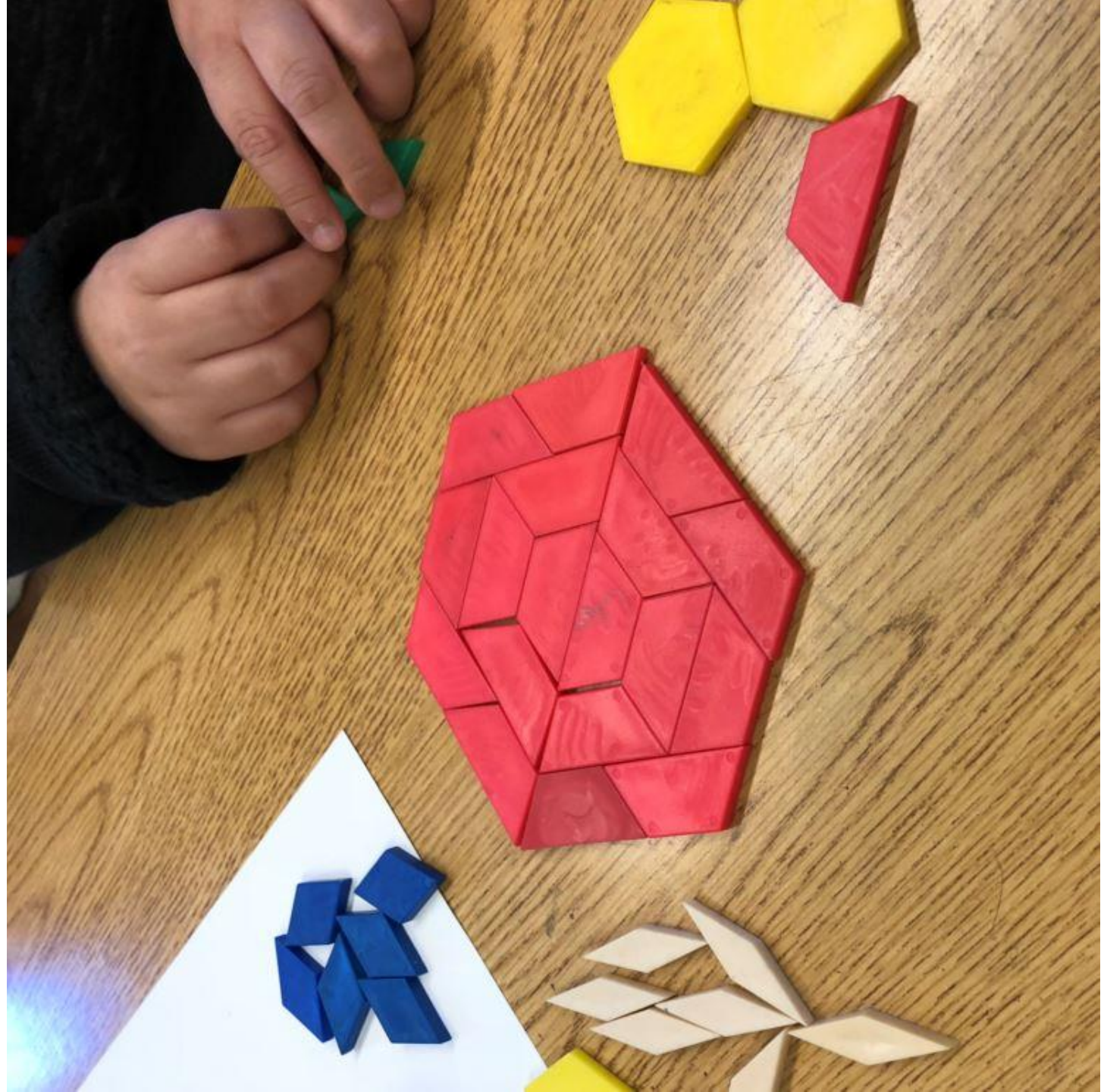
NCTM Annual Meeting 2018, Thursday, April 26, 4:30 pm

Story #1:

Connecting numerical reasoning and geometry

Exploring with Pattern Blocks





Try These:

① Make a straight road. Try doing this with only 1 block. Can you make a straight road with each block?

② Build larger shapes. Build a large triangle with only triangles. Build a large square with only squares. Can you do this with all the shapes?

③ Cover your entire paper. Try doing this with different blocks. Can you do it with just one block? Will all blocks work?

- Yellow Hexagon
- Cream Rhombus
- Red Trapezoid
- Orange Square
- Green Triangle
- Blue Rhombus

Mohamed: Two shapes can equal one shape. Two triangles equal 1 blue rhombus. Like two halves equal 1 whole.

Jason: 3 blue rhombus equal one hexagon.

Sofia: It's like fractions! Each blue rhombus is $\frac{1}{3}$ of a hexagon.

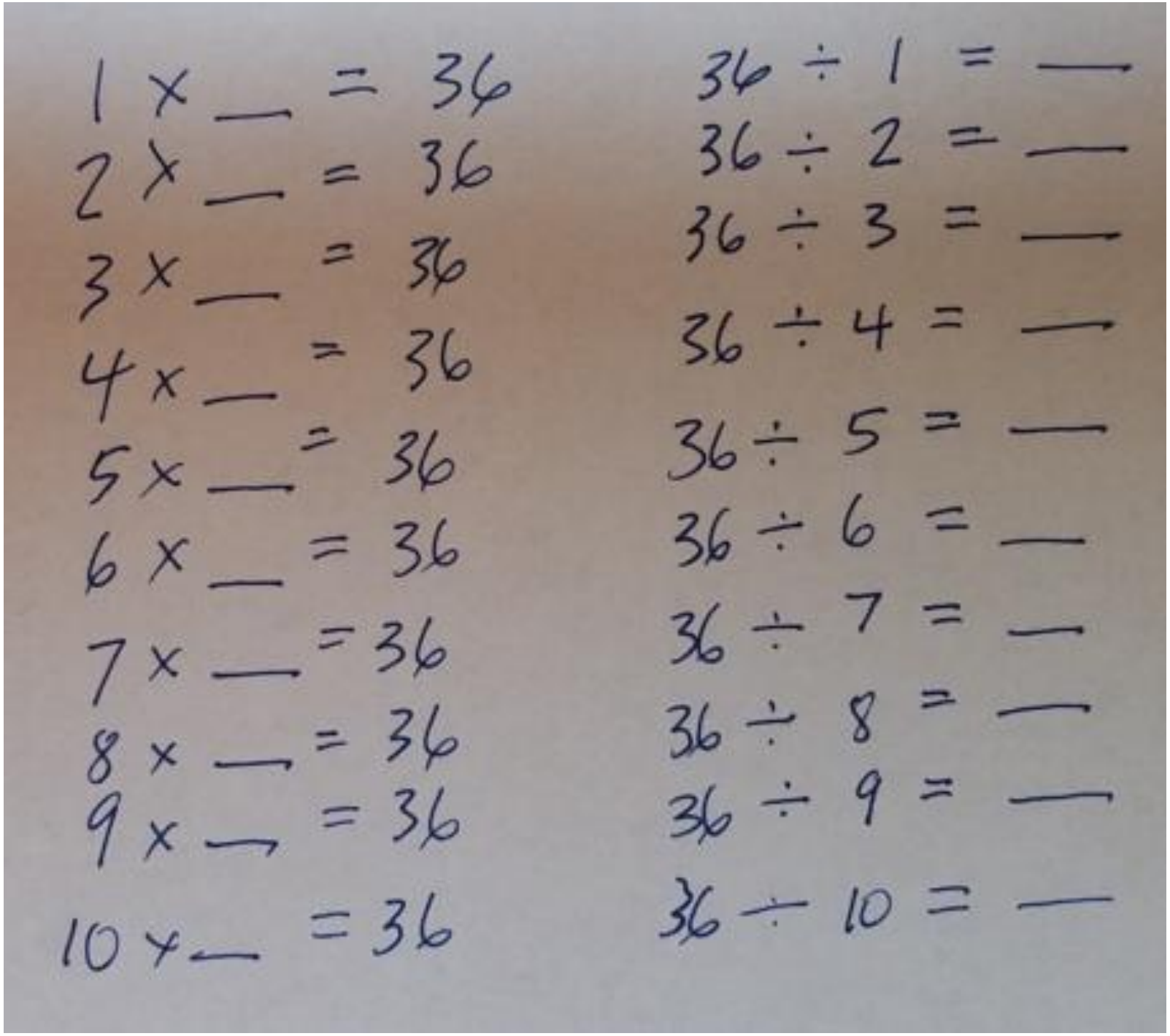
Joseph: 2 Red trapezoids make a hexegon. 1 red trapezoid is $\frac{1}{2}$ of a hexagon.

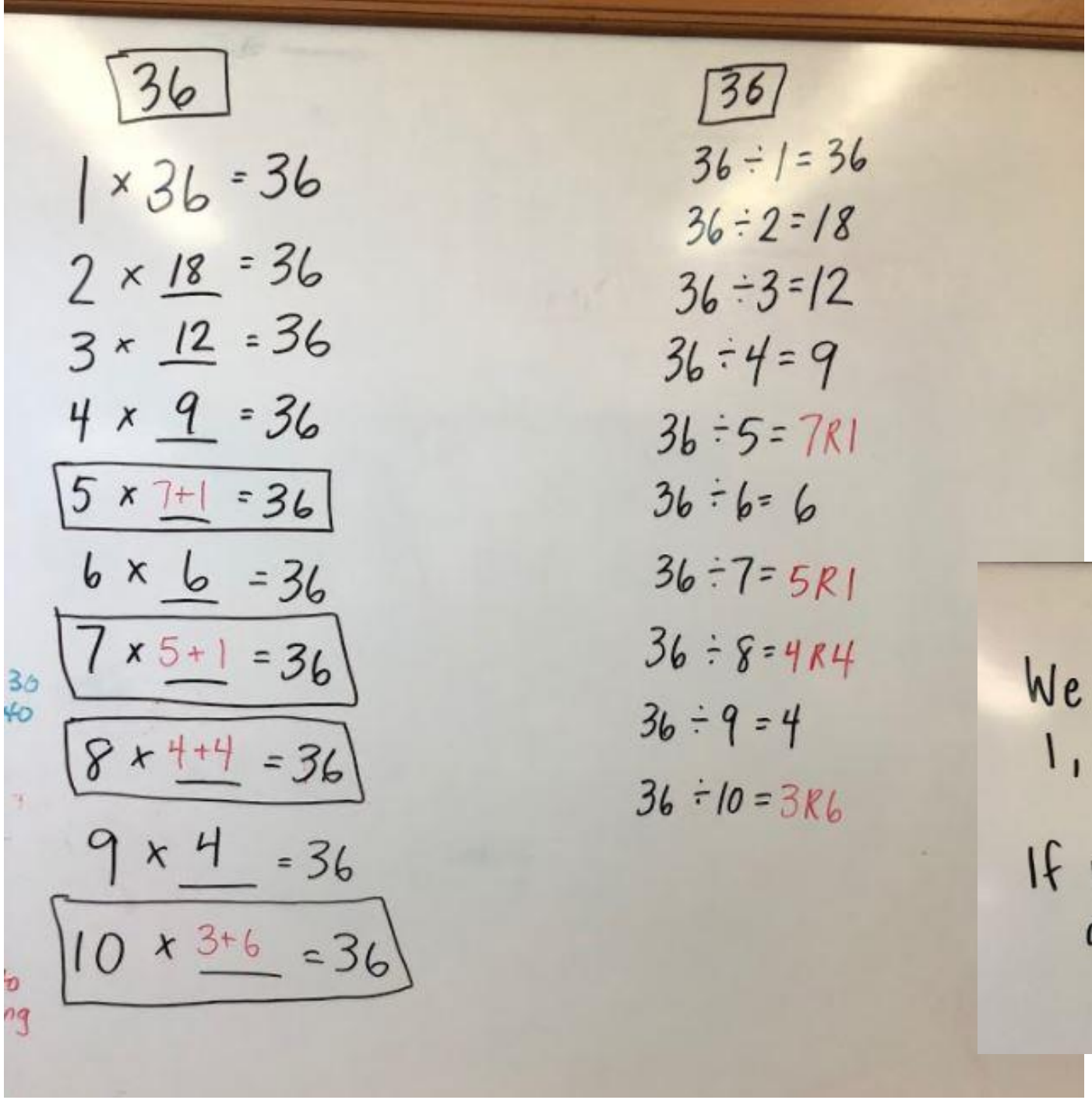
D'Sir/Jorge: 6 green triangles make a hexagon. 1 green triangle is $\frac{1}{6}$ of a hexagon.

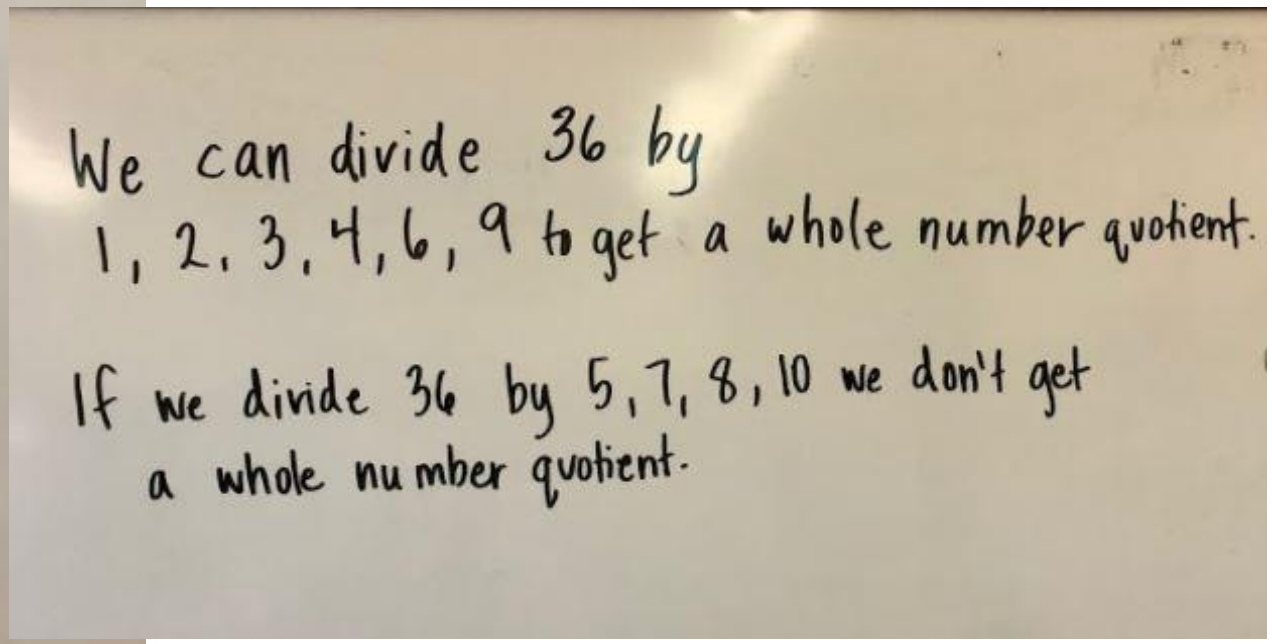
today I learned that many
Shapes are equal to another
Shapes its almost like change
Fraction to equivalent fraction also I
hoticed that 6 green triangle
are equal to 1 hexagon and
a triangle have 3 Points but
two red trapezoied are equal
to one hexagon and it
has four Points so why
does the square has four Points
and it dosent work in a
hexagon? ??? ? ? ? ?

A Number Talk—

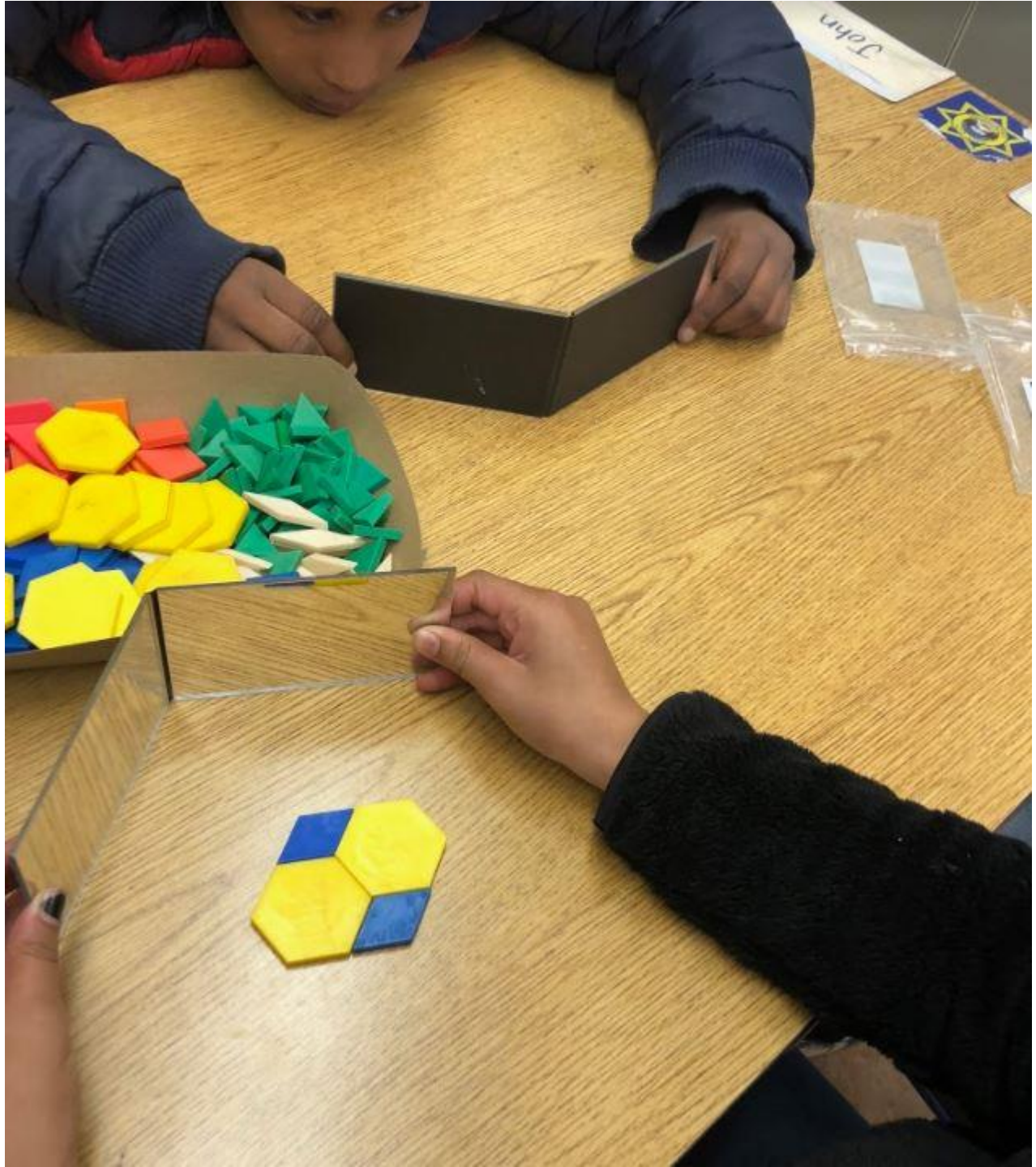
Finding the whole number factors of 36 for the numbers from 1 to 10

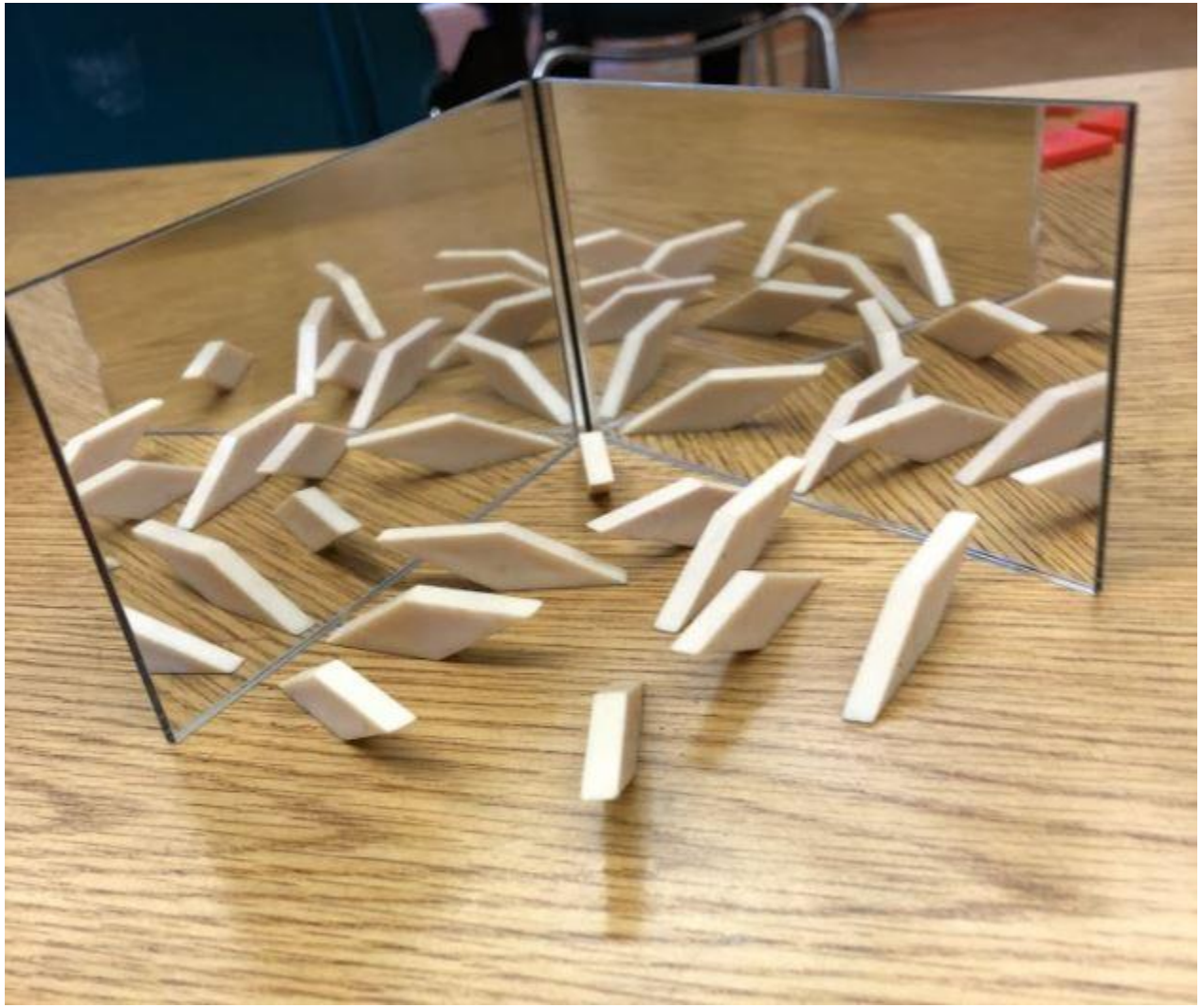


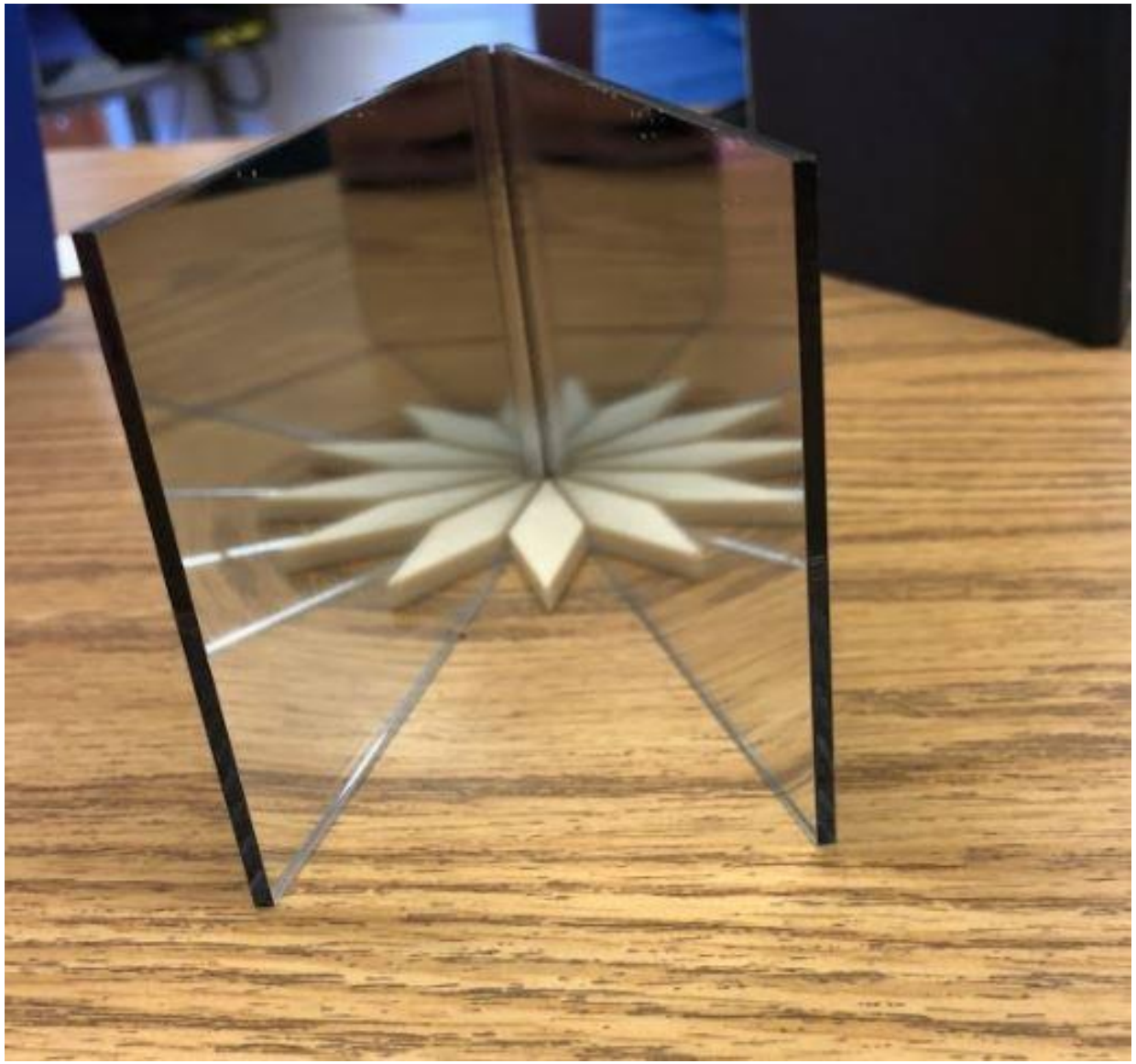

36
1 × 36 = 36
2 × 18 = 36
3 × 12 = 36
4 × 9 = 36
5 × 7+1 = 36
6 × 6 = 36
7 × 5+1 = 36
8 × 4+4 = 36
9 × 4 = 36
10 × 3+6 = 36
36
36 ÷ 1 = 36
36 ÷ 2 = 18
36 ÷ 3 = 12
36 ÷ 4 = 9
36 ÷ 5 = 7R1
36 ÷ 6 = 6
36 ÷ 7 = 5R1
36 ÷ 8 = 4R4
36 ÷ 9 = 4
36 ÷ 10 = 3R6


We can divide 36 by
1, 2, 3, 4, 6, 9 to get a whole number quotient.
If we divide 36 by 5, 7, 8, 10 we don't get
a whole number quotient.

Exploring with Pattern Blocks & Hinged Mirrors

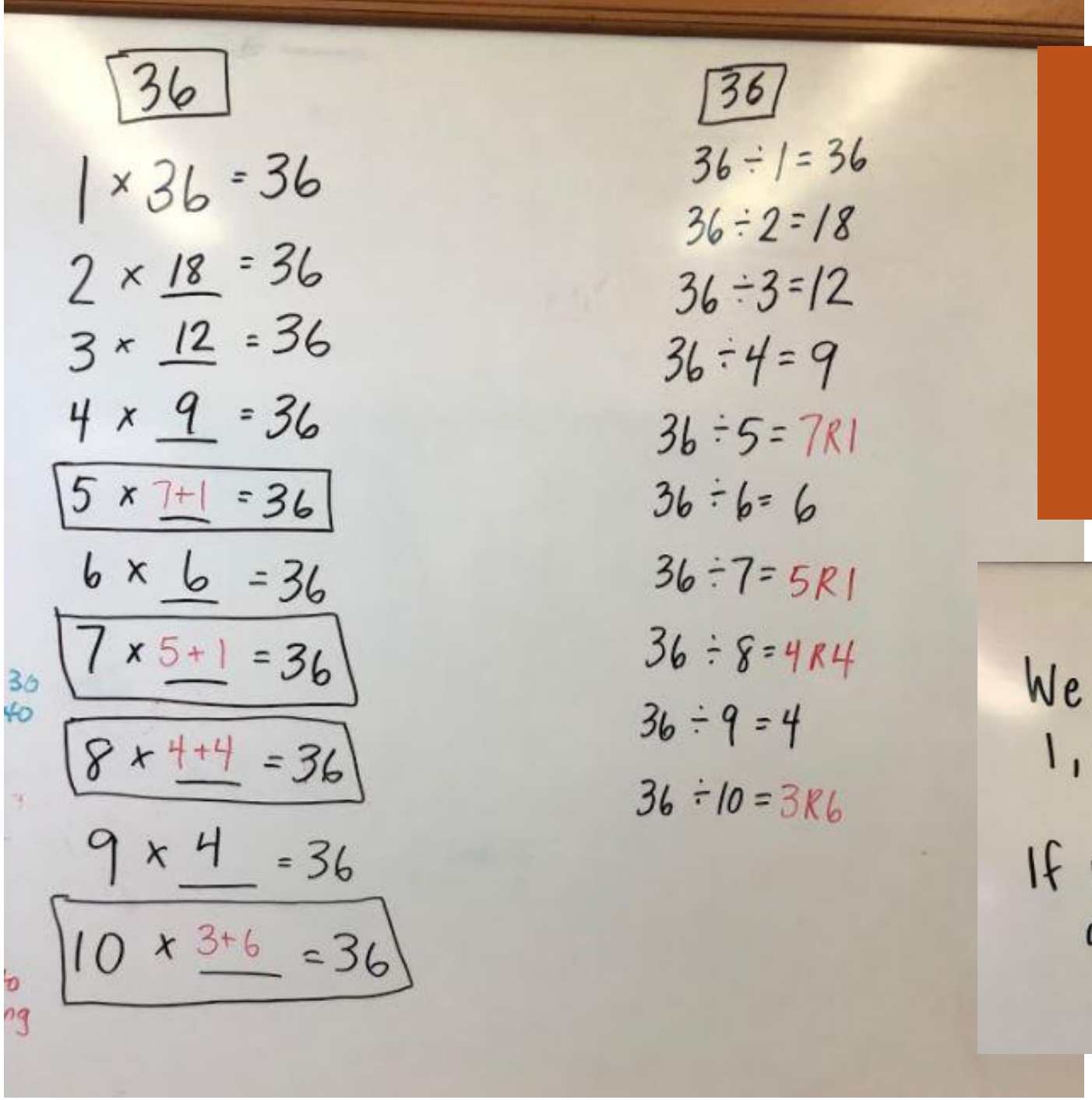

John

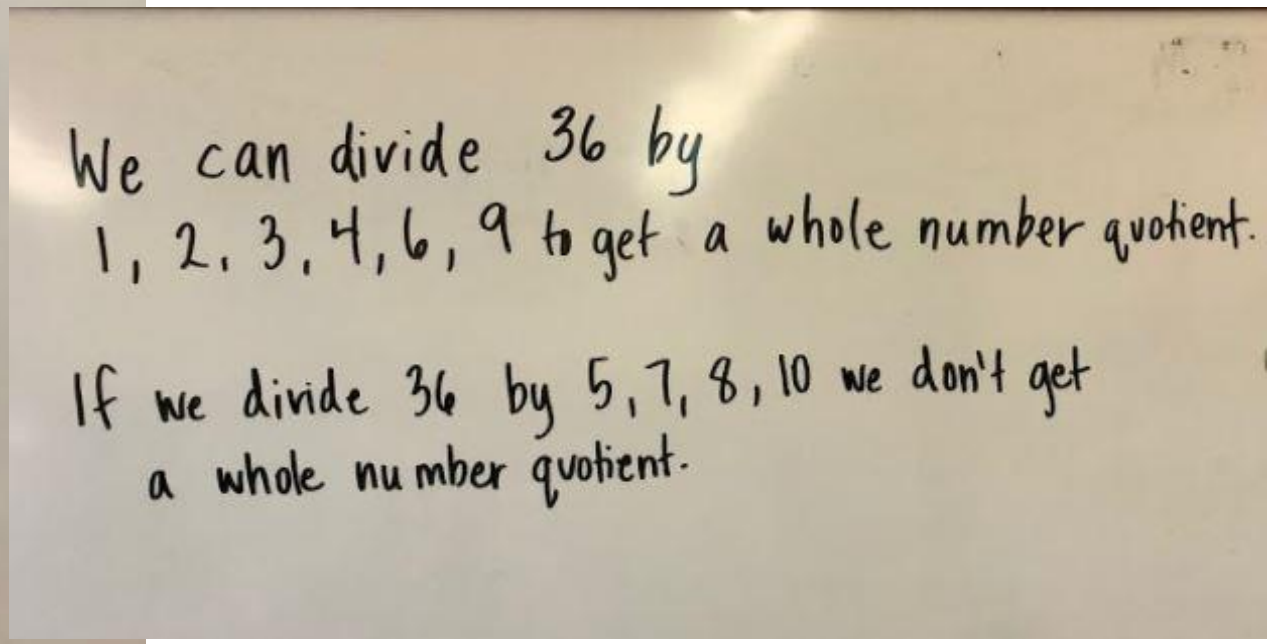


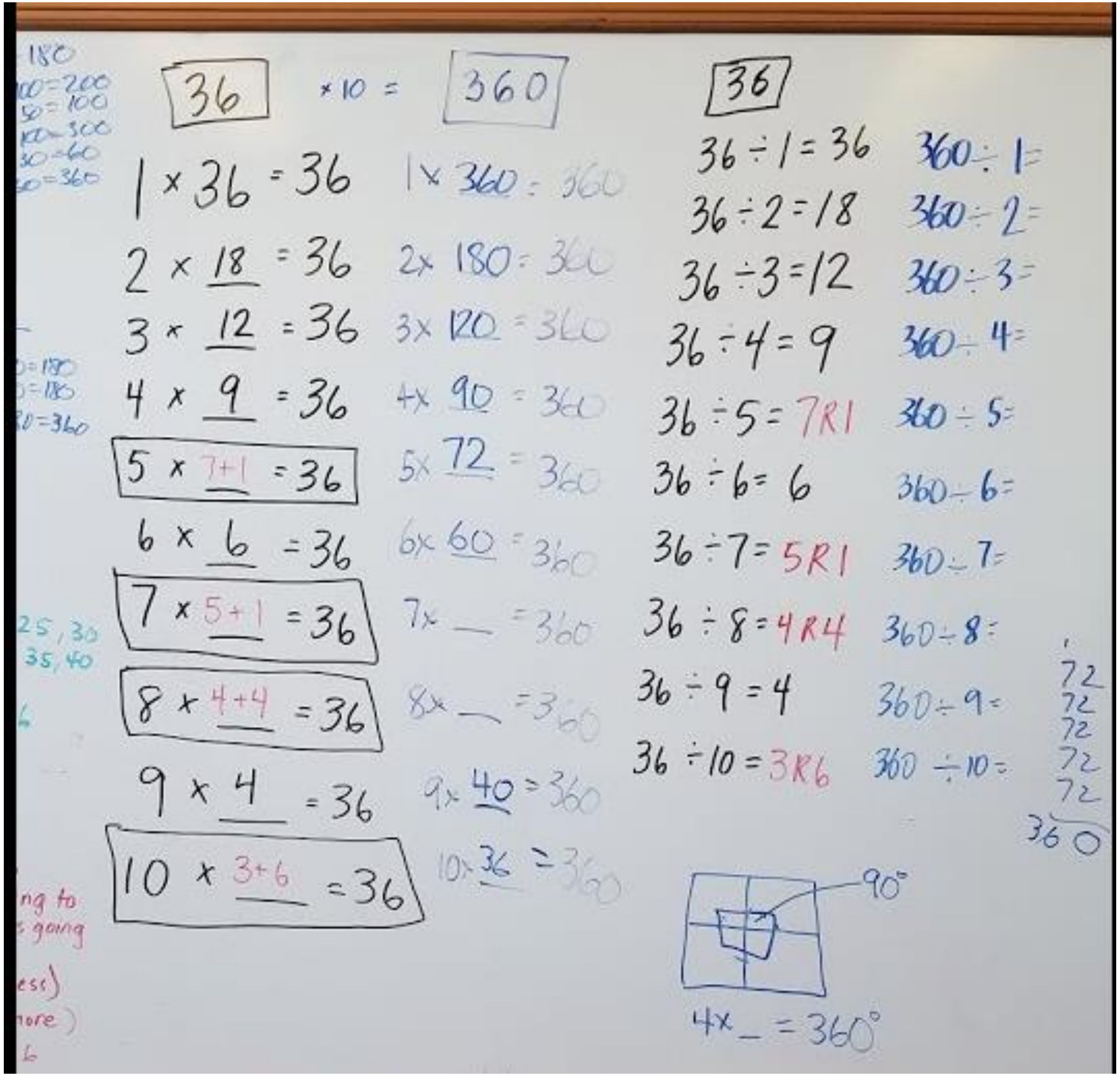


And the next day . . . first a number talk

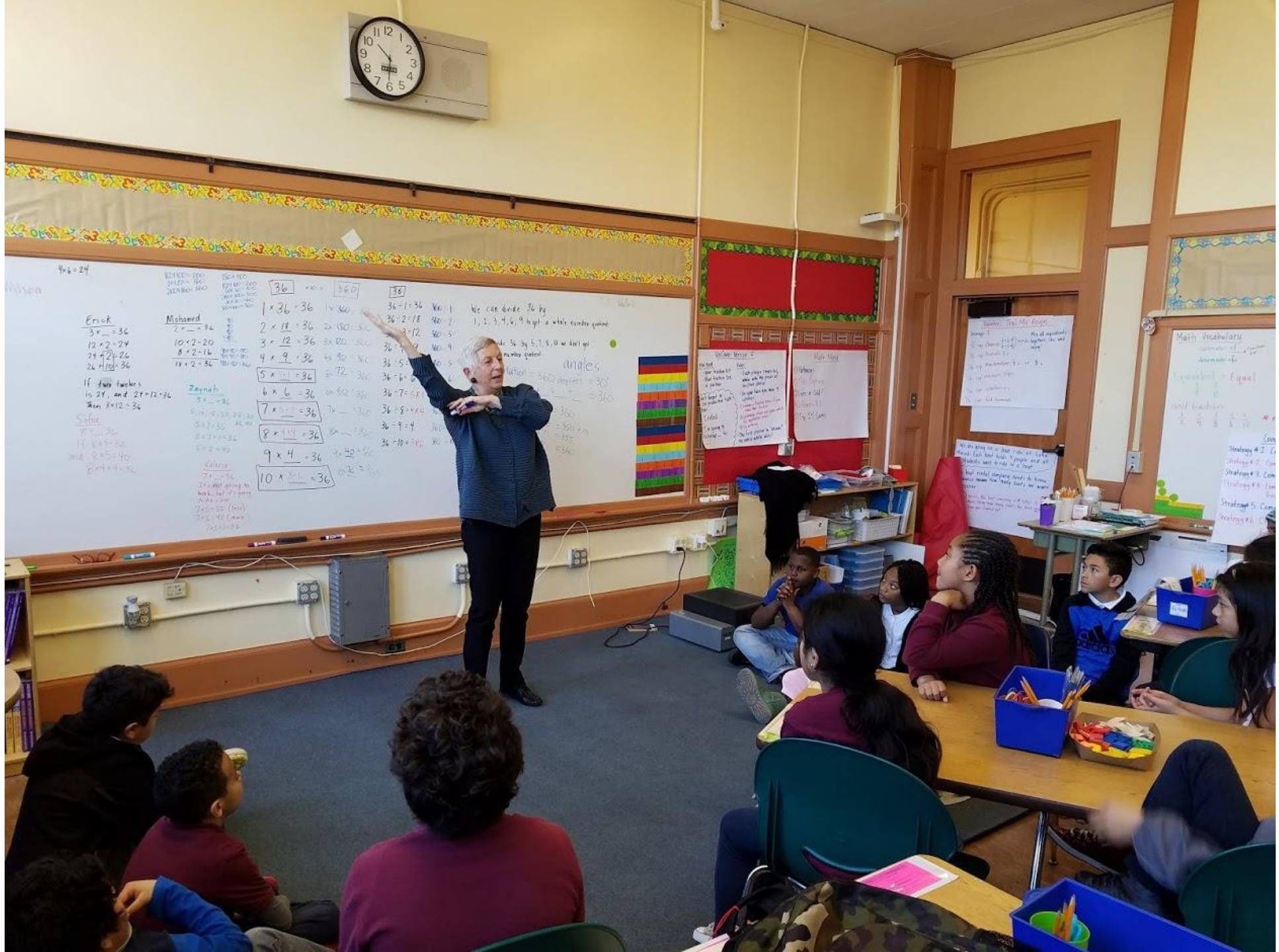
What if we change 36 to 360?

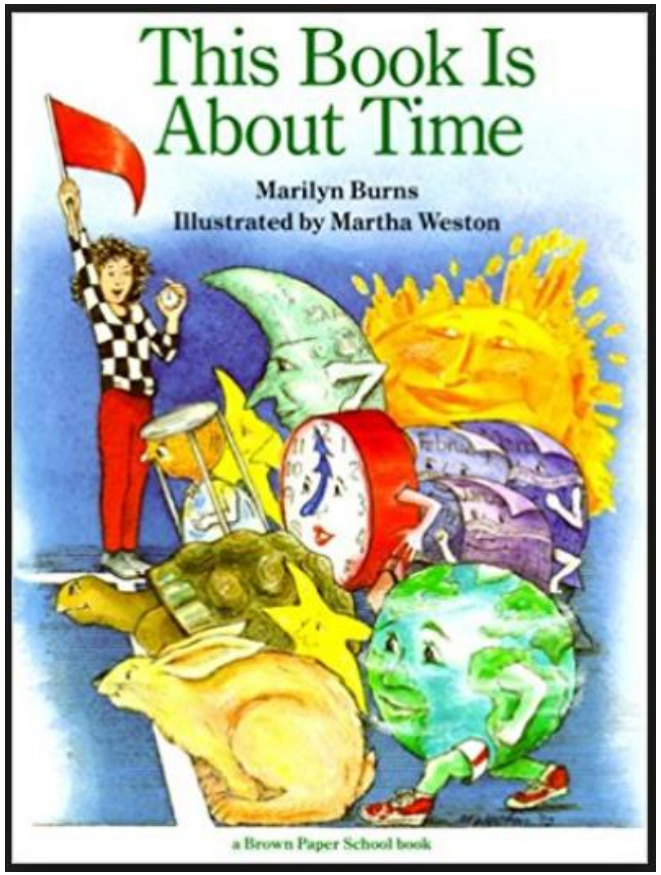






36 ×10 = 360
36
1 × 36 = 36
2 × 18 = 36
3 × 12 = 36
4 × 9 = 36
5 × 7+1 = 36
6 × 6 = 36
7 × 5+1 = 36
8 × 4+4 = 36
9 × 4 = 36
10 × 3+6 = 36
1× 360 = 360
2× 180 = 360
3× 120 = 360
4× 90 = 360
5× 72 = 360
6× 60 = 360
7× = 360
8× = 360
9× 40 = 360
10× 36 = 360
36 ÷ 1 = 36
36 ÷ 2 = 18
36 ÷ 3 = 12
36 ÷ 4 = 9
36 ÷ 5 = 7R1
36 ÷ 6 = 6
36 ÷ 7 = 5R1
36 ÷ 8 = 4R4
36 ÷ 9 = 4
36 ÷ 10 = 3R6
360 ÷ 1 =
360 ÷ 2 =
360 ÷ 3 =
360 ÷ 4 =
360 ÷ 5 =
360 ÷ 6 =
360 ÷ 7 =
360 ÷ 8 =
360 ÷ 9 =
360 ÷ 10 =
72
72
72
72
72
360
90°
4× _ = 360°


4×6=24
Erick
3×__=36
12×2=24
24+12=36
24+12=36
If two teachers is 24, and 24+12=36 Then 8×12=36
Mohamed
2×__=36
10×2=20
8×2=16
18+2=36
Zaynab
Valeria
36
360
1×36=36
2×18=36
3×12=36
4×9=36
5×__=36
6×6=36
7×__=36
8×__=36
9×4=36
10×__=36
35
36÷1=36
36÷2=18
36÷3=12
36÷4=9
36÷5=
36÷6=6
36÷7=5R1
36÷8=4R4
36÷9=4
36÷10=3R6
We can divide 36 by 1, 2, 3, 4, 6, 9 to get a whole number quotient
angles
Math Vocabulary
Equivalent = Equal

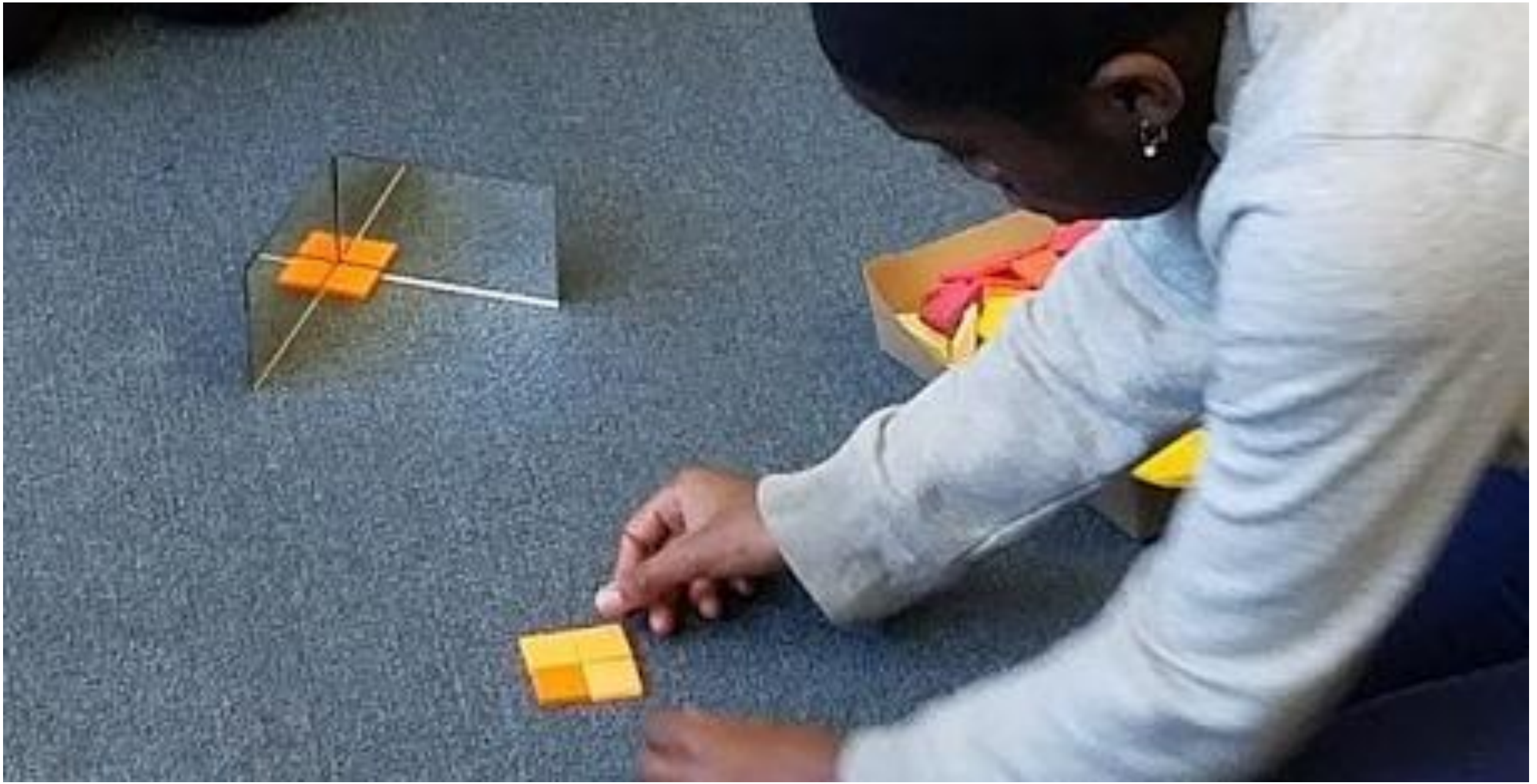

This Book Is About Time
Marilyn Burns
Illustrated by Martha Weston
a Brown Paper School book

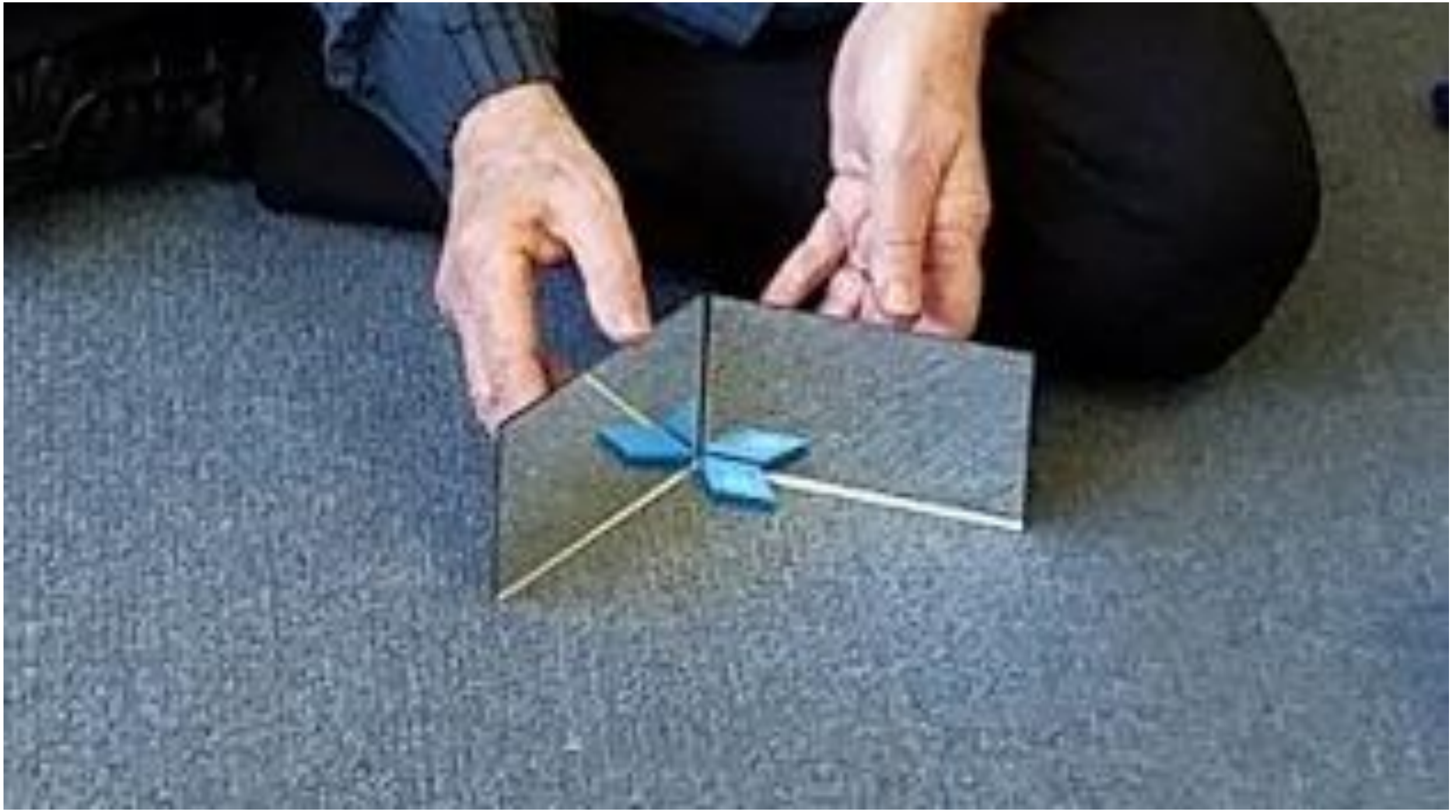

If two twelves is 24, and 2x+12=36 Then 3x12=36
Sofia
8x__=36
If 8x4=32 and 8x5=40 8x4+4=36
Zaynab
5x__=36
5,10,15,20,25,30 35,40
5x7=35
5x7+1=36
5x8=40
Valeria
7x__=36
It's not going to work, but it's going to be close
7x5=35 (less)
7x6=42 (more)
7x5+1=36
5 x 7+1 =36
6 x 6 =36
7 x 5+1 =36
8 x 4+4 =36
9 x 4 =3
10 x 3+6
5x72=360
6x60=360
7x__=360
8x__=360
40=360
=360
36÷6=6
36÷7=5R1
36÷8=4R4
36÷9=4
36÷10=3R6
360-6=
360-7=
360-8=
360-9=
360-10=
72
72
72
72
72
360
angles
1 rotation = 360 degrees = 30°
80 ___+___+___+___+___ = 360
5x70+10
5x__=360
70+70+70+70+70 =350+10
71+71+71+71+71 =355
72+72+72+72+72 =360
A COLLECTION OF MATH LESSONS

A COLLECTION OF MATH LESSONS

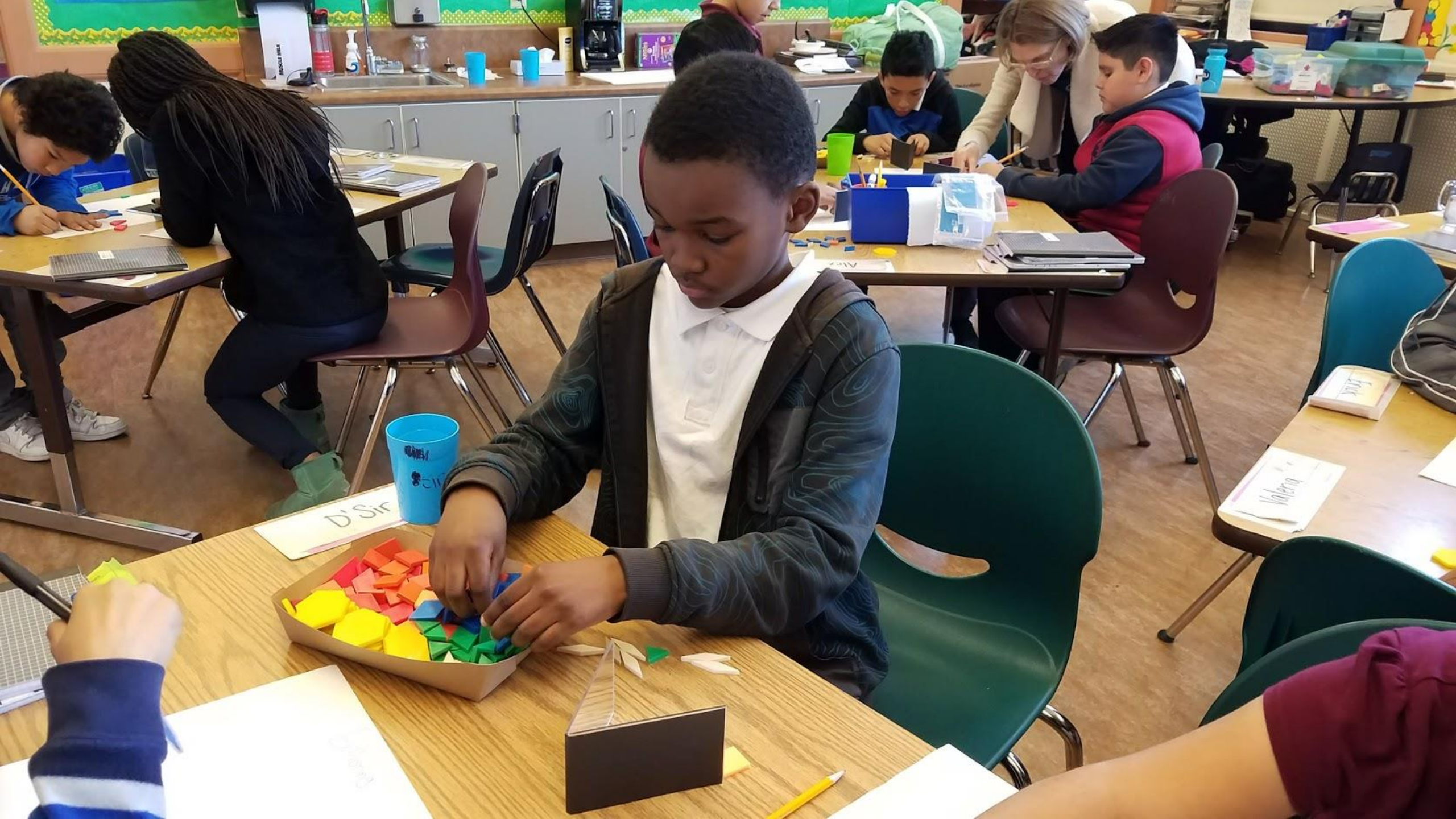
FROM GRADES 6 THROUGH 8

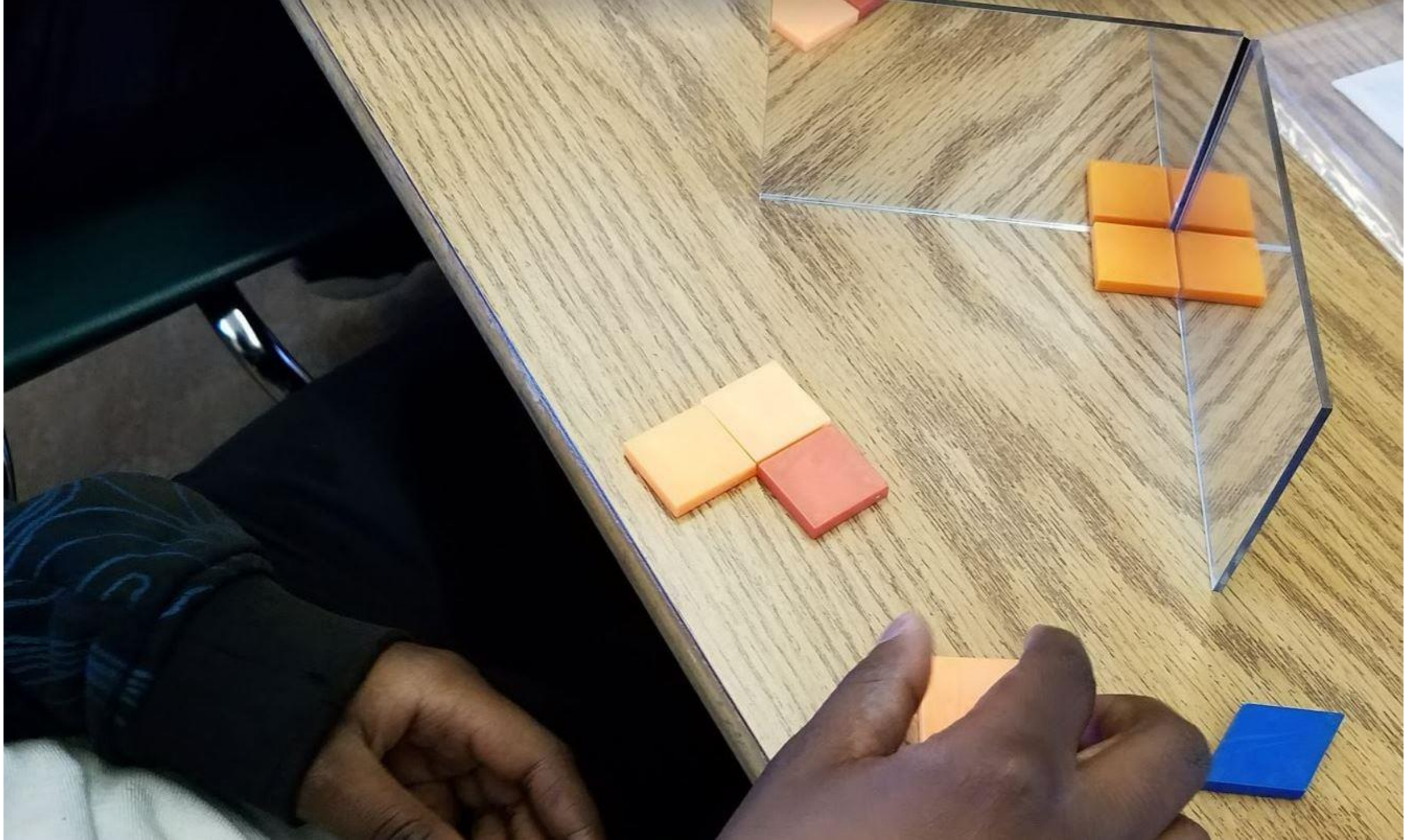
BY MARILYN BURNS
AND CATHY HUMPHREYS

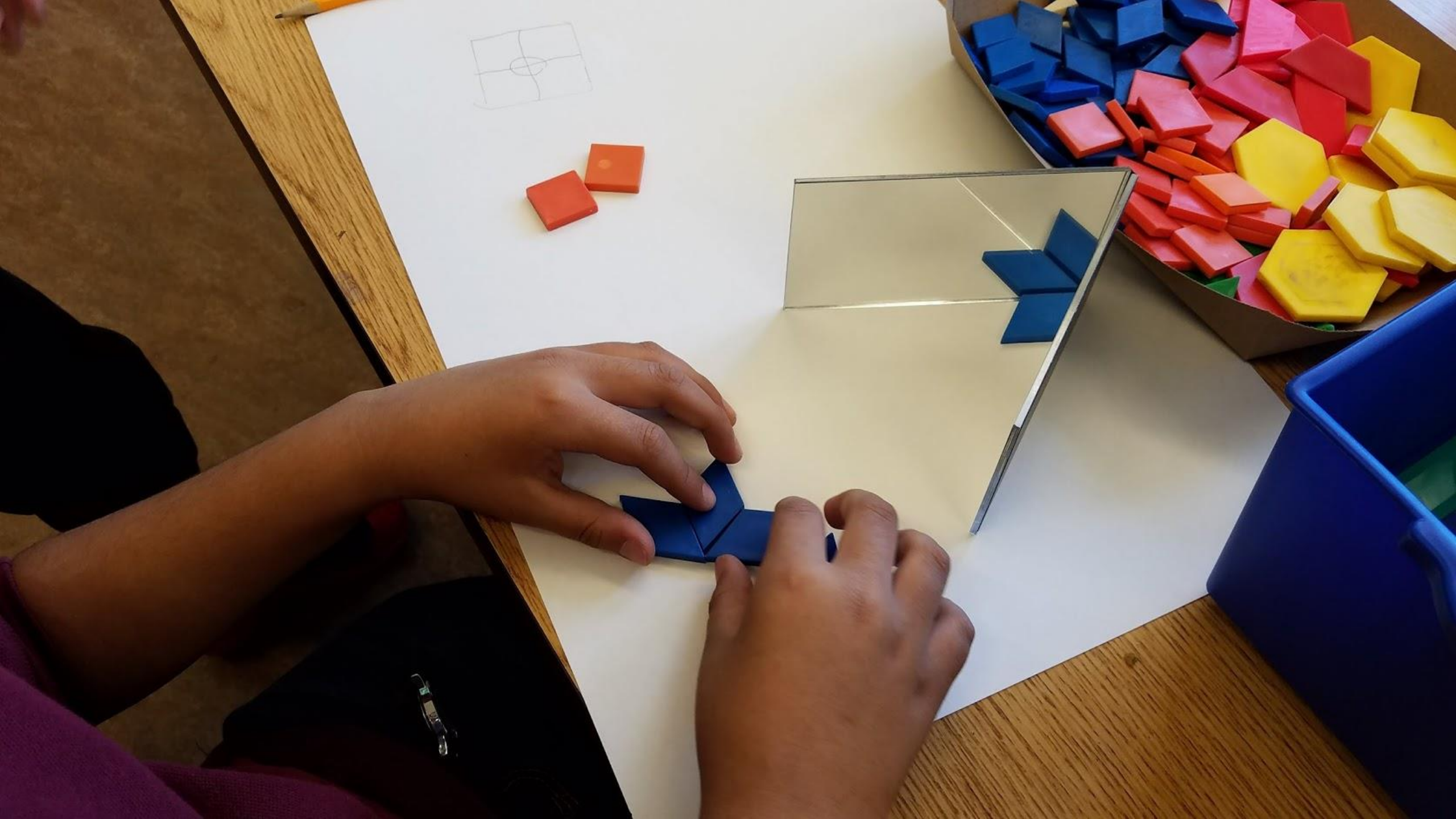




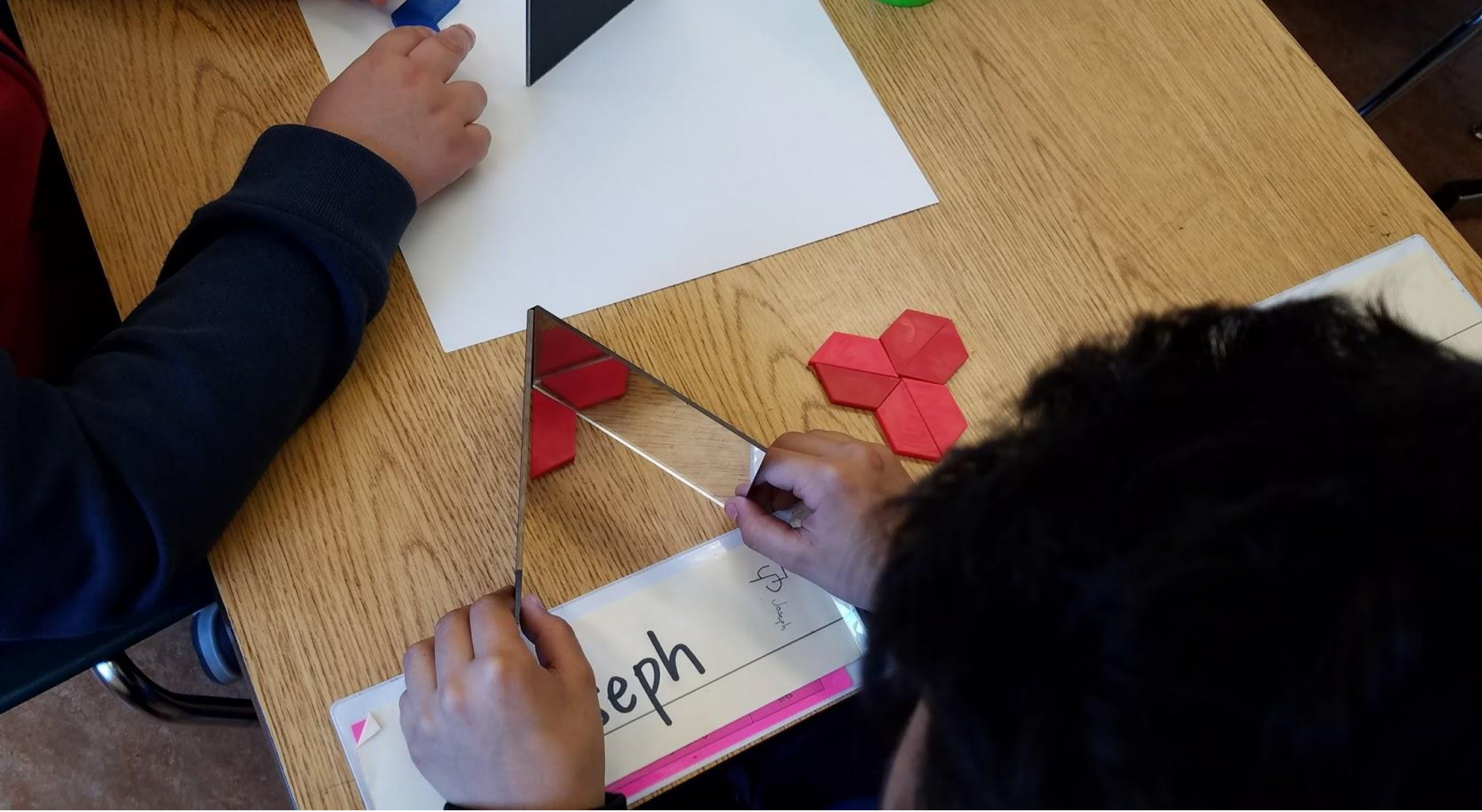

Making
Math a Story

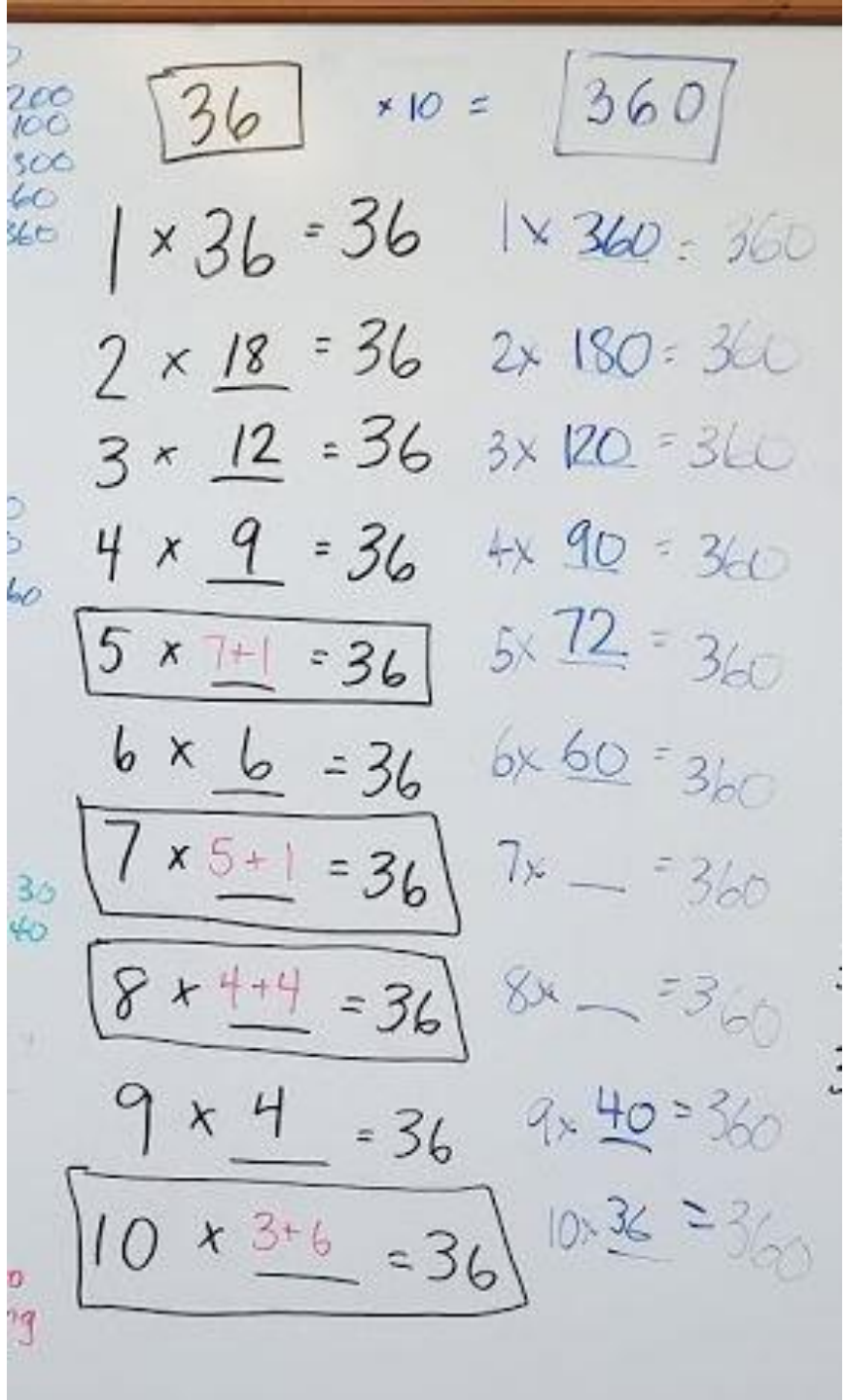


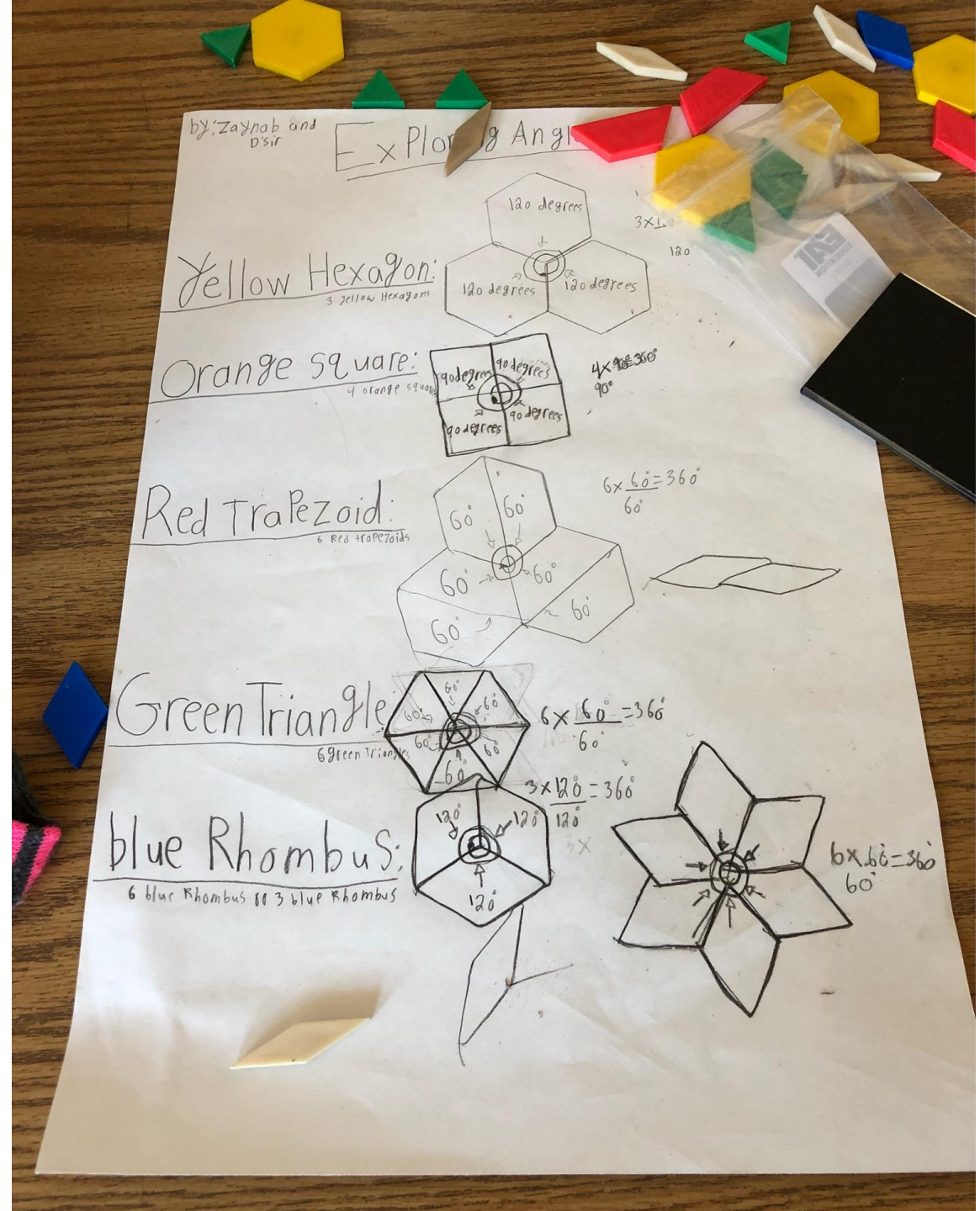





Hexagon

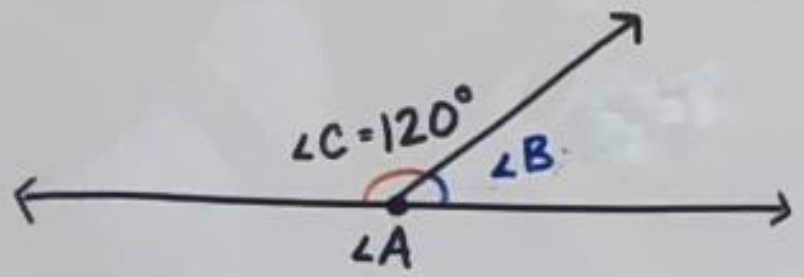

eph
Joseph


36 × 10 = 360
1 × 36 = 36
1 × 360 = 360
2 × 18 = 36
2 × 180 = 360
3 × 12 = 36
3 × 120 = 360
4 × 9 = 36
4 × 90 = 360
5 × 7+1 = 36
5 × 72 = 360
6 × 6 = 36
6 × 60 = 360
7 × 5+1 = 36
7 × ___ = 360
8 × 4+4 = 36
8 × ___ = 360
9 × 4 = 36
9 × 40 = 360
10 × 3+6 = 36
10 × 36 = 360


by: Zaynab and D'sir
Yellow Hexagon:
3 yellow Hexagons
120 degrees
120 degrees
120 degrees
120 degrees
Orange square:
4 orange squares
90 degrees
90 degrees
90 degrees
90 degrees
Red Trapezoid:
6 Red trapezoids
60°
60°
60°
60°
60°
60°
Green Triangle:
6 Green Triangles
blue Rhombus:
6 blue Rhombus or 3 blue Rhombus
120°
120°
120°

Date: 3/8/18

Problem: How can we find the measurement of angle B?



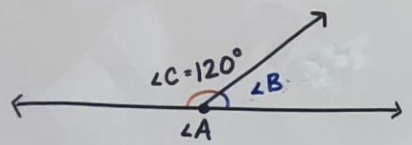
We can use what we know about straight lines, right angles, and protractors. But we do not need a protractor!

$\angle B = 60°$

Date: 3/8/18

Problem: How can we find the measurement of angle B?

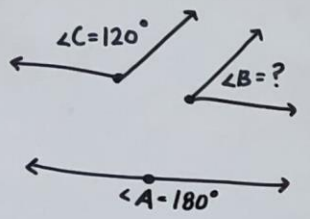
We can use what we know about straight lines, right angles, and protractors. But we do not need a protractor!



∠B=60°

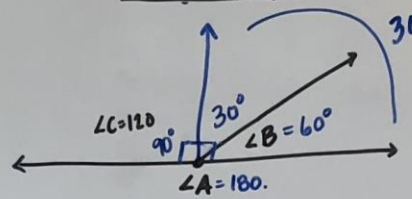
Jason

I split the angles into:



Then I estimated, and I think ∠B is about 60° or 70°.

Zaynab/Nadeah

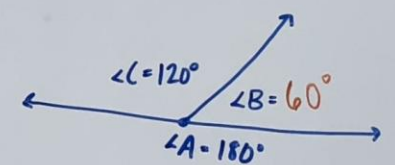


I split ∠A in half, then I had 2 right angles. A right is 90°.

I knew 30+60=90, I needed to add 30 to 60 to complet the right angle.

Sofia

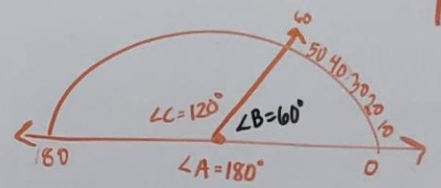
I notice ∠A was 180°, because it is a straight line. Then I looked at ∠C and it was 120°.



Then I thought 120+__=180. So if 120+60=180, then ∠B must be 60°.

Alex

I knew a protractor measu
180°.



Then I knew 120+60=180.

Summary:

You can use math you've already learned to solve problems that are new math.

Finally, 30+60+90=180.

Summary:

You can use math you've already learned to solve problems that are new math.

Story # 2

Differentiating Instruction

First:
One-on-One Interviews with 2nd graders

7 + 8

Add and subtract within 20.

Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

What am I listening for?

Does the student know the answer?

What am I listening for?

Does the student know the answer?

Also, does the student . . .

- **count on?**
- **use benchmark of 10?**
- **apply the commutative property?**
- **relate to doubles?**
- **use other reasonable strategy?**

7 + 8

What did I learn from Anna?

Did she know the answer?

Also, which best describes her reasoning. . .

- **count on?**
- **use benchmark of 10?**
- **apply the commutative property?**
- **relate to doubles?**
- **use other reasonable strategy?**

Add and subtract within 20.

Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know from memory all sums of two one-digit numbers.

Does Anna know 7 + 8 from memory?

Is Anna fluent???

What did I learn from Lydia?

Did she know the answer?

Also, which best describes her reasoning. . .

- **count on?**
- **use benchmark of 10?**
- **apply the commutative property?**
- **relate to doubles?**
- **use other reasonable strategy?**

What did I learn from Daryl?

Did he know the answer?

Also, which best describes his reasoning. . .

- **count on?**
- **use benchmark of 10?**
- **apply the commutative property?**
- **relate to doubles?**
- **use other reasonable strategy?**

What did I learn from Jonathan?

Did he know the answer?

Also, which best describes his reasoning. . .

- **count on?**
- **use benchmark of 10?**
- **apply the commutative property?**
- **relate to doubles?**
- **use other reasonable strategy?**

What did I learn from Gabriela?

Did she know the answer?

Also, which best describes her reasoning. . .

- **count on?**
- **use benchmark of 10?**
- **apply the commutative property?**
- **relate to doubles?**
- **use other reasonable strategy?**

50 + 70

Use place value understanding and properties of operations to add and subtract within 1000.

What am I listening for?

Can the student figure out the answer?

What am I listening for?

Can the student figure out the answer?

Also, does the student . . .

- **break numbers apart?**
- **make use of 100?**
- **use the commutative property?**
- **use the standard algorithm?**
- **use other reasonable strategy?**

50 + 70

Gabriela

Math Reasoning Inventory™

Sign Up Log In

Home | About the Assessments | Resources | MRI Manager | About Us | Support | Contact Us

Find out what your students really understand about math

- Focus on how students think and reason
- Uncover students' strategies, understandings, and misconceptions
- Learn how students respond to questions the Common Core expects all middle school students to answer successfully

Sign up for a **FREE** account and try it today!

SIGN UP FOR FREE | LEARN MORE

Why is Marilyn Burns so excited about MRI?

The Assessments

Math Reasoning Inventory (MRI) is an online formative assessment tool designed to make teachers' classroom instruction more effective.
Learn More

The Reports

MRI instant reports can be used to inform instruction, monitor progress, identify students who would benefit from intervention, and communicate with parents.
Learn More

Reasoning Strategies

The MRI Interview reveals the strategies students use to reason with whole numbers, decimals, and fractions.
Learn more

"In just a few minutes, I was able to gain valuable awareness about my math students and adjust my lessons accordingly."

— **Diana Jones**
Grade 6 Teacher
SLCUSD, California

1 Question 1

1000 – 998

I'm going to show you some problems one at a time. Try to figure out the answers in your head. The first problem is 1000 minus 998.

2 Answer

- Correct (2)
- Incorrect
- Self-corrected (2)
- Did Not Answer

3 Explanation

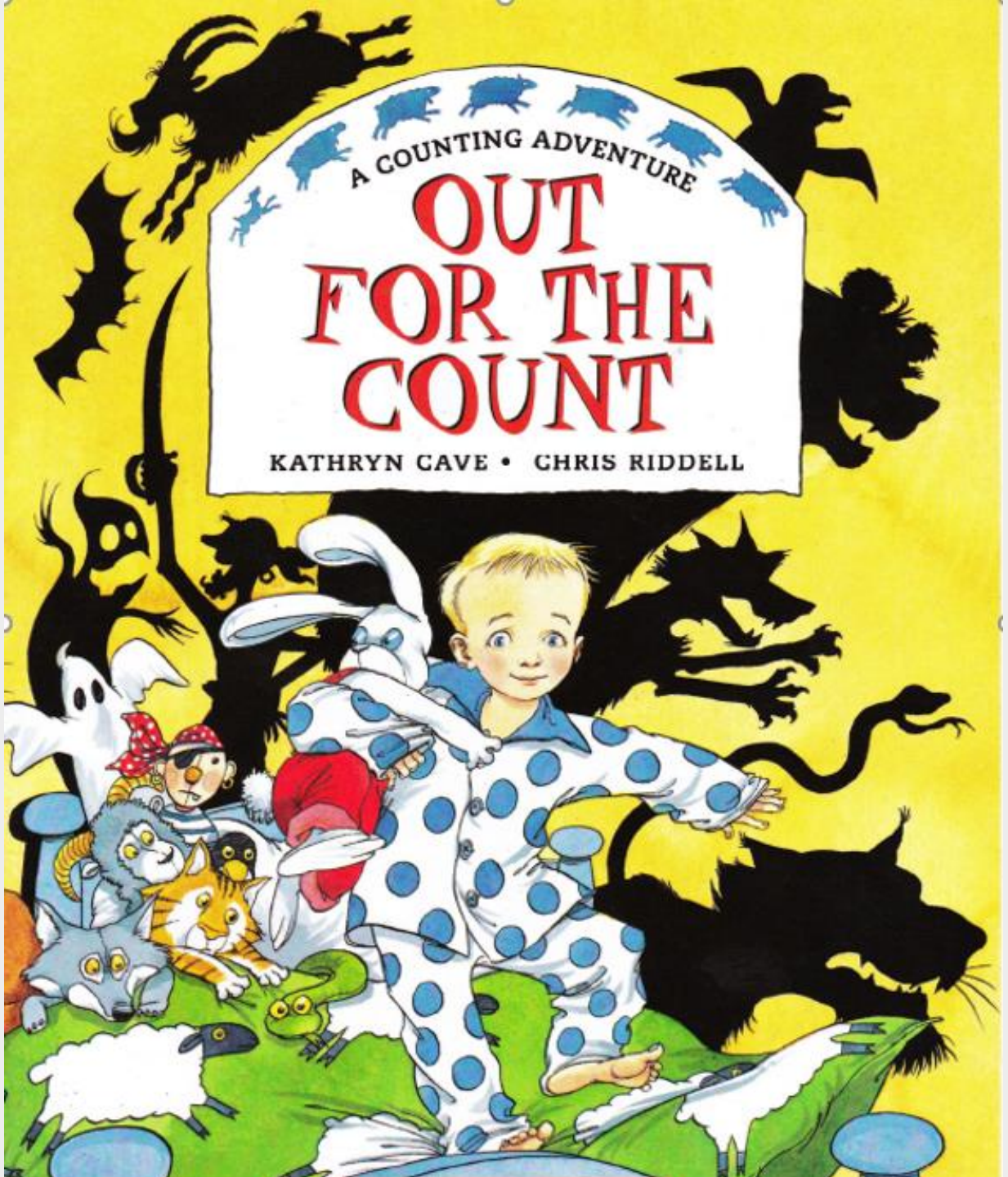
How did you figure out the answer?

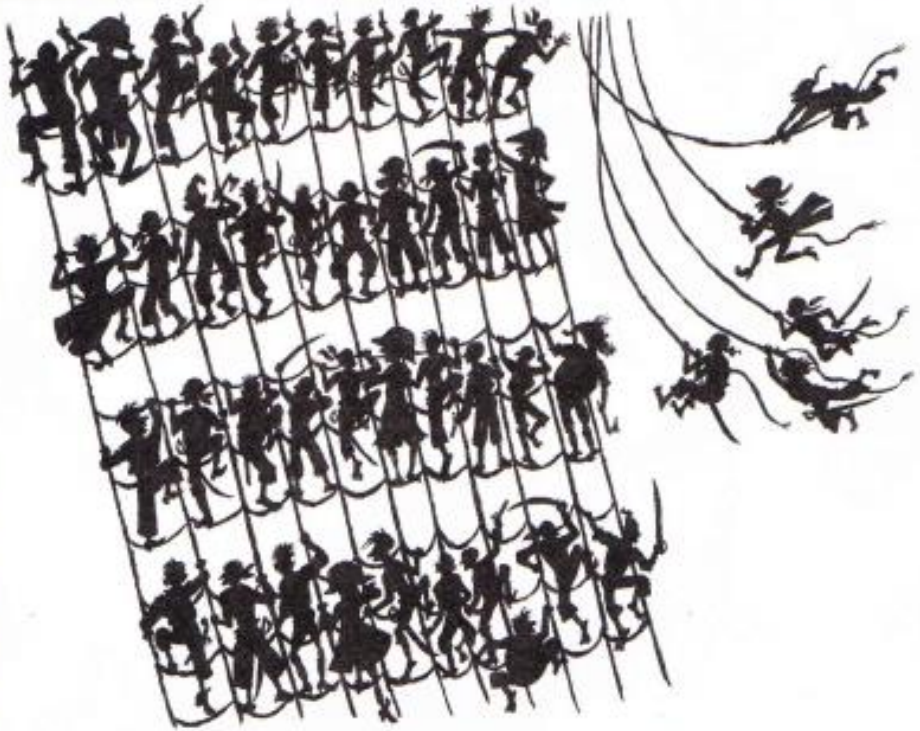
- Used standard algorithm to subtract
- Counted up or added 2
- Added 98 + 2 and applied to 998 + 2
- Gave other reasonable explanation
- Guessed, did not explain, or gave faulty explanation

Notes

record student response

Now:
Into the Classroom


A COUNTING ADVENTURE
OUT FOR THE COUNT
KATHRYN CAVE • CHRIS RIDDELL



Approaching from the east Tom saw
a band of pirates armed for war,
with pistols, cutlasses and axes,
looking for some target practice.

No time to hide, no place to run,
outnumbered 45 to one,
Tom ducked beneath the gangplank fast
and tripped them up as they went past.

STUFFED ANIMALS

Penguin $13	Bear $35
Sheep $18	Wolf $38
Ghost $26	Pirate $39
Python $29	Tiger $47
Goat $34	Rabbit $49

Solve Three Ways

- Use equations
- Add going down (stacking)
- Use an open number line.

1 Stuffed animals: Ghost and Bear

Problem 25 + 26 = 61

Use equations.

35 + 26 =

20 + 30 = 50

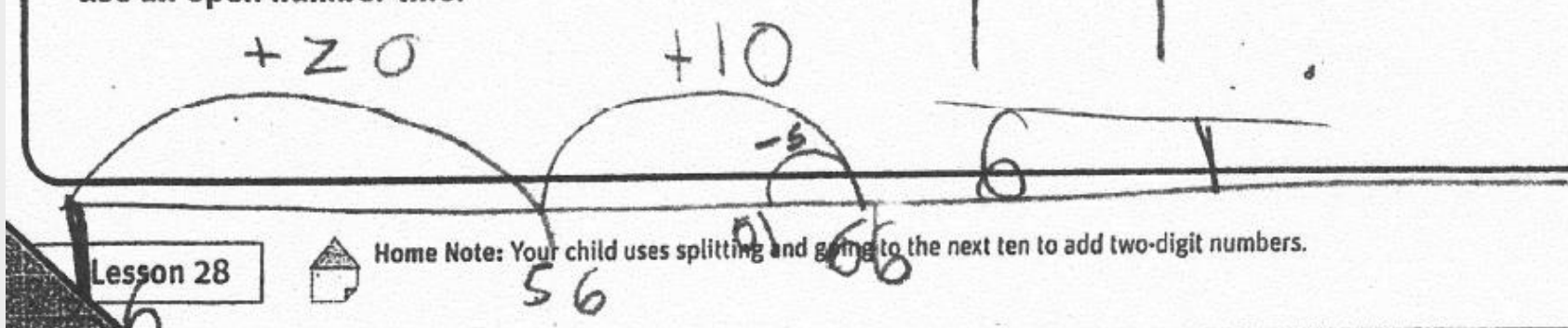
5 + 6 = 11

50 + 11 = 61

Add going down.

2	5
+ 3	6
+ 5	0
1	1

Use an open number line.



Lesson 28

Home Note: Your child uses splitting and going to the next ten to add two-digit numbers.

③ Stuffed animals: tiger and penguin and ghost and goat

Problem 47+13+26+34

Use equations.

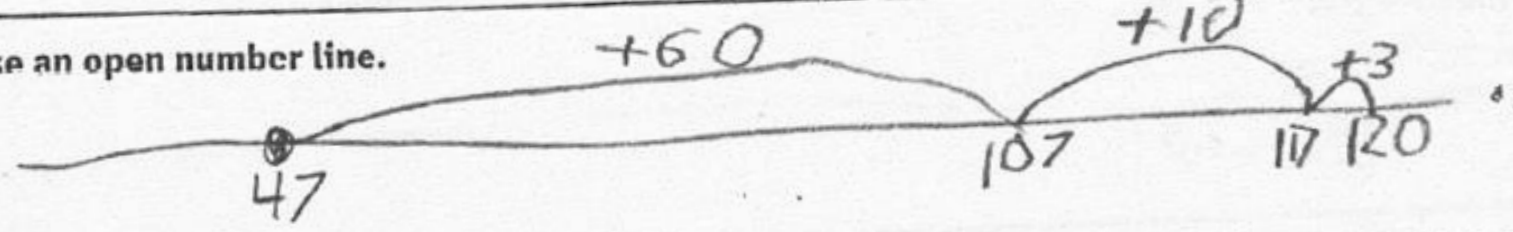
47+13+26+34

60+60=120

Add going down.

47
+26
+13
+34
100
20
120

Use an open number line.



SPECIAL SALE!

STUFFED ANIMALS

Penguin	~~$13~~ $6	Bear	~~$35~~ $8
Sheep	~~$18~~ $9	Wolf	~~$38~~ $10
Ghost	~~$26~~ $10	Pirate	~~$39~~ $6
Python	~~$29~~ $15	Tiger	~~$47~~ $5
Goat	~~$34~~ $12	Rabbit	~~$49~~ $20

SPECIAL SALE

STUFFED ANIMALS

Penguin	~~$13~~	$10	Bear	~~$35~~	$30
Sheep	~~$18~~	$15	Wolf	~~$38~~	$30
Ghost	~~$26~~	$20	Pirate	~~$39~~	$35
Python	~~$29~~	$25	Tiger	~~$47~~	$50
Goat	~~$34~~	$30	Rabbit	~~$49~~	$50

SPECIAL SALE

STUFFED ANIMALS

Penguin	~~$13~~ $10	Bear	~~$35~~ $30
Sheep	~~$18~~ $15	Wolf	~~$38~~ $30
Ghost	~~$26~~ $20	Pirate	~~$39~~ $35
Python	~~$29~~ $25	Tiger	~~$47~~ $40
Goat	~~$34~~ $30	Rabbit	~~$49~~ $45

Ghost-20
Goat-30
Tiger-40
~~Python-25~~
Penguin-10
30
+40
70
+10
80
+20
100

Combinations

Wolf-30 — 30
Bear-30 — 30
Goat-30 — 30
Penguin-10 — +10
100

Rabbit-45
Sheep-15
Penguin-10
Bear-30
45
+15
60
+10
70+25

Ghost-20 — 45
Rabbit-45 — +20
Pirate-35 — 65
+35
100

~~$~~

100

SPECIAL SALE

STUFFED ANIMALS

Penguin	~~$13~~ $10	Bear	~~$35~~ $30
Sheep	~~$18~~ $15	Wolf	~~$38~~ $30
Ghost	~~$26~~ $20	Pirate	~~$39~~ $35
Python	~~$29~~ $25	Tiger	~~$47~~ $40
Goat	~~$34~~ $30	Rabbit	~~$49~~ $45

Rabbit + Pirate + Ghost

45 + 35 + 20

40 + 30 + 20 = 90

5 + 5 = 10

90 + 10 = 100

45
+35
20
90
10
100

+30 +5 +5
45 75 80

Bear Wolf and tiger

30 + 30 + 40 = 100

30 + 30 + 40

30 60 100

30
130
40
100

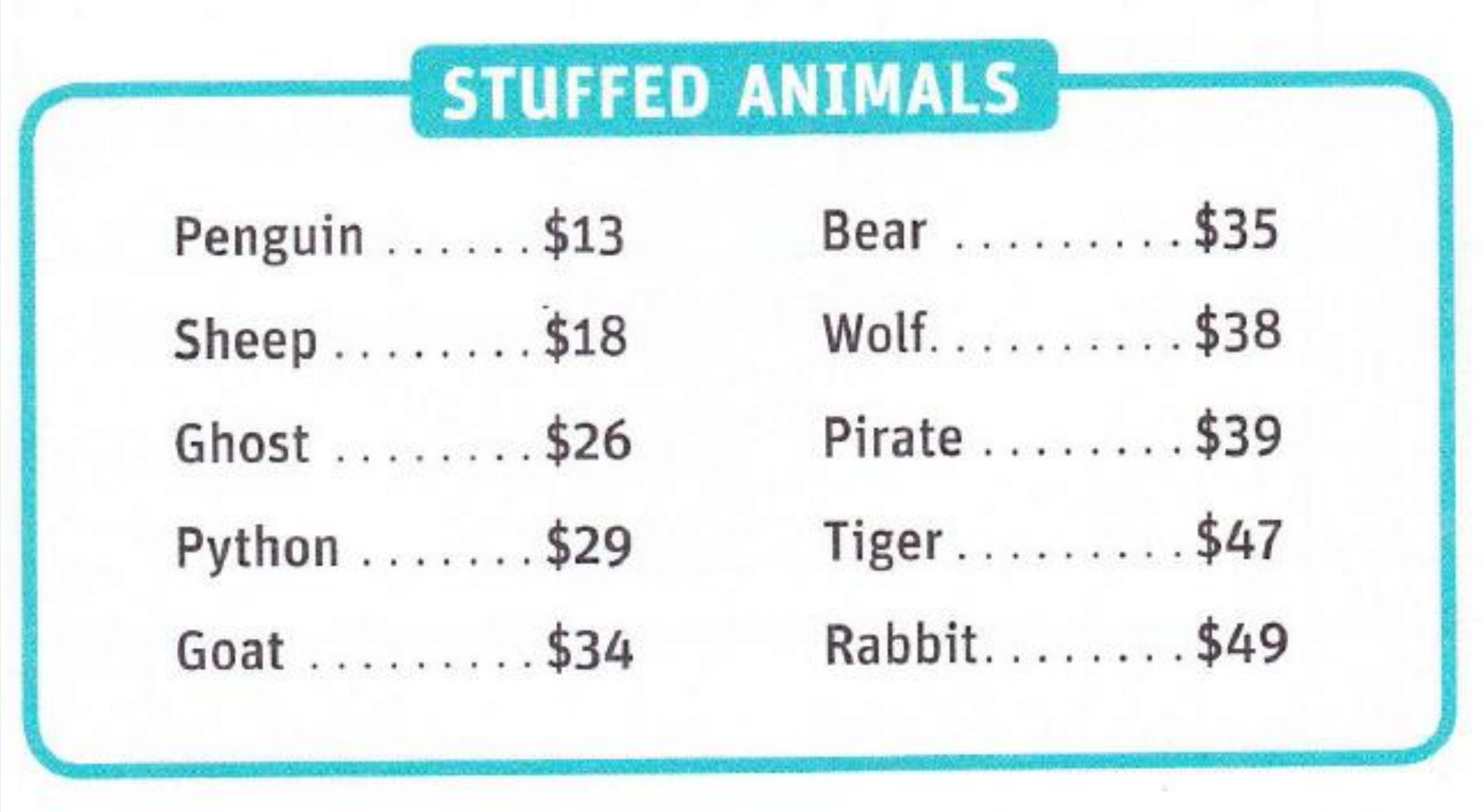
penguin sheep Ghost Python Goat

10 + 15 + 20 + 25 + 30

10 + 10 + 20 + 20 + 30 = 90

5 + 5 = 10 90 + 10 = 100

Can you spend exactly $100?



STUFFED ANIMALS

Penguin	$13	Bear	$35
Sheep	$18	Wolf	$38
Ghost	$26	Pirate	$39
Python	$29	Tiger	$47
Goat	$34	Rabbit	$49

Penguin+
Penguin=
36

36
49
75
+13

Rabbit + Wolf + penguin

49+38
40+30=70
9+8=17

70+17=87
87+13=100

yes

STUFFED ANIMALS

Penguin	$13	Bear	$35
Sheep	$18	Wolf	$38
Ghost	$26	Pirate	$39
Python	$29	Tiger	$47
Goat	$34	Rabbit	$49

shelp
Python
Bear
Sheep

13
+18
31
+49
80

18
+29
47
+35
82
+18
100

1 Rabbit + 1 Python + 1 Penguin
49 + 29 + 18 + 13 = 91
if I do this one I would have nine dollars of change.

1 Rabbit + Python + 1 Sheep
49 + 29 = 78 + 18 = 94 I would have six dollars left.
+16

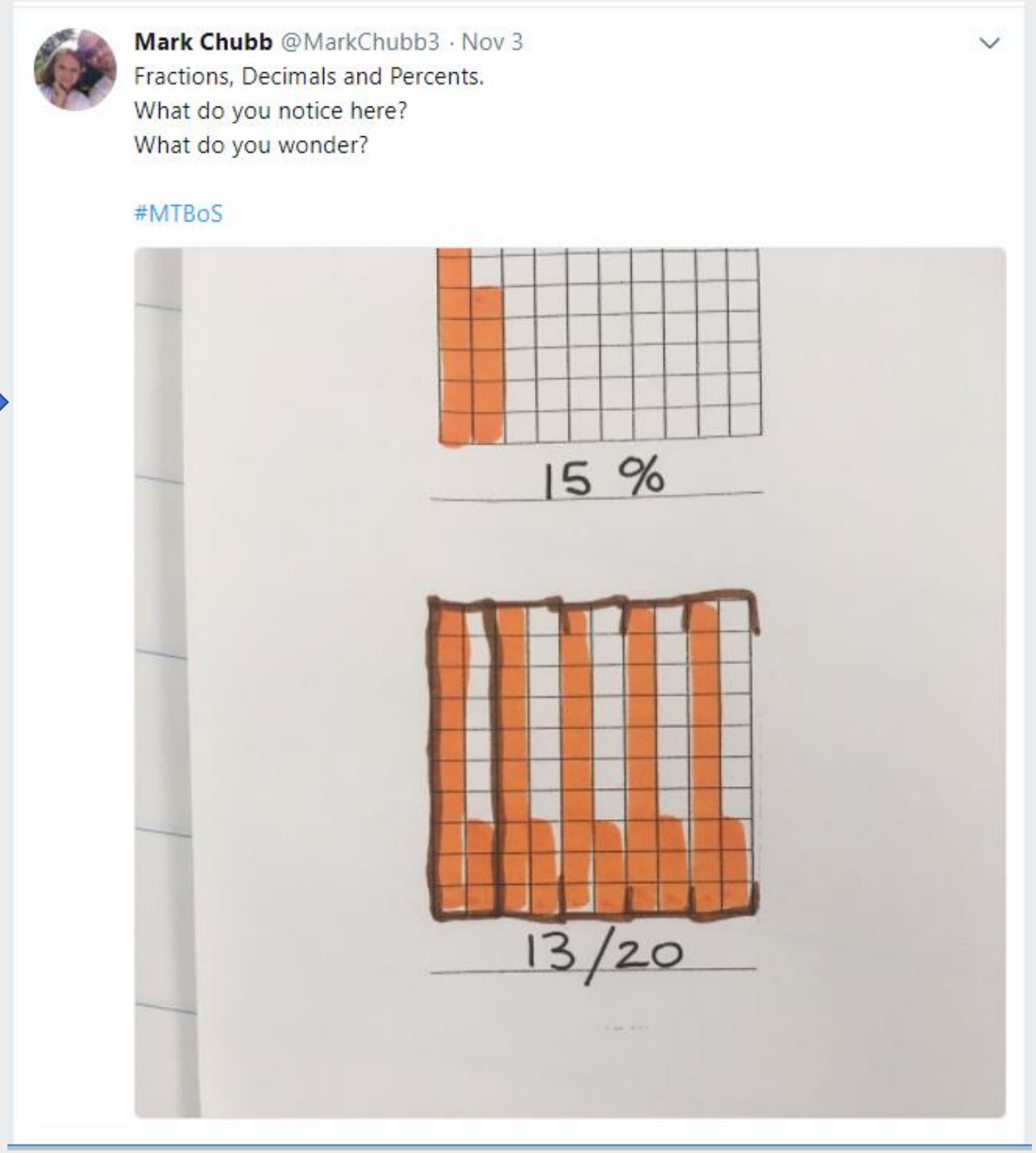
Addition/Subtraction Module A

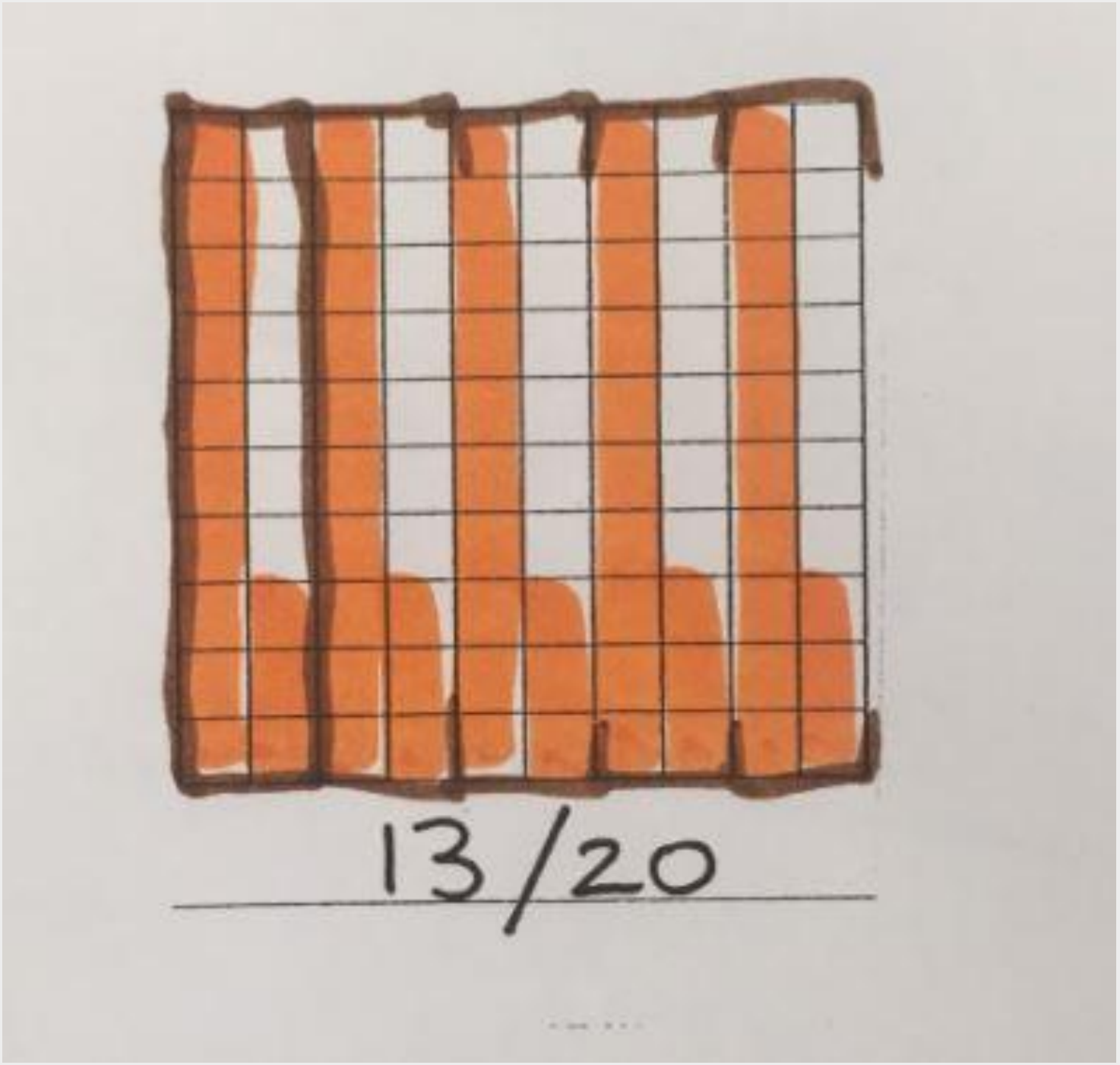
Lessons 26, 27, 28

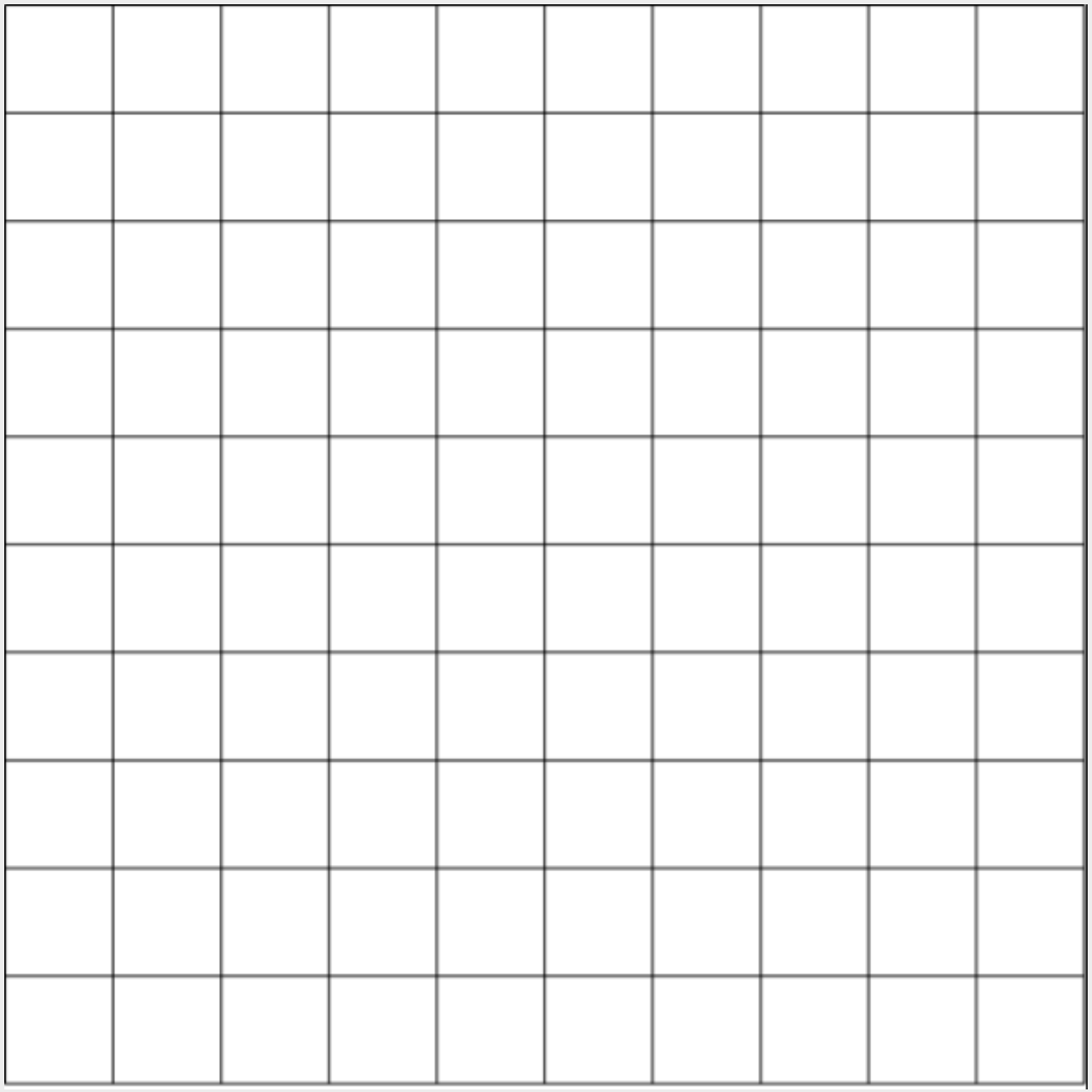
Story # 3

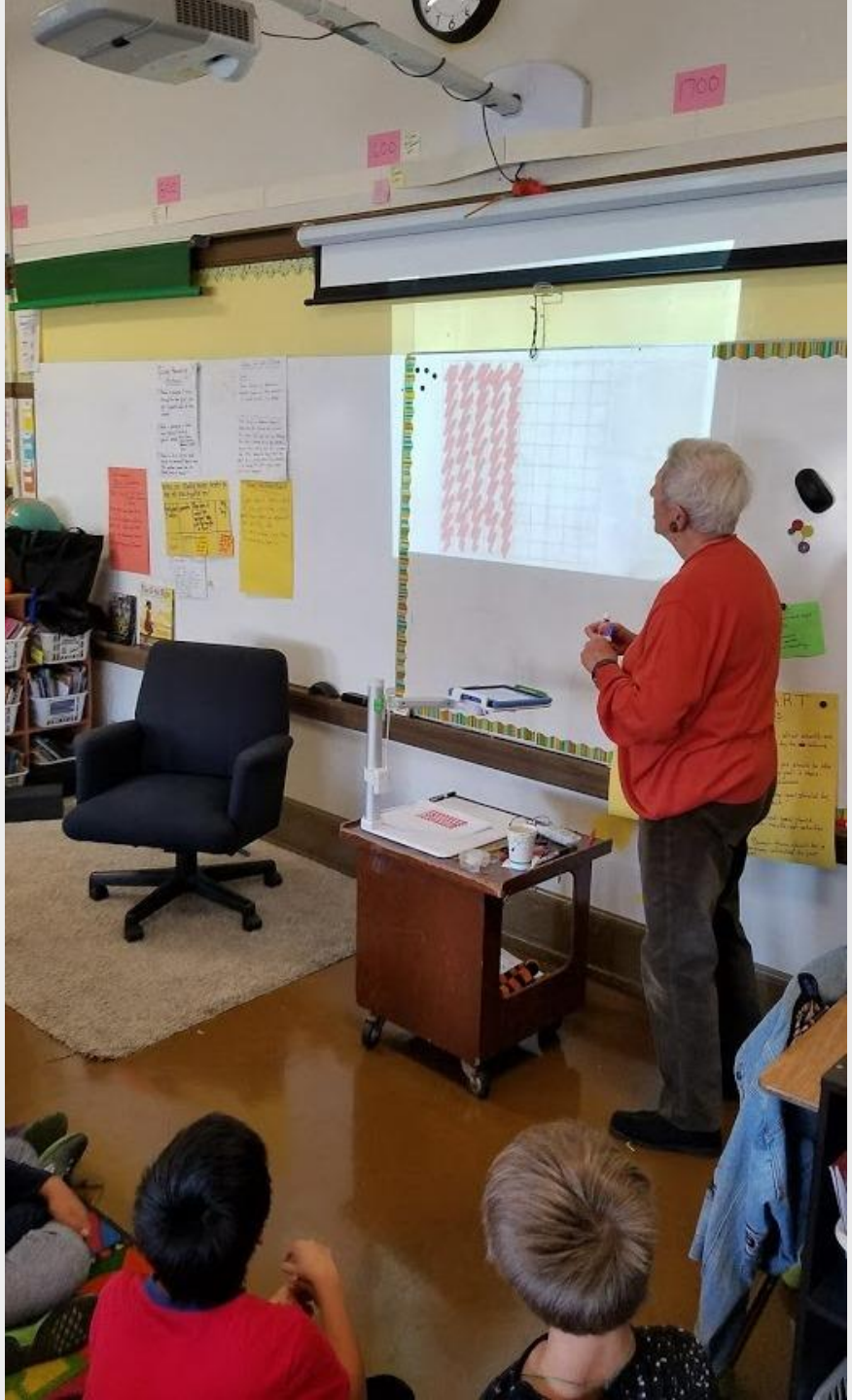
A Twitter Inspired Fraction/Decimal Lesson

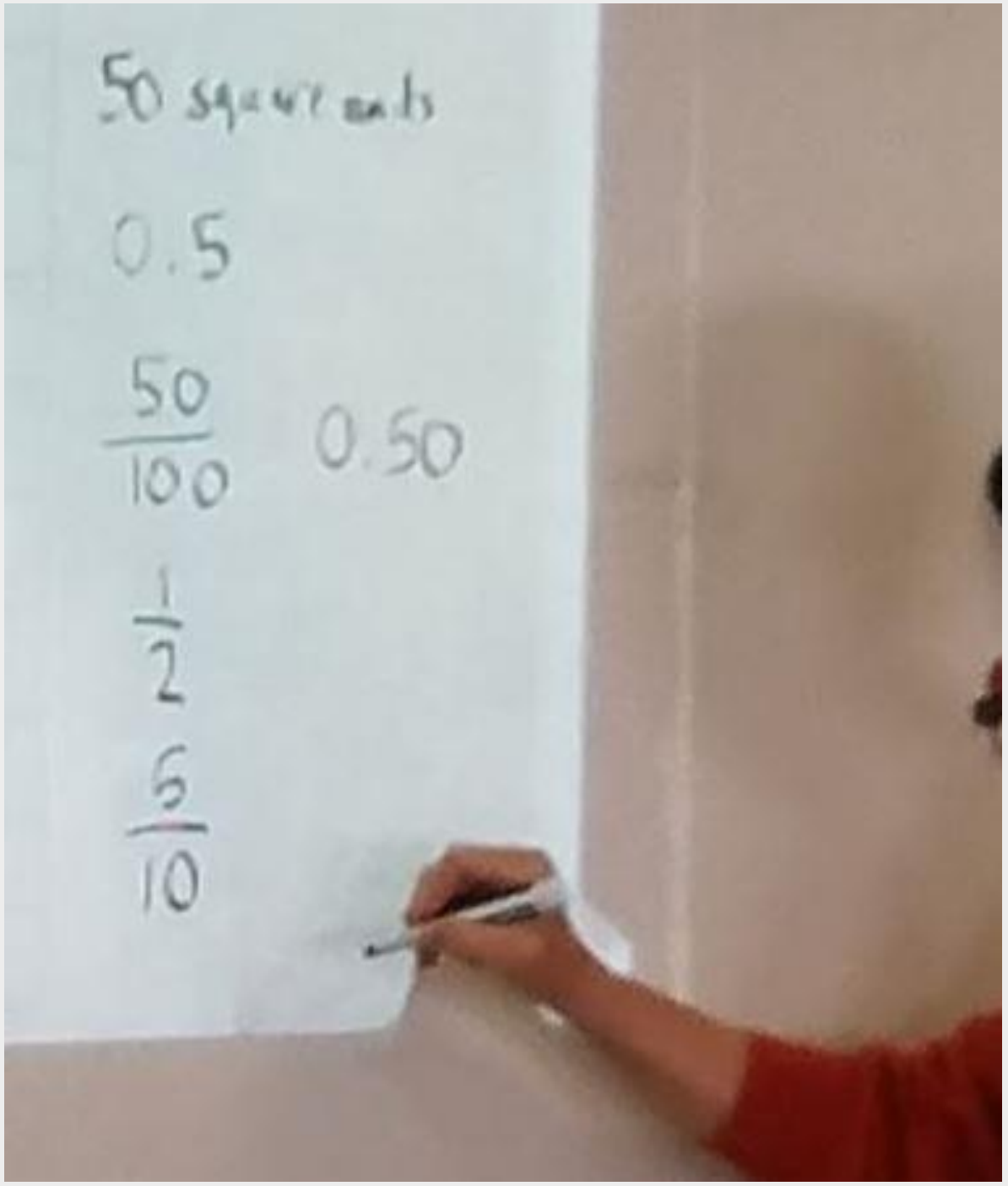
From Twitter, Nov 3

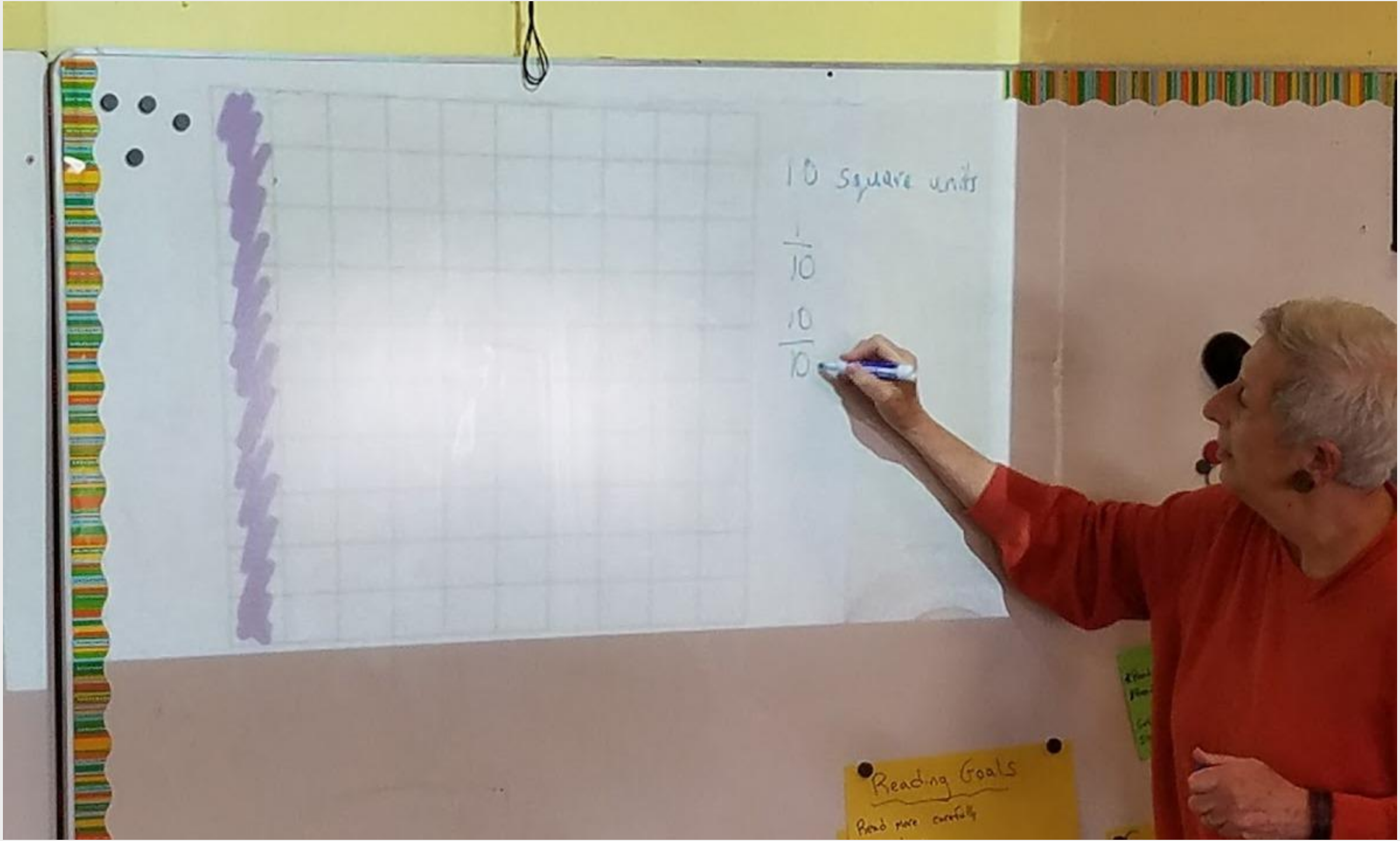


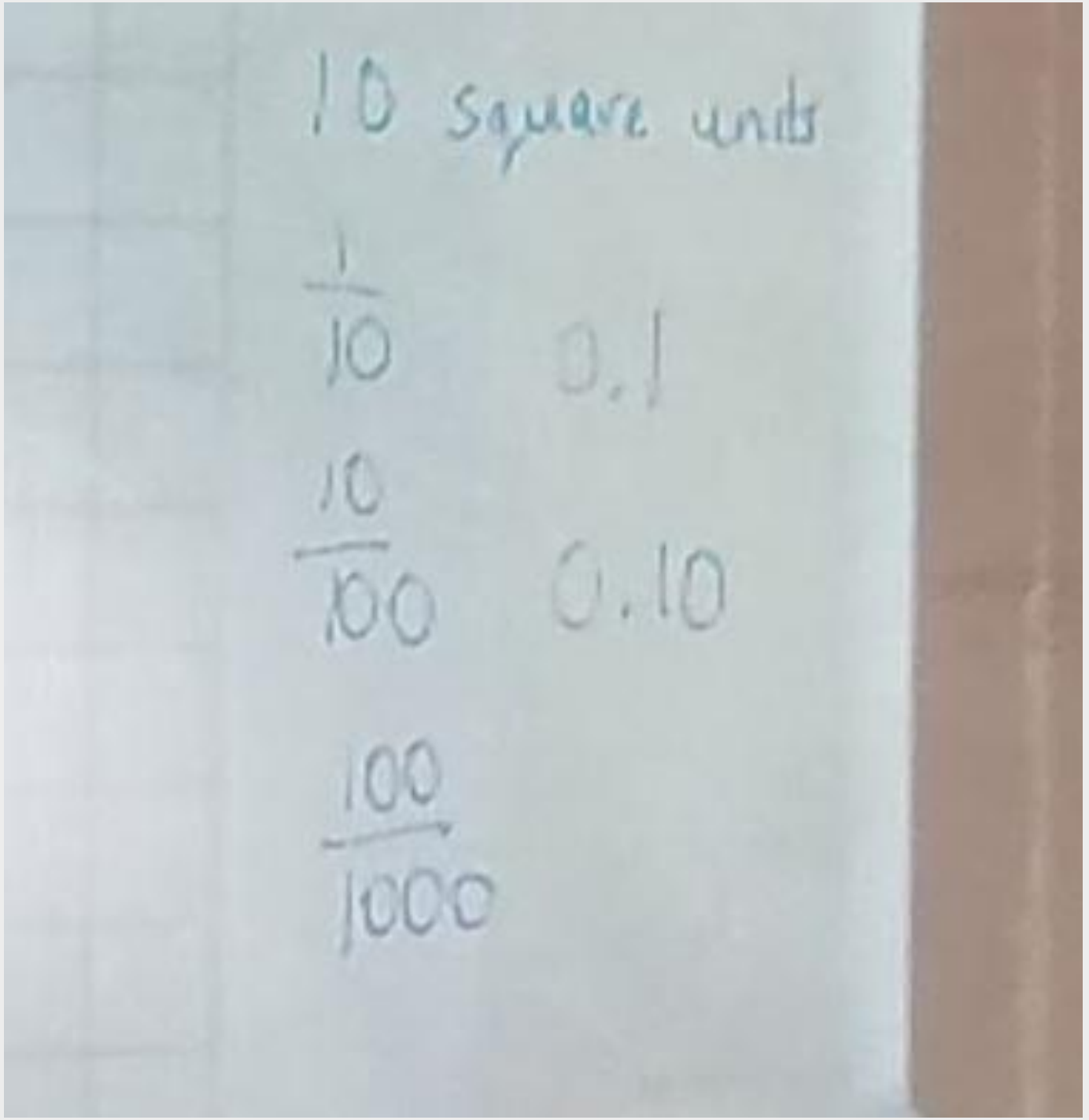

13/20

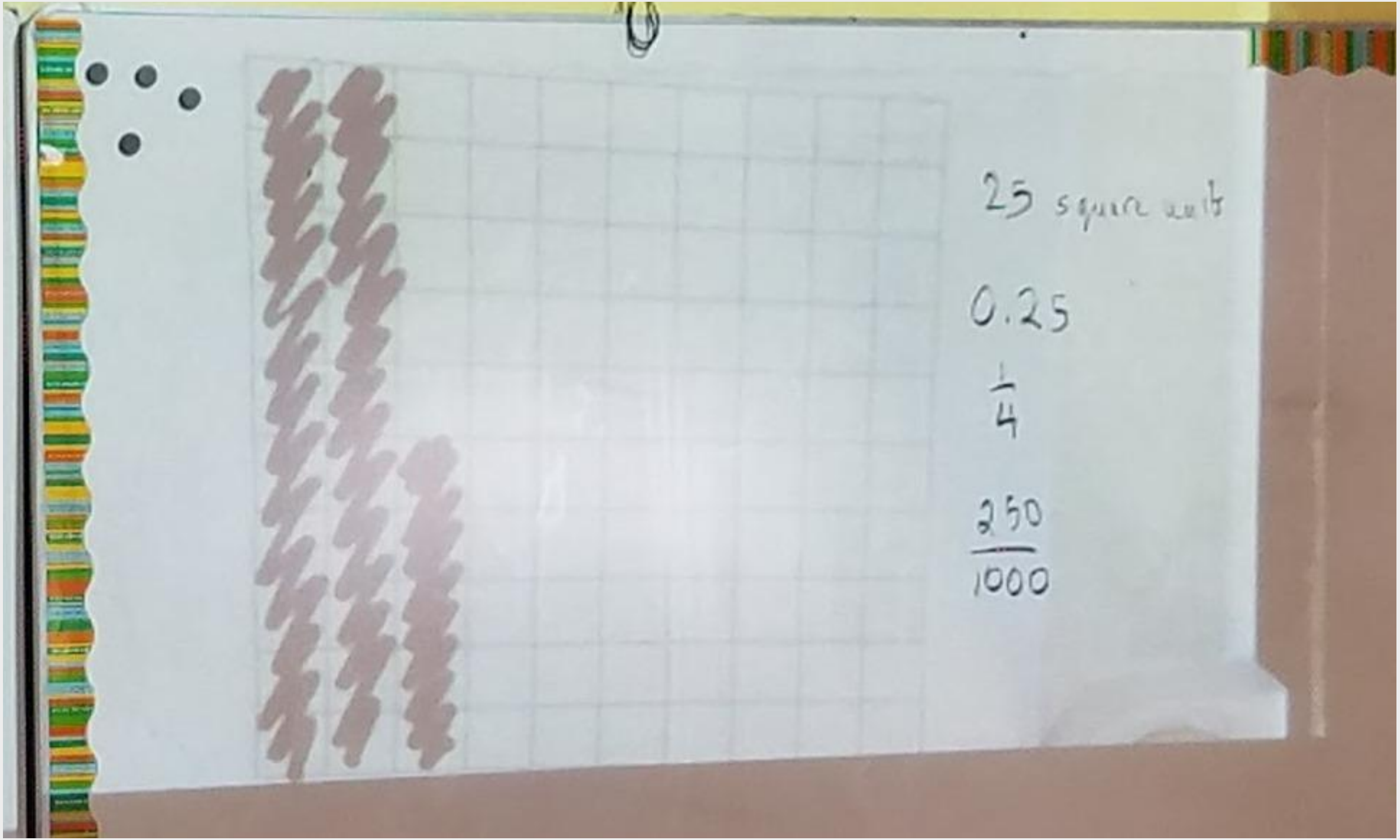


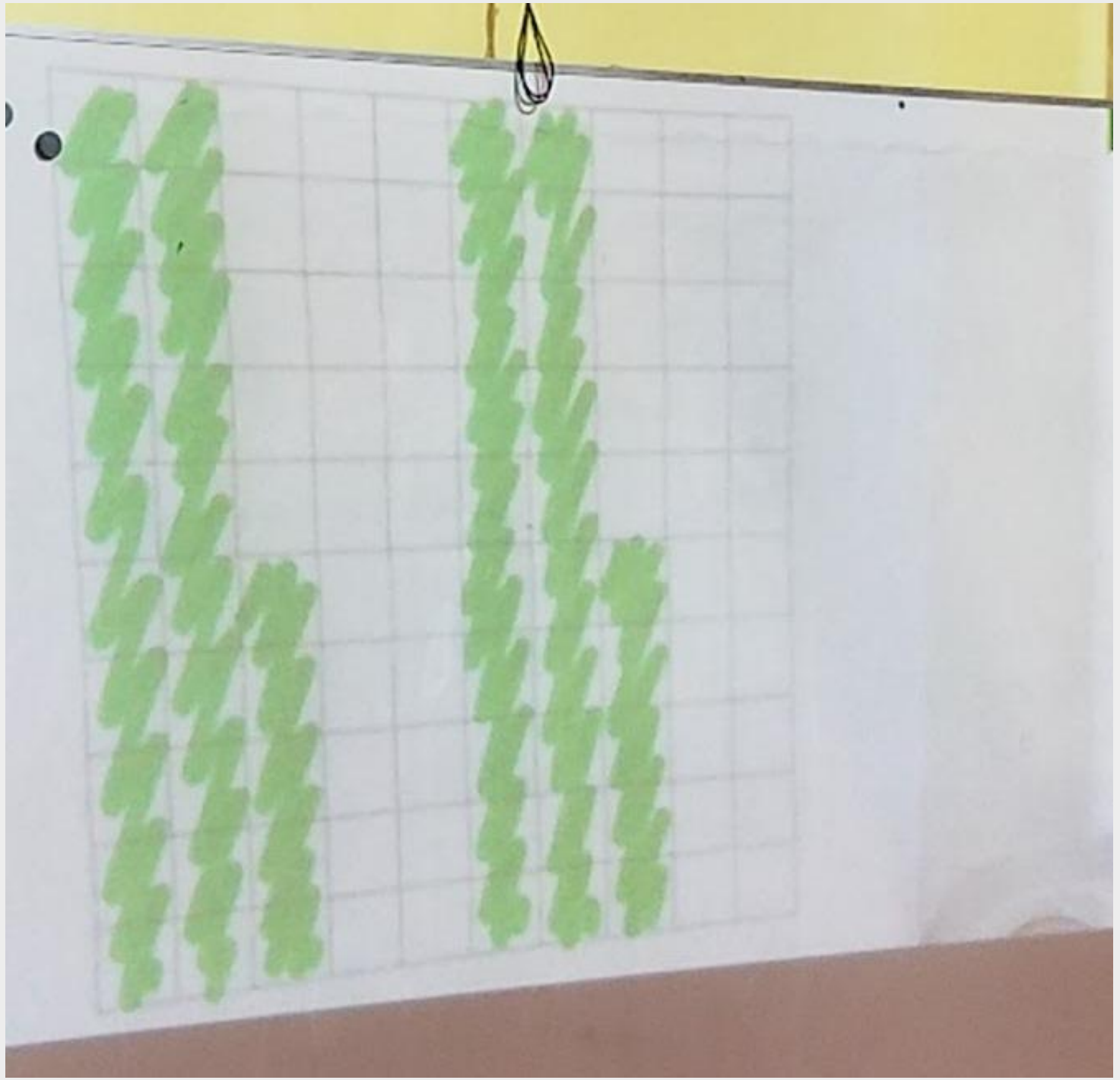


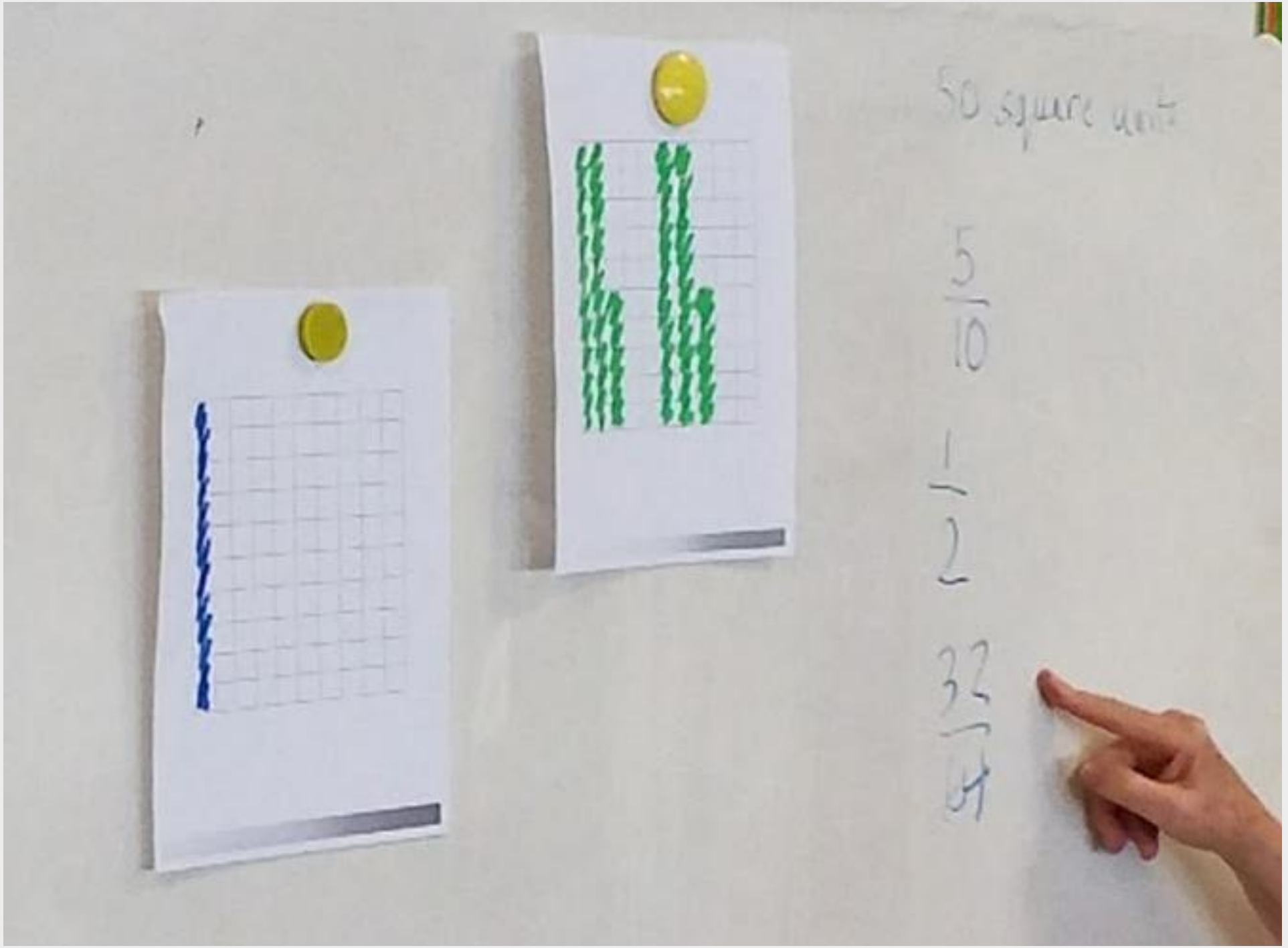

50 squares
0.5
50/100
0.50
1/2
5/10

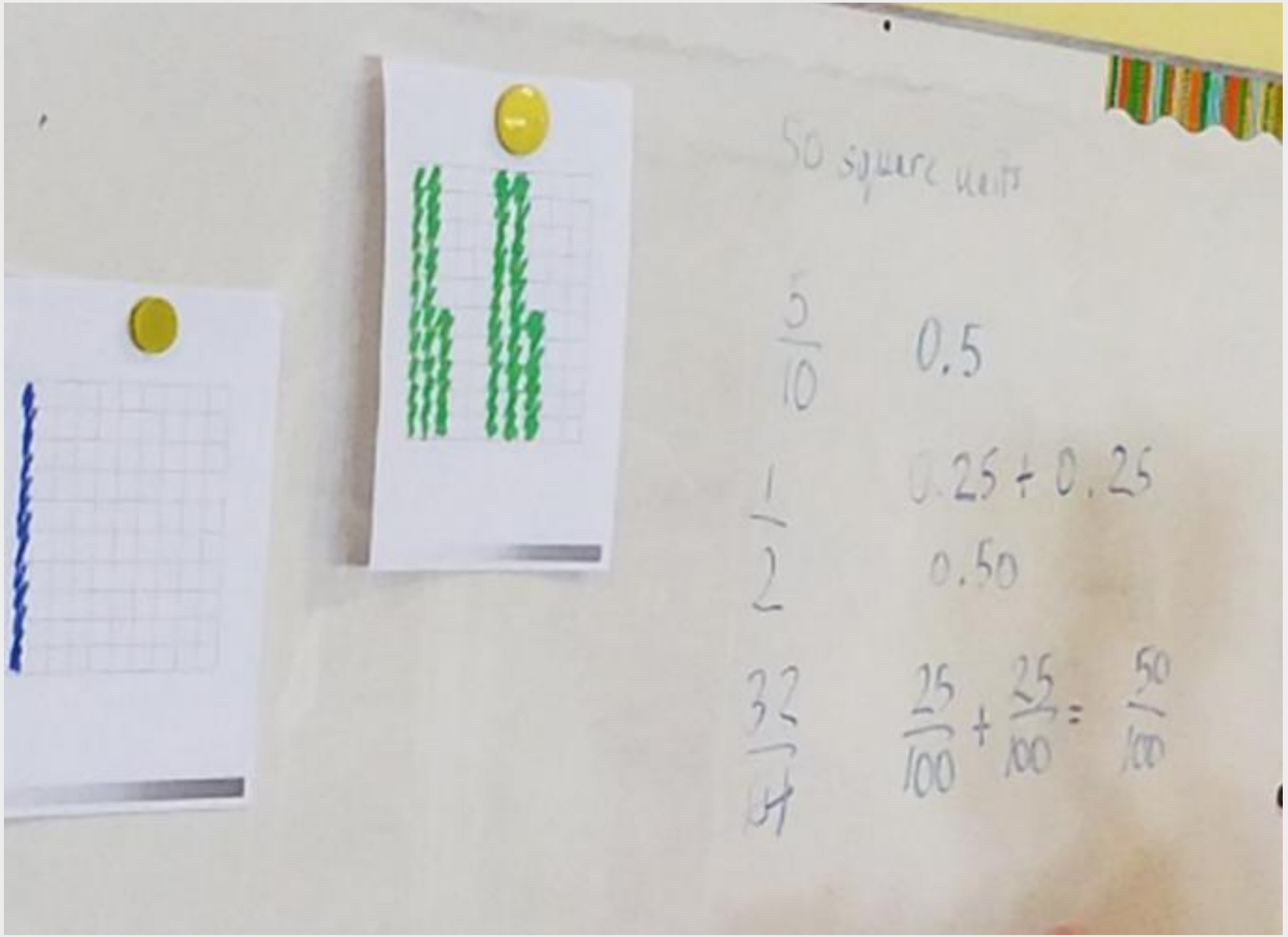

10 square units
1/10
10/10
Reading Goals

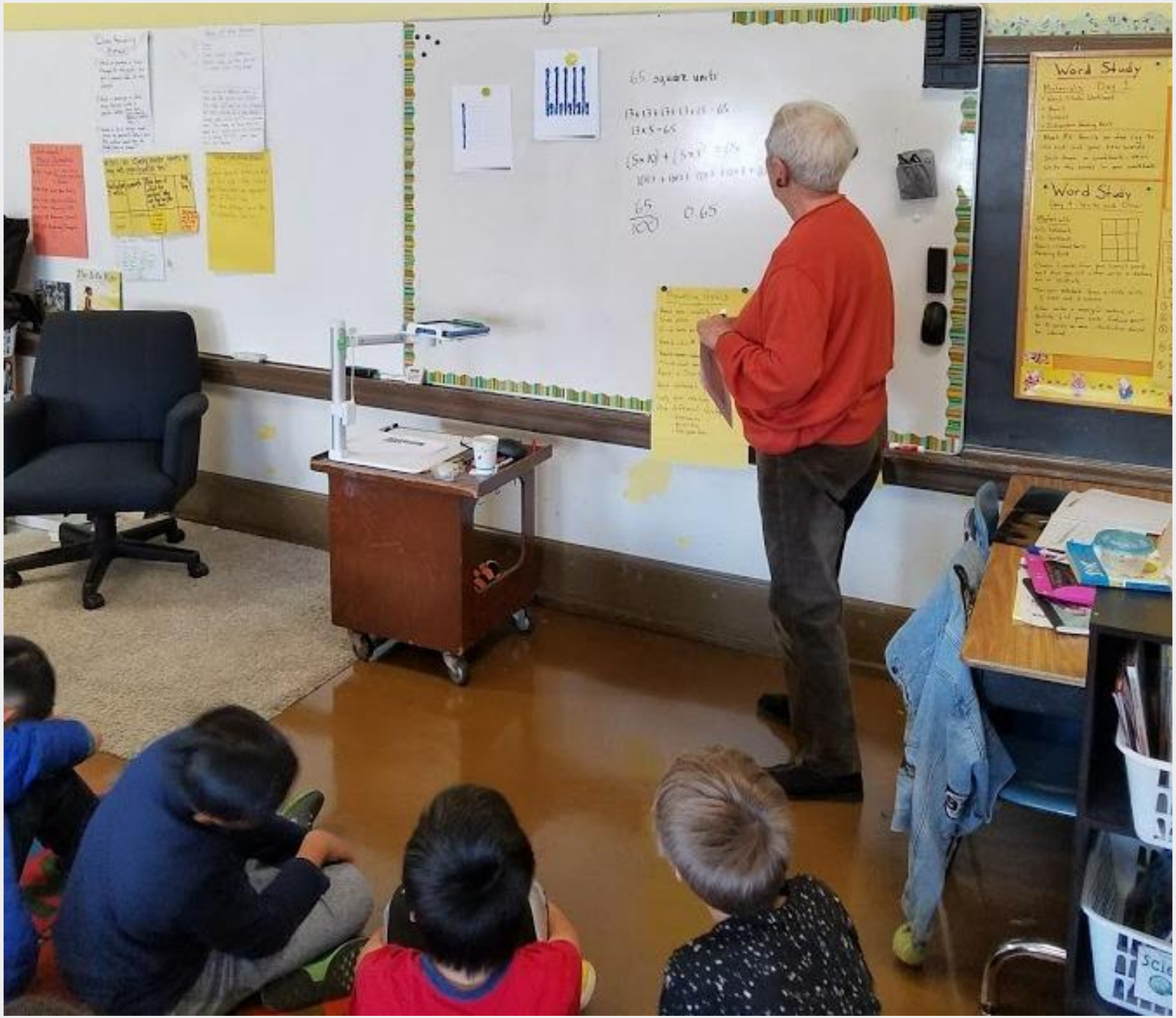

10 square units
1/10 0.1
10/100 0.10
100/1000

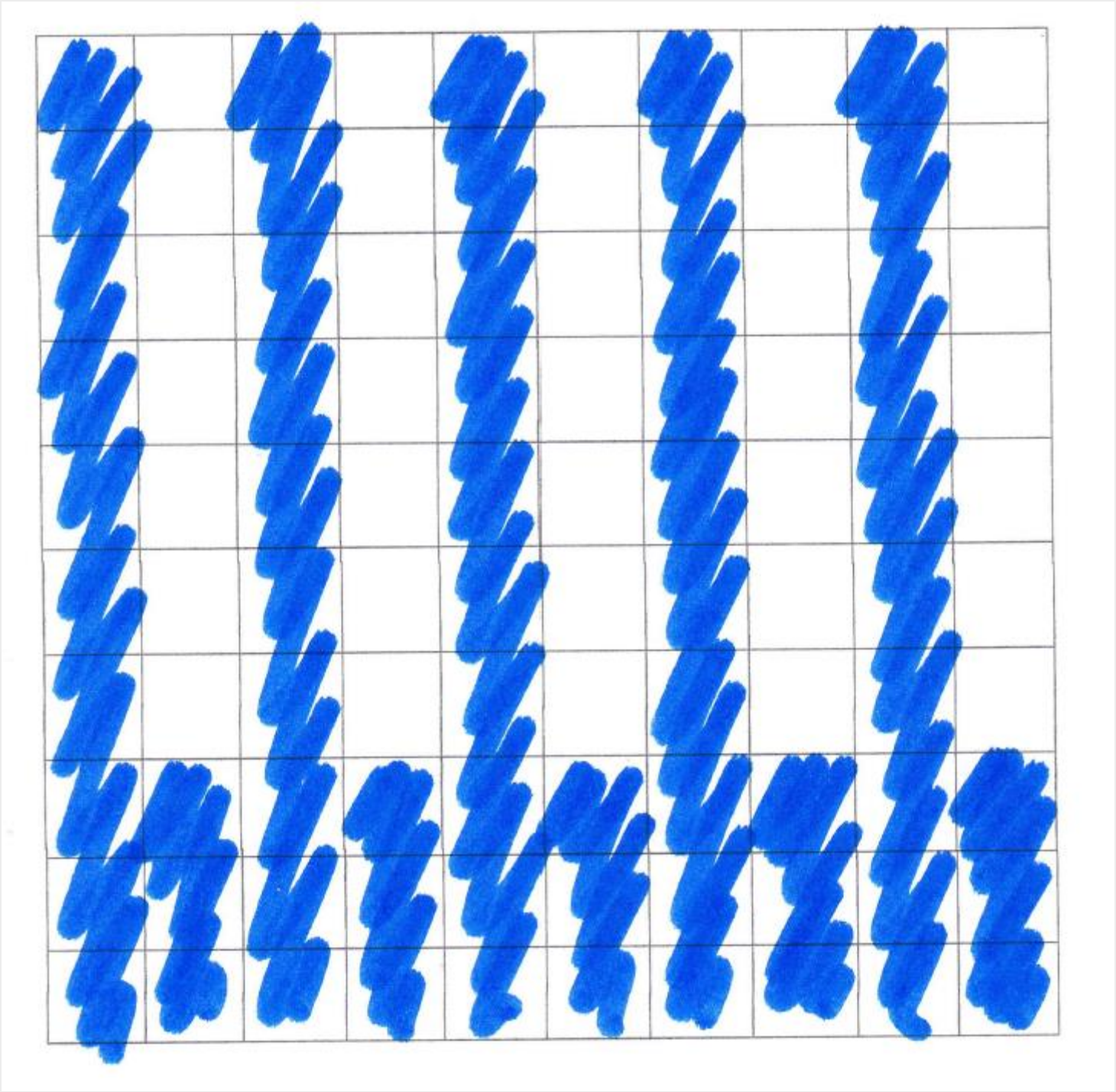

25 square units
0.25
1/4
250/1000

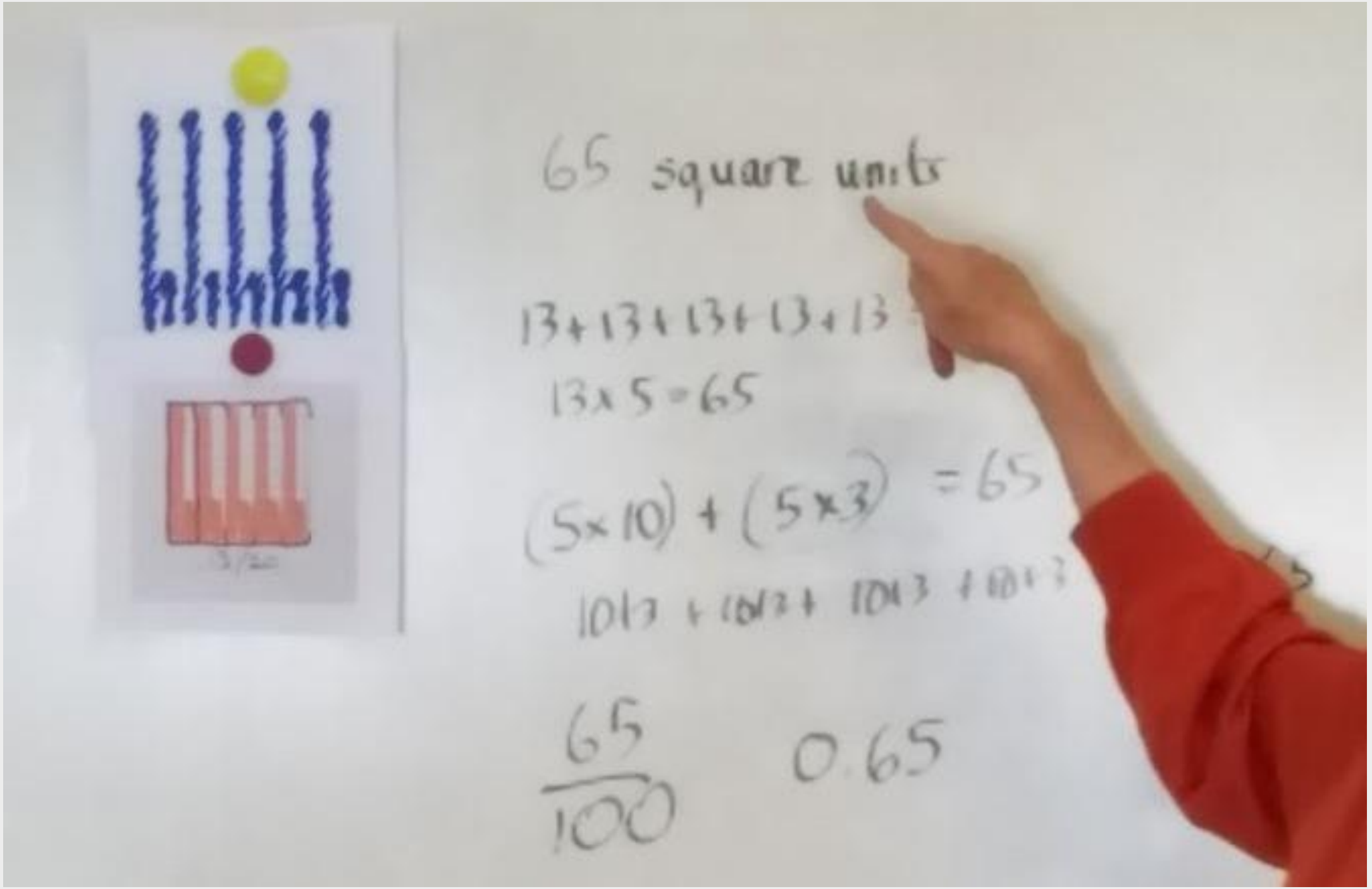


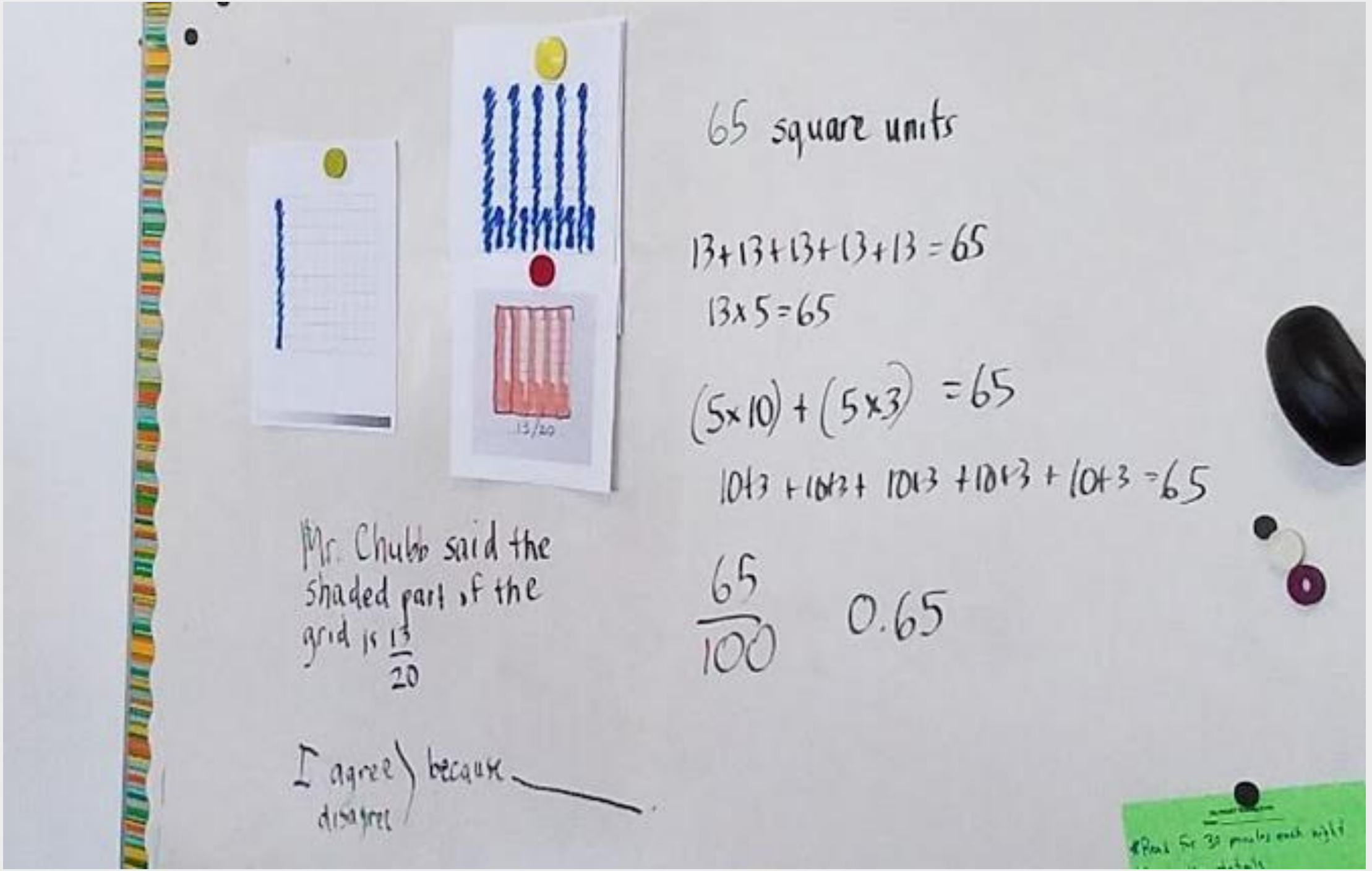

50 square units
5/10
1/2
32/64

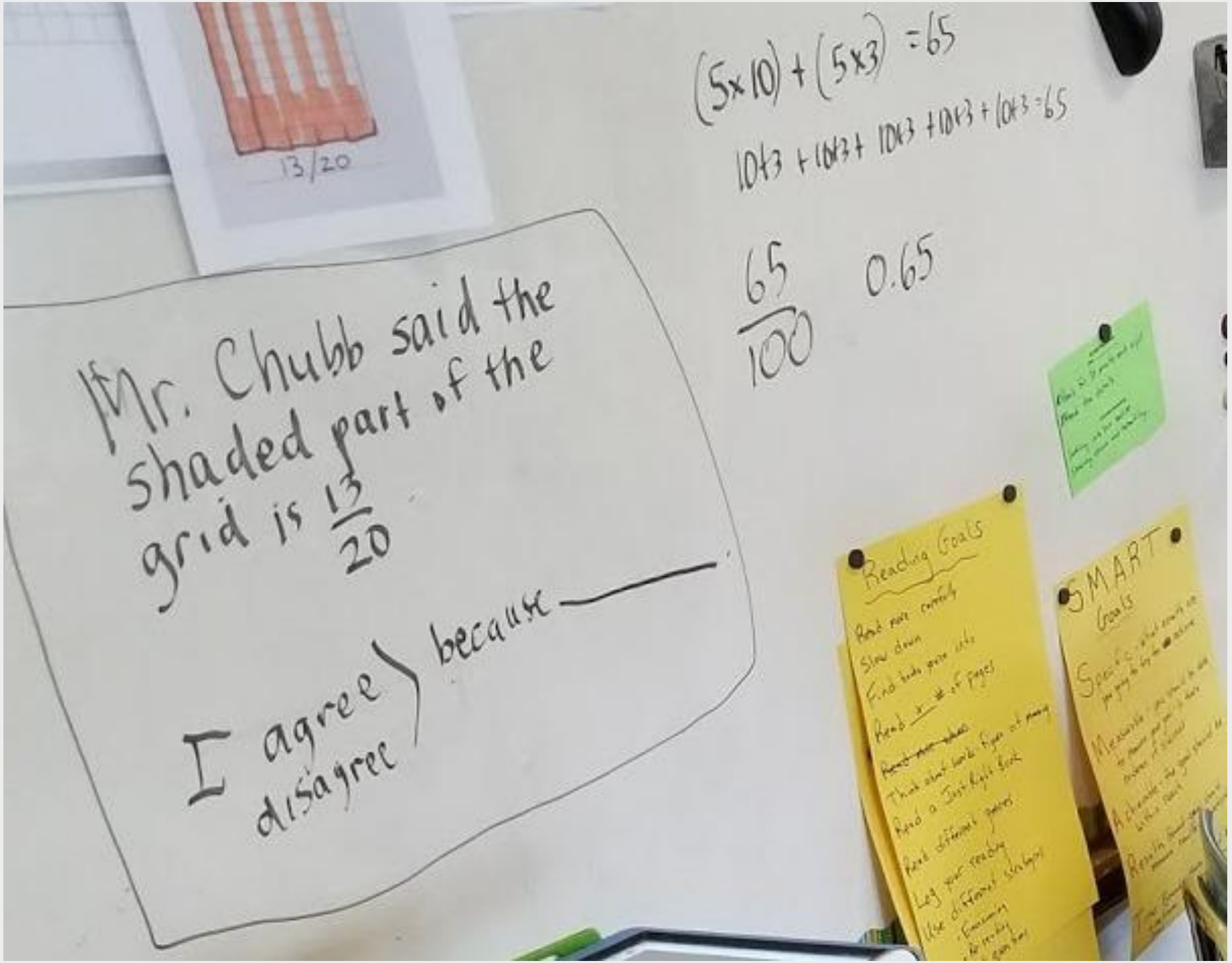

50 square units
5/10 0.5
1/2 0.25 + 0.25
0.50
32/64 25/100 + 25/100 = 50/100

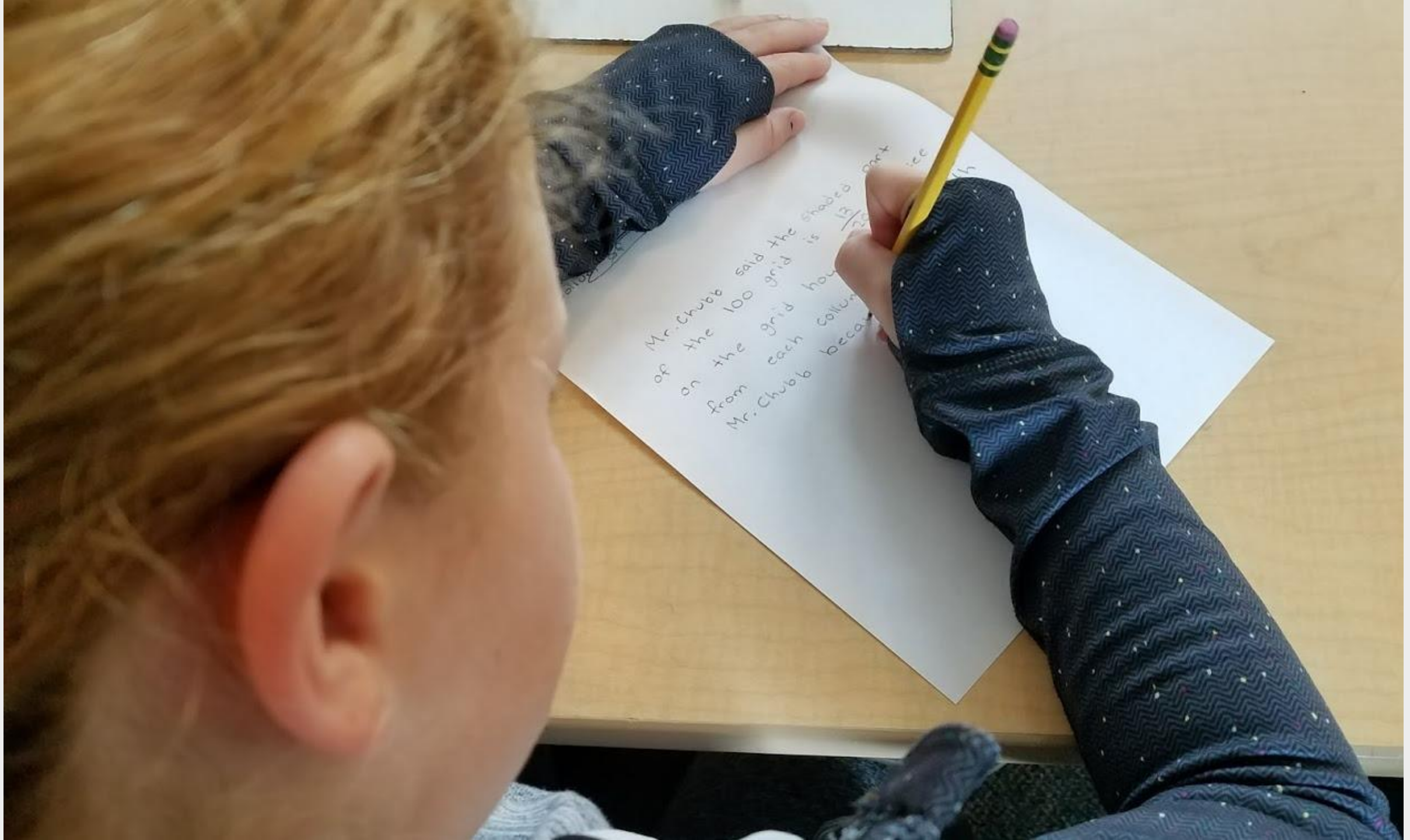

Word Study
Word Study
65 square units
65
0.65

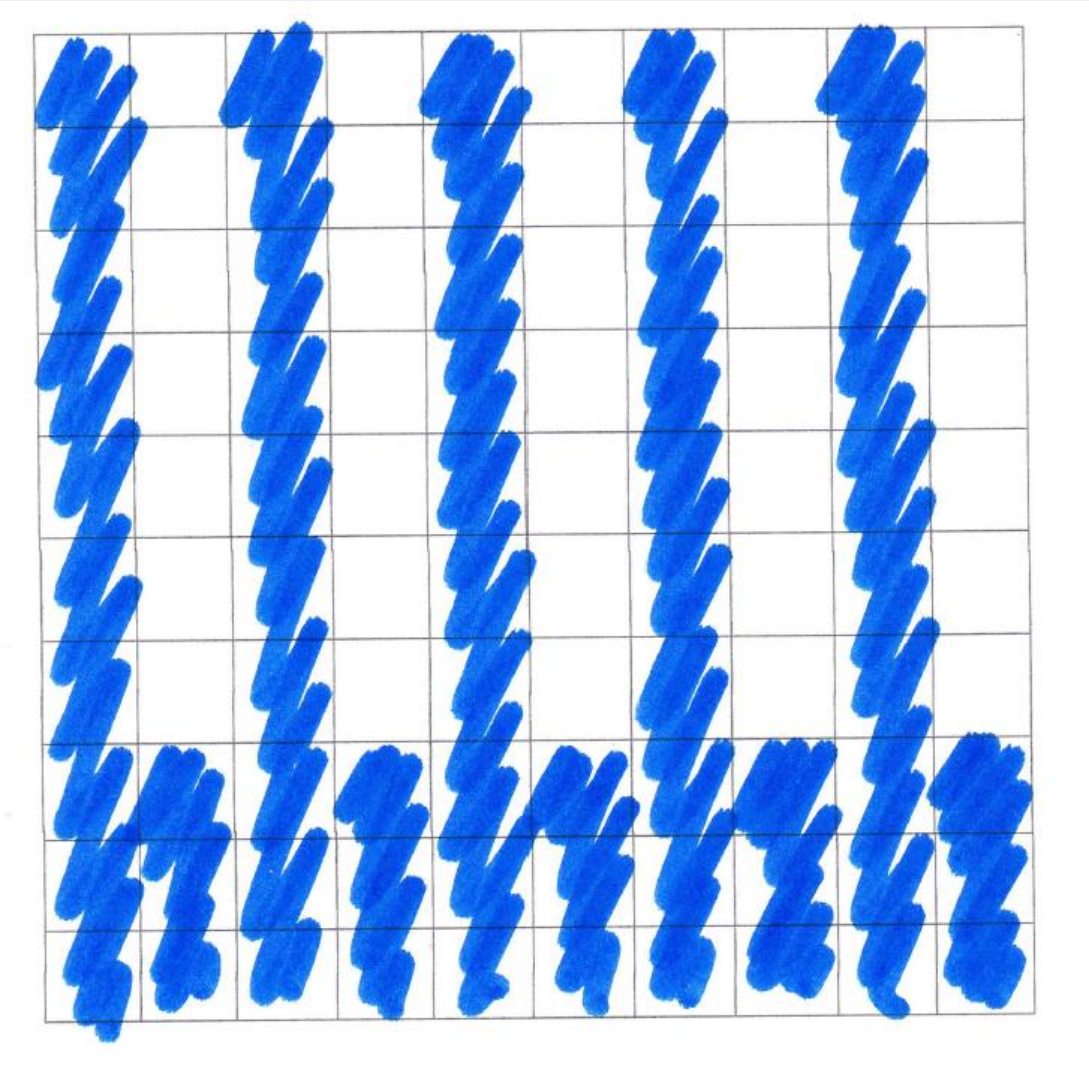


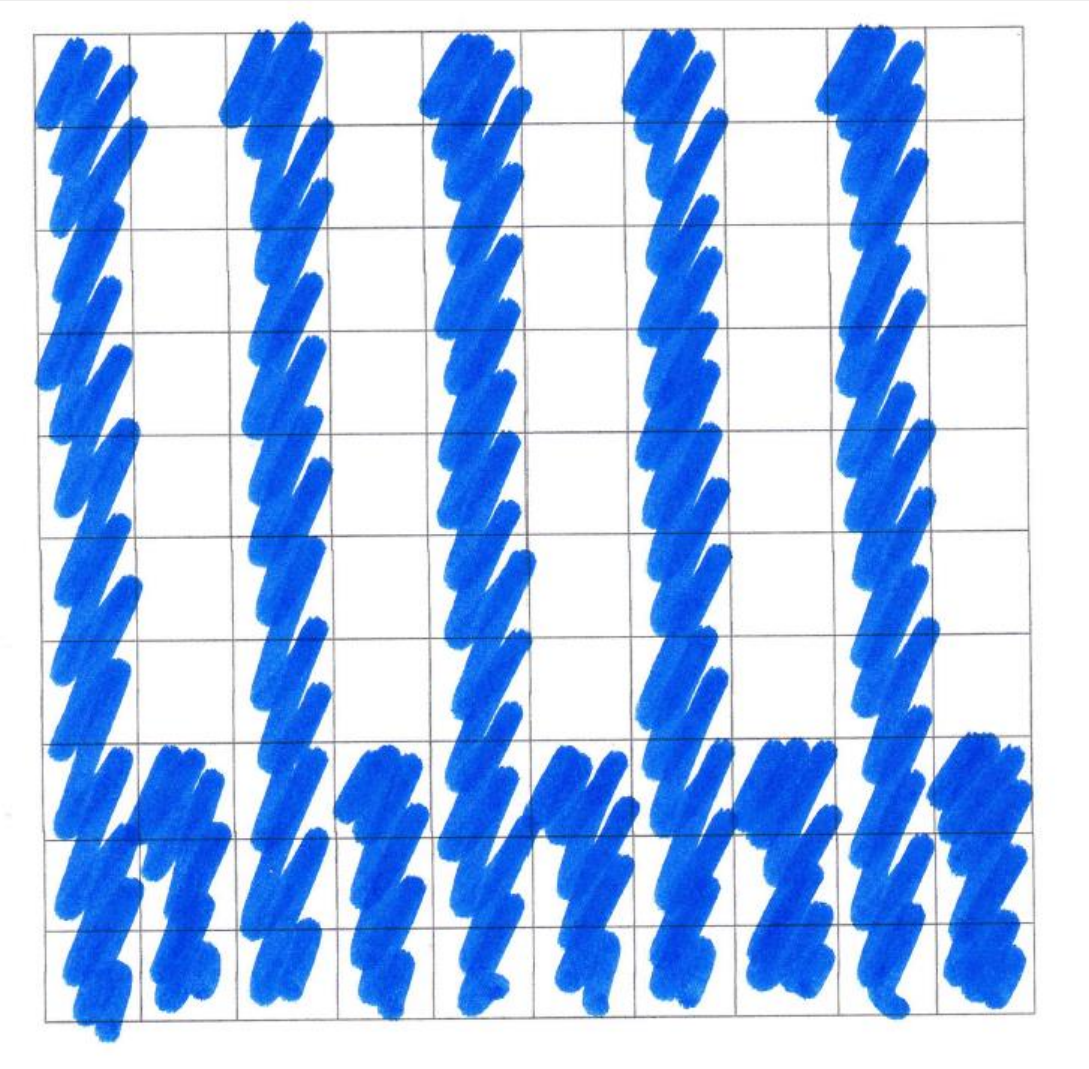

65 square units
13+13+13+13+13
13x5=65
(5x10)+(5x3) =65
65/100
0.65

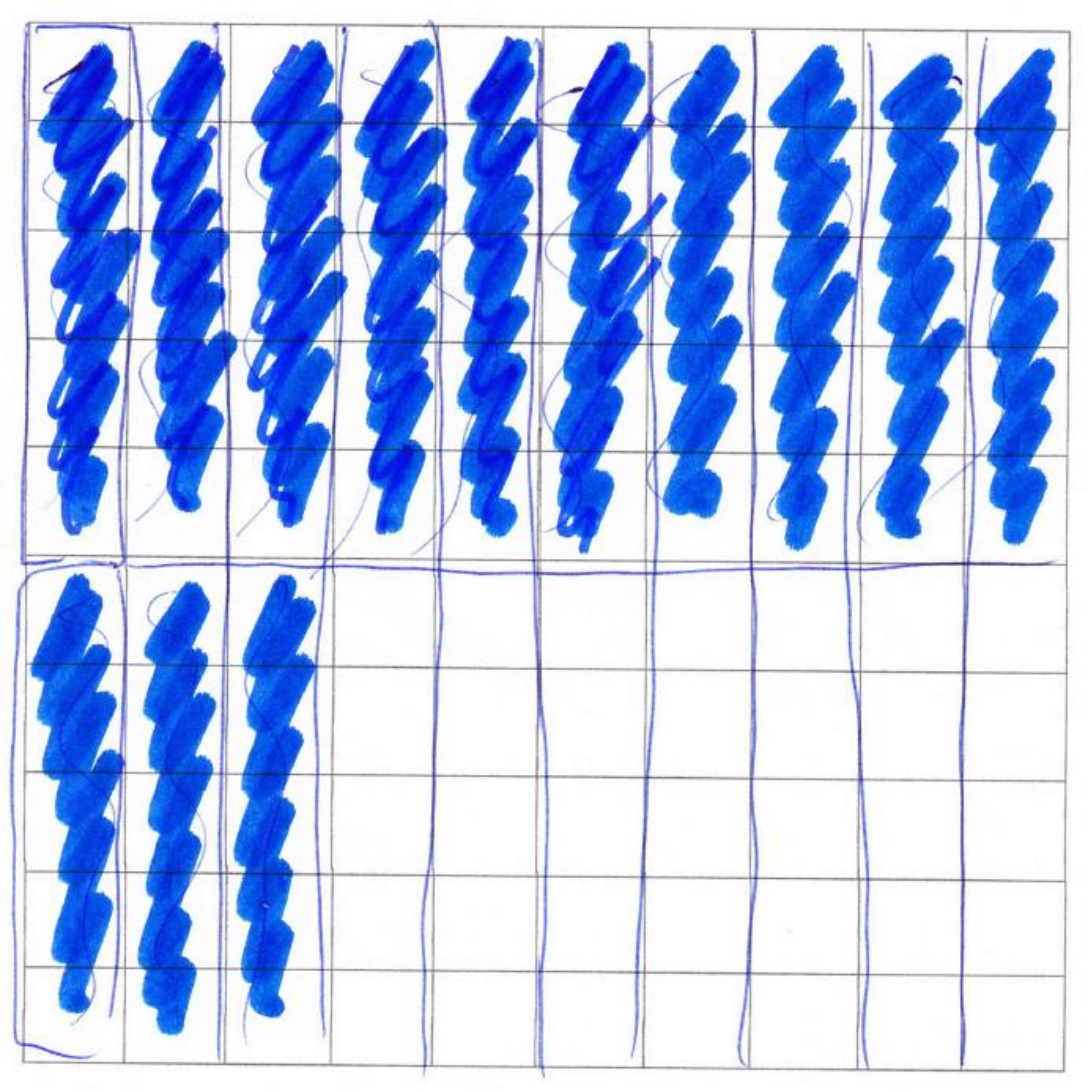

65 square units
13+13+13+13+13=65
13x5=65
(5x10)+(5x3) =65
10+3+10+3+10+3+10+3+10+3=65
65/100
0.65
Mr. Chubb said the shaded part of the grid is 13/20
I agree / disagree because
13/20


13/20
(5x10) + (5x3) = 65
10+3 + 10+3 + 10+3 + 10+3 + 10+3 = 65
65/100
0.65
Mr. Chubb said the shaded part of the grid is 13/20
I agree / disagree because
Reading Goals
SMART Goals


Mr. Chubb said the shaded part
of the 100 grid is
on the grid
from each
Mr. Chubb







Our teaching goal is not to *cover* the Standards . . .

Our teaching goal is not to *cover* the Standards . . .

but to *uncover* them.

Do The Math
Created by Marilyn Burns

Transforming Our School's Math Instruction with Marilyn Burns and *Do The Math*

Math educator Sara Liebert explains the positive impact Marilyn Burns and her instructional support have had on Title 1 students in San Francisco, California.

John Muir Elementary School in San Francisco, California, is a Title 1 elementary school. I work at John Muir as the Instructional Reform Facilitator, focusing on supporting teachers in math [illegible] a fifth [illegible] first [illegible] year [illegible] 3–5.

I had the opportunity to meet Marilyn Burns when she observed a guest teacher in my fifth-grade classroom during the spring of 2015. She reached out to me

centers, but I had never been able to keep up with the organizational component that they required. (For infor[illegible]bout math menus, two resources fro[illegible] article she wrote [illegible]e recent **blog post**. [illegible]students to engage [illegible]extension activities. [illegible] way to differentiate instruction and better [illegible] all of my students' needs.)

A Professional Breakthrough: Understanding "Why," "What," and "How

After 10 years of classroom teaching, I felt that this year I was finally teaching math successfully. The structure of *Do The Math* gave me much

Hear from John Muir Elementary School Teachers

Lisa Alley, a second-grade teacher with 28 years of classroom experience, discussed what she appreciated about the *Do The Math* lessons.

Justin Stoddard, a third-grade teacher with 11 years of classroom experience, also reflected on what he appreciated about *Do The Math*.

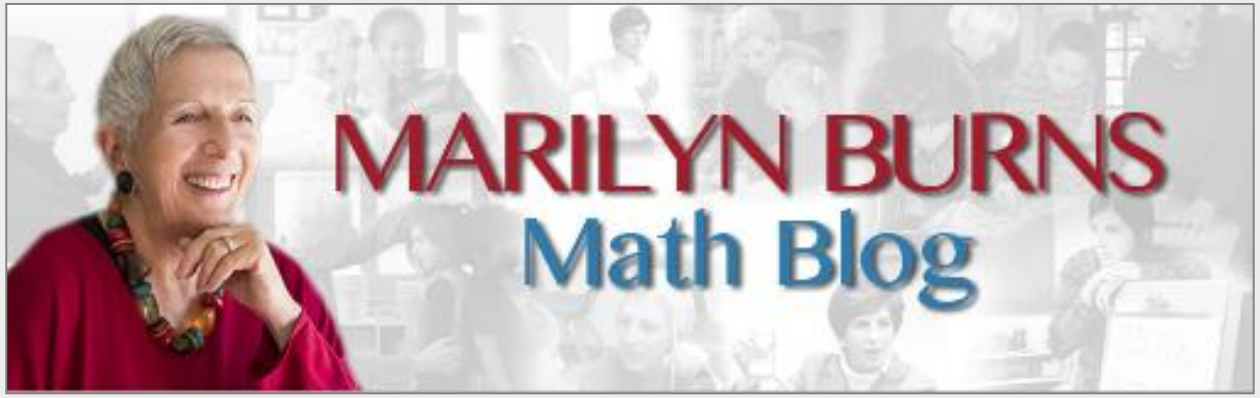
Kristin Matthews, a second-grade teacher with 9 years of classroom experience, commented on the features of *Do The Math* that she found particularly helpful.

Lisa Cowles, a second-grade teacher with 10 years of classroom experience, was skeptical at first, but then was surprised at her experience using the lessons.

Rashida Carter, a first-year teacher teaching fourth grade, especially liked the timeliness of these assessments.

Joe Mannarrino, a first-year teacher teaching fifth grade, had a different view of the scripted lessons.

https://goo.gl/TeHVLT



@mburnsmath

Math Solutions.
FOUNDED BY MARILYN BURNS

Math Reasoning Inventory™