CHAPTER ELEVEN
SHARING COOKIES

Overview

In this lesson, students share "cookies" among three, four, and six people. The cookies are actually paper circles that students can fold and cut as they explore how to divide them equally. From this exploration, students learn about dividing different quantities into equal shares and also have the opportunity to see relationships among halves, thirds, fourths, and sixths.

Materials

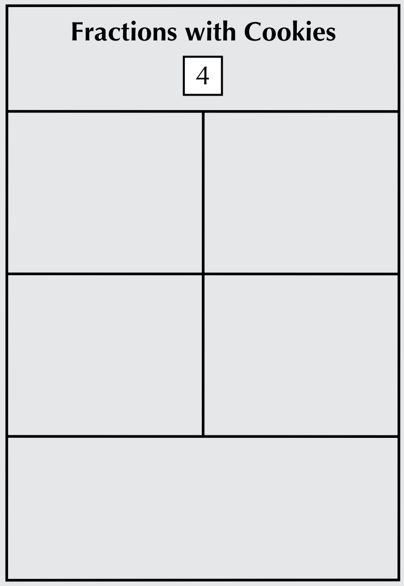
- *Circles for Cookies* worksheet, duplicated on white paper, approximately 2 sheets per student (see Blackline Masters)
- *Sharing Cookies* worksheets for sharing among 3, 4, and 6 people, each duplicated on a different color paper (not white), approximately 2 sheets of each per student (see Blackline Masters)
- optional: *Fractions with Cookies* worksheet, 1 per pair of students (see Blackline Masters)
- scissors
- paste or glue

Time

- two class periods

Teaching Directions

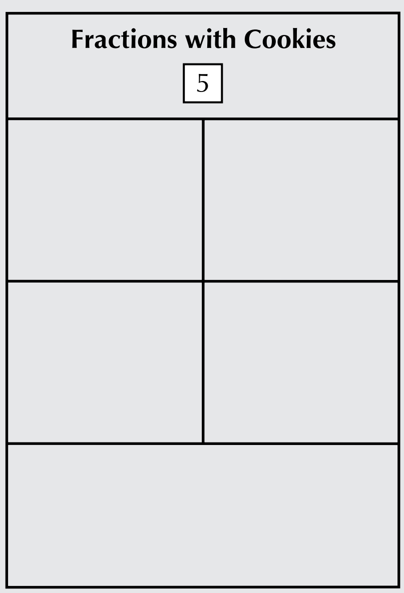
1. Draw on the board a replica of the sheet for sharing cookies among four people. In the box for the number of cookies to share, write a *4.* Also, draw four circles on the board to represent the four cookies.



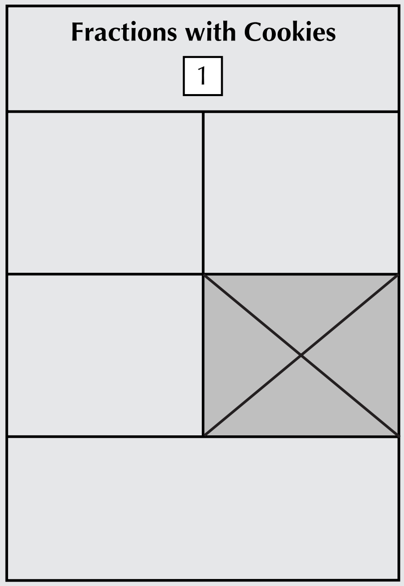
2. Ask: "How would you share four cookies among four people?" The problem will be trivial for the class. Model for the students how to record by drawing a circle in each of the spaces on the board, checking off one of the "cookies" you drew as you do so. Also, record below: *Each person gets one cookie.*

3. Then ask: "How would you share five cookies among four people?" Redo the worksheet, write a *5* in the box, and draw five circles for the cookies. Ask students to explain their thinking, giving more than one student the chance to respond if each reasoned differently. Record again by drawing a circle and a fourth of a circle in each space. Record below in several ways:

Each person gets 1 cookie and $\frac{1}{4}$ of a cookie. Each person gets $1\frac{1}{4}$ cookies. Each person gets $\frac{5}{4}$ cookies.

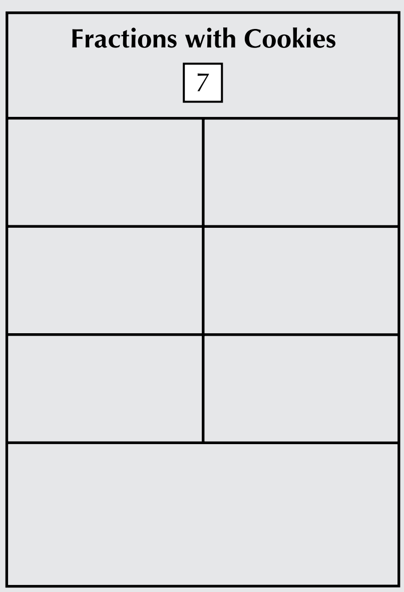


4. Repeat for sharing one cookie among three people. Cross out one of the boxes on the worksheet on the board so that there are only three places to place cookie shares. Talk with the class about how to share the cookie and have students explain their reasoning. Then draw one-third of a cookie in each space and record: *Each person gets $\frac{1}{3}$ of a cookie.*



5. Next ask: "How can you share four cookies among three people?" Again, have students explain, draw the cookie shares, and record: *Each person gets $1\frac{1}{3}$ cookies.*

6. Model with one last problem: "How can you share seven cookies among six people?" Alter the worksheet on the board so that it has six spaces. For this example, not only record the answers but also record explanations from at least two of the students to model for the class how they are to write when they work on their own.



7. Show the class the three versions of the worksheet for sharing among three, four, and six people. Explain the activity. Write the directions on the board or distribute copies:

Fractions with Cookies

1. Do five worksheets, one of each version and two others of your choice.
2. Use no more than 20 cookies for any one worksheet.
3. Cut "cookies" into equal shares and paste them in the appropriate places.
4. Record how much each person gets. Explain your reasoning.
5. For each sheet, choose a number of cookies that is not a multiple of the number of people.

Discuss the last direction with the class to be sure that the students understand what multiples are.

Teaching Notes

A version of the *Sharing Cookies* activity appears in *A Collection of Math Lessons, Grades 3–6*, a book I wrote more than ten years ago. In that version, third graders concentrated on sharing cookies among only four people. That's also the way I've introduced the activity with fourth graders who were just beginning to think about fractions.

I decided to restructure the activity into a more complex version for these fifth graders because they had more prior experience with fractions than the third and fourth graders I previously did the activity with. They had already worked with fraction kits and pattern blocks, and this exploration gave students a different context in which to confront the ideas they were learning about fractions.

The Lesson

DAY 1

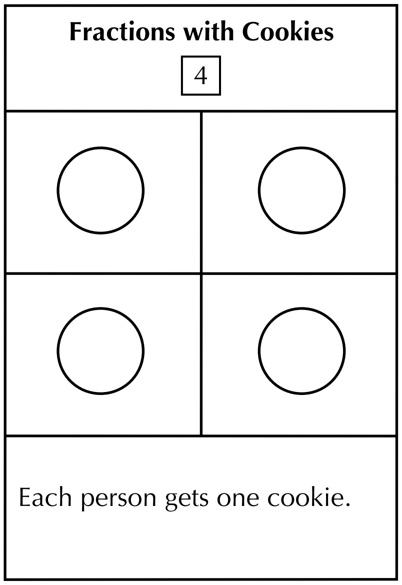
In preparation for the lesson, I duplicated three versions of a worksheet for sharing cookies among three, four, and six people. I duplicated each version on a different color paper, making enough copies so that there was one for every two children. Also, I duplicated circles that would be the cookies on white paper. Using colors for the activity sheets would make the white circles clearly visible and, therefore, easier to use for later class discussion.

To begin the class, I drew on the board a replica of the sheet for sharing cookies among four people. In the box for the number of cookies to share, I wrote a *4.* I also drew four circles on the board to represent the four cookies.

"How would you share four cookies among four people?" I asked. Hands shot up and there were titters and comments of surprise at this easy question. I called on Shannon.

"They'd each get a cookie," she said.

"I agree," I said. As I drew a circle in each of the spaces on the board, I checked off one of the cookies I had drawn on the board. Then I recorded below: *Each person gets one cookie.*



While this example was trivial mathematically, I used it to establish the routine for how students were to use the worksheets when they worked on their own. Explicit directions and modeling when introducing something new help avoid confusion later, and I've learned to err through overkill rather than make assumptions that lead to later distractions.

I erased the check marks I had made next to the cookies and what I had recorded on the worksheet. I added a circle to the four on the board so I had five cookies represented. "How would you share five cookies among four people?" I asked. Again, I chose a question that I thought would be easy. Most hands shot up, but not immediately from all of the students.

"You can't do that," Robert said.

"Yes, you can," Lara countered. "Just divide a cookie."

"Oh yeah," Robert said.

"Oh yeah," Dan also said.

This was a quick exchange, done softly, so I didn't reprimand the students for speaking out, even though I'd been struggling to get them to raise their hands during lessons and wait to be called on. Instead, I waited until the room was quiet and all hands were raised. I called on Claudia.

"They each get a cookie and a quarter of a cookie," she said. Others agreed. I drew one cookie in each space, again checking off these circles I had drawn on the left. One cookie was left. I looked at Claudia.

"Just divide that one into fourths and put a fourth in each space," she said.

I did what Claudia suggested. As I drew a fourth of a circle in each space, I said, "You won't have to draw when you do this activity. You'll have paper cookies to cut and paste." Then I asked, "What should I write for how much each person gets?" I called on Delia.

"Each person gets one cookie and a fourth," she said.

"A fourth of what?" I asked. I'm always pushing the children to define the whole.

"A fourth of a cookie," she added. I wrote:

Each person gets 1 cookie and $\frac{1}{4}$ *of a cookie.*

"I know a shortcut way to write each person's share," I said. "When you have a whole number and a fraction, you can write it like this." I wrote:

Each person gets $1\frac{1}{4}$ *cookies.*

Dan raised his hand. "I have another way to write it," he said. "Each person gets five-fourths." I recorded on the board:

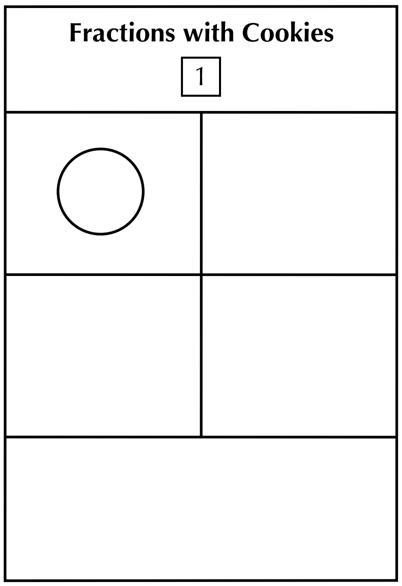
Each person gets $\frac{5}{4}$ *cookies.*

"Explain how you got that," I said to Dan.

His response was precise. "If you cut up one cookie, you get four fourths, and one more makes five fourths."

"That works," I agreed. "Any comments or questions?" There were none.

I erased what I had recorded, again leaving just the worksheet. I then did two examples with sharing cookies among three people—first sharing one cookie and then sharing four cookies. I crossed out one of the boxes on my replica of the worksheet so that there were only three places to draw cookie shares, just as I had done on the worksheets they were going to use. For the first example, I drew one circle to the side and wrote a *1* in the box on the worksheet.

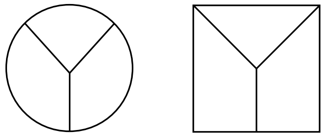


"How would you share one cookie among three people?" I asked.

After all hands were raised, I called on Grant.

"They each get a third," he said.

I drew lines to divide the circle I had drawn into thirds and then drew a third in each of the spaces, commenting aloud that drawing the letter *Y* was a good way to approximate thirds. I've found that students often are initially confused by dividing a circle into thirds and it warrants discussion. (Beware, however, that some students try to draw the letter *Y* when dividing a square into thirds! See page 141 for *The* Y *Problem,* an assessment that relates to this.)



Some of the students criticized that the thirds I drew didn't "look right." I responded, "It would be easier if I cut and pasted paper circles, as you will. It's hard to draw." I recorded on the board:

Each person gets $\frac{1}{3}$ *of a cookie.*

For my second example, sharing four cookies among three people, I drew four circles on the board. I purposely chose this example to avoid having them figure out how to share two cookies among three people, as they would have to do with sharing two or five cookies. I wanted to have the students tackle thinking about sharing two cookies among three people when they were working in pairs and had the circles in hand. While whole-class instruction is useful for helping students understand what they are to do, it's not the best setting to have students grapple with a problem that might be difficult without having, as in this instance, the opportunity to get their hands on their own cookies. (In retrospect, it might have been a good pedagogical strategy to ask them to share two cookies among three people and guide them to see how that answer should be twice as much as the answer for sharing one cookie among three people.)

"How would you share four cookies among three people?" I asked.

From the quick response of hands, I assessed that this was also easy for them. I called on Libby to answer, and she did so correctly. I quickly drew the cookies in the spaces and recorded: *Each person gets* $1\frac{1}{3}$ *cookies.*

Next I introduced the problem of sharing seven cookies among six people. I altered the worksheet on the board so that it had six spaces. For this example, not only did I record their answers, but I also recorded explanations from two of the students to model for the class what I expected them to write when they worked on their own. Also, writing verbatim what the students say is a way to help students realize that what they write can be directly linked to how they explain their reasoning.

From Delia's explanation, I wrote: *Each person gets $1\frac{1}{6}$ cookies. There are 6 cookies, so each person gets 1. Then divide the last cookie into sixths and each person gets $\frac{1}{6}$.*

From Davey's explanation, I wrote: *Each person gets $\frac{7}{6}$. Each person gets 1 whole and $\frac{1}{6}$, and 1 whole is 6 sixths, and another sixth is $\frac{7}{6}$.*

I left these two explanations on the board for students to refer to when they were working independently.

I then showed the class the three versions of the worksheet and explained the activity. As I gave the directions, I wrote them on the board:

Fractions with Cookies

1. *Do five worksheets, one of each version and two others of your choice.*
2. *Use no more than 20 cookies for any one worksheet.*
3. *Cut "cookies" into equal shares and paste them in the appropriate places.*
4. *Record how much each person gets. Explain your reasoning.*
5. *For each sheet, choose a number of cookies that is not a multiple of the number of people.*

"What do I mean by this last direction?" I asked. "What's a multiple?"

Some hands went up. I called on Maggie.

"A multiple is like two, four, six, eight, like that," she said.

"I agree that the numbers you said—two, four, six, eight—are all multiples of two," I responded. "So what's a multiple?"

"It's something you get when you multiply," Josh said.

I nodded and paraphrased. "A multiple of a number is the product of multiplying the number by some number. Who remembers what we call the numbers you multiply?" Several students remembered "factor."

"When you share cookies among four people, what numbers aren't allowed?" I asked.

Jennifer answered, "Four, eight, twelve, sixteen, twenty."

"That's right," I said. "Those are multiples. If you can divide a number into another evenly, without a remainder, then the larger number is a multiple of the smaller one. Four can be divided into all of the numbers Jennifer said. I don't want you to use four, eight, twelve, sixteen, or twenty cookies for this sheet because then there wouldn't be any need to think about fractions. Pick numbers that aren't multiples so that you'll have to use fractions to share all the cookies evenly."

I distributed one of each color sheet and a sheet of cookies to each pair of students, put the rest of the sheets on the supply table, and had the students begin work.

Observing the Students

I circulated as the students worked. I observed Josh and Robert sharing four cookies among six people. Josh had cut all of the cookies into halves and had glued a half in each of the six spaces. There were two halves left.

"Oh, I know," he said. "Let's cut each of these into thirds." Robert agreed and they each snipped a half into three pieces.

"They're each a sixth," Josh said to me, noticing I was paying attention to what they were doing. "They each get one-half and one-sixth."

"How do you know each of those small pieces is one-sixth?" I asked.

"Because three make a half and three make the other half, so there are six from a whole," he answered.

Robert was frowning. "What are you thinking?" I asked him.

"We should have started by cutting all of the cookies in thirds," he said. "Then each person would get two-thirds."

Josh stopped gluing the sixths in place to consider this. "Yeah," he said. "That would work."

"So is two-thirds the same as one-half plus one-sixth?" I asked.

Both boys were stopped by this question.

"It has to be," Josh said after a moment.

"What do you mean?" Robert asked.

"Can you explain why they have to be the same?" I asked Josh.

Josh shook his head. "I don't think so."

"Oh, look," Robert said. "It's the same. They're both four-sixths. The half is three-sixths and one more is four-sixths. And two-thirds is four-sixths if I cut them each in half."

"Oh," Josh said, nodding. (See Figure 11–1.)

I next went to check on Joseph, who was working with Sean. I knew that Joseph was having difficulty with fractions. In general, Joseph learns at a slower pace than the others and seems comforted when he knows that someone will help him if he has difficulty. He's a willing student, but he needs encouragement as well as a good deal of extra assistance. His partner today, Sean, is a clever student but is prone to play whenever he can. He seems younger than the others, but he is a good worker and is invested in doing well.

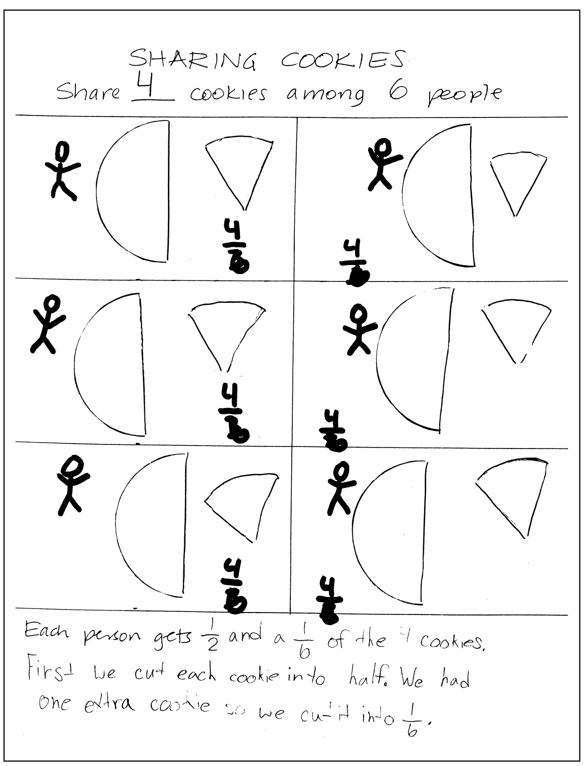


Figure 11–1 *Robert and Josh figured out that $\frac{1}{3}$ of $\frac{1}{2}$ is $\frac{1}{6}$.*

Joseph and Sean were on their first problem, working on sharing ten cookies among four people. They had already divided the cookies by pasting two cookies and an additional half cookie in each space. Sean was busily writing at the bottom of the sheet.

Sean looked up and said to me, "Joseph glued and I'm writing." I've worked hard with students to be sure that both students are involved when working in pairs, and Sean wanted to assure me that this was the case.

"How did you decide on using ten cookies?" I asked.

Sean kept on writing and Joseph answered. "It was Sean's idea."

"How did you feel about using ten cookies?"

"Okay," Joseph said, in a noncommittal tone. I wasn't sure how he felt.

"Can you explain how you divided up the cookies?" I probed.

"We put the whole ones in the spaces and had two left over. Then we made halves," Joseph answered.

I looked at their paper and noticed that the halves had pencil lines on them, as if the boys had contemplated dividing them into fourths. I commented on what I noticed. Sean looked up from his writing.

He said, "Well, at first I thought we should cut them into fourths since there were four people, but then I realized we didn't have to, that we could just make halves and we would have four of them."

I looked at Joseph's face but still got no clue about his thinking. "Can you tell me how much each person's share is?" I asked Joseph.

He looked down at the paper and answered, "Two cookies and a half of a cookie." Sean was now through writing. I picked up the paper.

"Joseph, how about reading aloud what Sean wrote so you and I can be sure we agree with it?" Joseph did this. Sean had written:

> *Each person gets 2 cookies and $\frac{1}{2}$ of a cookie. We went around to each square and put one cookie in each untill we had 2 cookies in each square, but we had 2 cookies left wich we cut $\frac{1}{2}$'s and we had 4 $\frac{1}{2}$'s left. We put the $\frac{1}{2}$'s in the squares.*

"That's what we did," Joseph said after reading.

"Let's do sharing with six people next," Sean said, reaching for another worksheet. "How many cookies should we use, Joseph?"

"You pick," Joseph said.

Sean thought for a minute. Then he grinned and said, "Let's do nine cookies."

"How come nine?" I asked.

"I know I can do that," he said. "I can tell it will be easy."

"Do you know how much each person will get?" I asked.

"I think one and a half, but I'm not sure," he said. "Let's cut out the cookies, Joseph."

"Remember to talk about what you're doing," I said. "And after one of you does the writing, the other one should read it aloud so that you can both check that you agree." I left the boys to work.

Even though I didn't get much information from Joseph about his thinking, Sean's comments helped me see how he was thinking and that he knew how to choose a problem he was sure of being able to do.

I noticed Jennifer and Davey working together on their first problem, sharing sixteen cookies among three people. "We thought a problem with a big number would be interesting," Davey told me. They worked efficiently, cutting out circles quickly and overlapping one on the other on their sheets until they had pasted five in each section. As Davey pasted them down, Jennifer cut the remaining circle into thirds. Jennifer then recorded:

> *Each person gets $5\frac{1}{3}$ cookies. There are sixteen cookies if you leave out one cookie thats 15 and 15 ÷ 3 = 5 and there's one cookie left cut it into thirds and it's even.*

I left once they decided on their next problem, to share eighteen cookies among four people.

Time for math was over, so I had the students gather up their work and have it ready for the next day. I said, "When we start math tomorrow, just get back to work where you left off."

DAY 2

At the beginning of class the next day, I told the students to get back to their cookie problems. I circulated to make sure they had all gotten started again, encouraging a few and solving the problem of Josh and Robert's missing work. (It was in Robert's cubby.)

Then I went to see how Dan and Grant were doing. In contrast to the large numbers of cookies that Jennifer and Davey had been choosing, Dan and Grant chose small numbers—first sharing two cookies among four people and then sharing two cookies among three people. Now they were completing work on sharing two cookies among six people. They had pasted down the cookies but hadn't recorded yet.

"How come you chose those problems?" I asked the boys.

"We wanted to see how they would come out," Dan said.

"And we thought they'd be easy," Grant said, grinning.

"How did they come out?" I asked. "Let's take a look." They placed the three sheets next to one another on the table.

"Look, there's two pieces in each part on all of them," Grant said.

"What do you mean?" Dan asked.

"See, on this one, they each get two-thirds," Grant said, pointing to the sheet on which they had shared two cookies among three people. He then added, pointing to the other worksheets, "And on this one they got two-fourths and on this one they got two-sixths."

"But the pieces are smaller when there are six people," Dan said.

"Does that make sense?" I asked.

Grant answered. "There are more people, so they get less."

I left the boys and turned to Maggie and Shannon. They were sharing six cookies among four people and had correctly pasted one circle and a half of a circle in each section. However, they had written:

> *Each person gets $1\frac{1}{4}$ cookies. We had 6 cookies and we gave a cookie to each person, then we had 2 cookies left and four people.*

I pointed to one of the sections on the worksheet with one whole cookie and half of a cookie pasted in it. "How much is in this section?" I asked.

"A cookie and a half," Shannon answered. I looked at Maggie and she nodded her agreement.

"I agree," I said. "But I'm confused by what you wrote below." The girls both read what they had written.

"Ohhh, I think it should be one and one-half," Shannon said.

Maggie had done the writing and defended what she had recorded, "But there are four people, so they each get a quarter of what there is."

"But they got a cookie and a half," Shannon insisted.

"See, they each get a quarter," Maggie said to us. "One and a half is a quarter of six cookies."

"Now I'm confused," Shannon said.

"You're both thinking right," I said. "But there's a problem with how you recorded your thinking mathematically, Maggie. If you look at what you pasted down, it shows that each person gets one and a half cookies. I agree that you divided the cookies into four equal shares so you have quarters. You could say each person gets one quarter of the cookies altogether."

"So I'm wrong?" Maggie asked.

"Your thinking is correct," I responded. "But what you wrote doesn't mathematically express what you explained."

"Oh, I know now," Maggie suddenly said. "They each get one and a half cookies." She reached for their paper to make the change.

"That's what I thought," Shannon said, watching Maggie make the change.

Later when I checked their work, I noticed that they had done another paper sharing eleven cookies among four people. In each section, they had pasted two whole cookies, a half of a cookie, and a quarter of a cookie. They had written:

> *Each person gets $2\frac{3}{4}$ cookies. We put 2 cookies, we had 3 cookies left over. We cut 2 cookies in $\frac{1}{2}$ and the last cookie in $\frac{1}{4}$'s.*

They had combined all of the pieces and correctly expressed that each person got two and three-fourths cookies. (See Figure 11–2.)

I left the girls and moved over to check on Jennifer and Davey, who were talking about sharing eighteen cookies among four people. They had shared the whole cookies, pasting four in each place, and had cut the two remaining cookies into halves.

"Let's cut them into fourths," Jennifer said.

"We don't have to," Davey said. "We have enough halves."

"But we're dividing them for four people," Jennifer said. "We need fourths."

"You don't have to have fourths," Davey protested.

"But you always have fourths if you divide something for four people," Jennifer stated.

Davey shrugged, overpowered for the moment by Jennifer's insistence. Jennifer cut the halves into fourths and placed two fourths in each space.

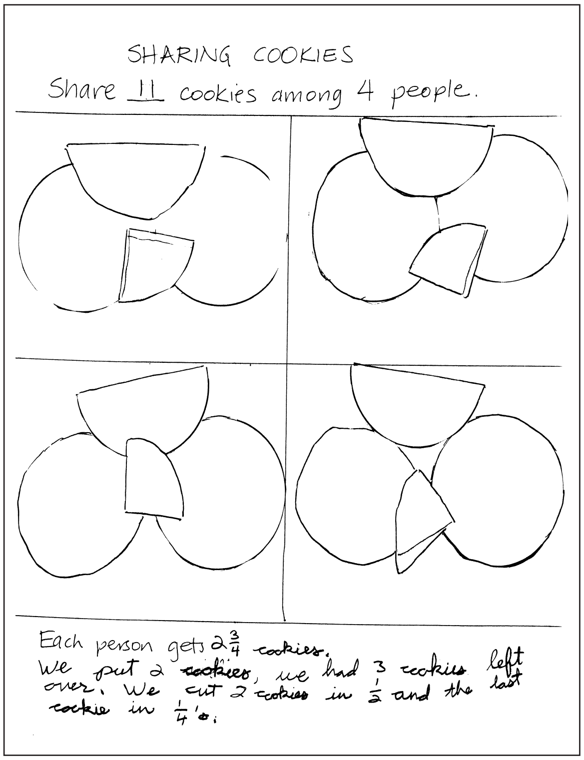


Figure 11–2 ***When they shared 11 cookies among 4 people, Shannon and Maggie realized that $\frac{1}{2}$ plus $\frac{1}{4}$ was the same as $\frac{3}{4}$.***

"Here, glue these down," she said, pointing to the circles while she started to write. She wrote:

> *Each person gets 4 cookies and $\frac{2}{4}$ cookies because 4 × 4 is 16. There [are] 2 cookies left over so we cut them in 4ths and it is equal.*

"I have a question," I said. Jennifer and Davey looked up. "If I said that each person gets four and a half cookies, would I also be right?"

"Yes," Jennifer said. "One-half is the same as two-fourths." I looked at Davey; he nodded his agreement.

"So I'm interested in why you cut the cookies into fourths instead of pasting down halves," I said.

"She just wouldn't leave them alone," Davey said.

"Well, what you've done is correct," I said. "But I'm curious about why you cut them."

"It's just clearer this way to me," Jennifer said. I realized that Jennifer's thinking that she needed four equal shares was mixed with her thinking about fourths of a circle. I wasn't concerned about this because she seemed clear that one-half and two-fourths were equivalent. (See Figure 11–3.)

I watched Jennifer and Davey talk about sharing four cookies among six people. They quickly cut three of the four cookies each in half and pasted one-half in each section. Then they took the last cookie, cut it into sixths, and pasted one-sixth in each section. Jennifer wrote:

> *Each person gets $\frac{1}{2}$ and $\frac{1}{6}$ of a cookie.*

"Can you find one fraction to write that means the same as one-half and one-sixth together?" I asked.

"That's hard," Davey said.

"No, it's not," Jennifer said. "It's like we did with the fraction kit and found one frac-

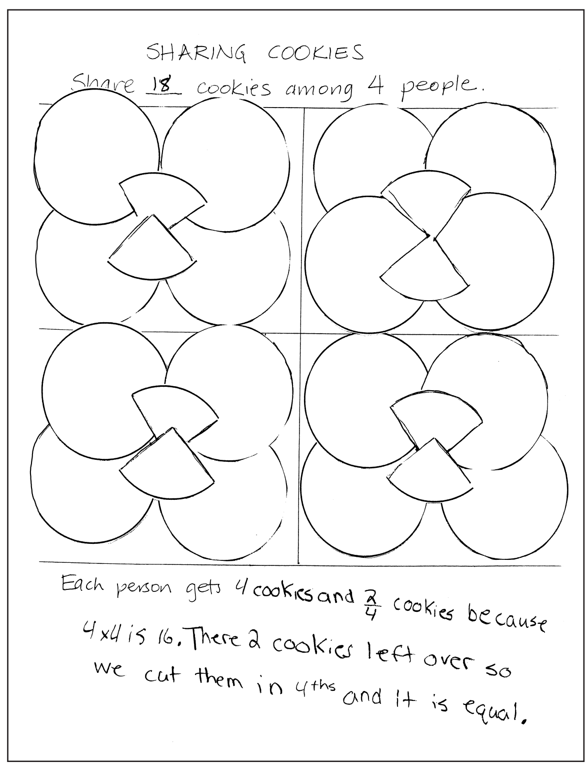


Figure 11–3 ***Jennifer and Davey were interested in problems with large numbers of cookies.***

tion for a long train. Look, a half has three-sixths, so it has to be four-sixths altogether."

"Oh, okay, I get it," Davey said.

"Explain it to me in your own words," I said to Davey, to be sure that he understood.

"There are six-sixths in a whole, so a half has three, and one more makes four-sixths," he responded. Jennifer added to the bottom of their paper: *This is the same as $\frac{4}{6}$ of a cookie.*

When I looked at Lara and Delia's work, I noticed a recording error on their paper. They had correctly shared eleven cookies among six people, pasting in each section one whole cookie, one-half of a cookie, and one-third of a cookie. They wrote:

> *Each person gets $1\frac{1}{2}$ $\frac{1}{3}$ cookies. Since each person gets $\frac{1}{1}$ there are 5 cookies. We cut 3 cookies in half and 2 cookies in 3rds. So each person gets $1\frac{1}{3}$ $\frac{1}{2}$ cookies.*

Their error wasn't from lack of conceptual understanding, but of incorrect use of fractional notation, or punctuation. They should have written either: *Each person gets 1, $\frac{1}{2}$, and $\frac{1}{3}$ cookies* or *Each person gets 1 + $\frac{1}{2}$ + $\frac{1}{3}$ cookies* or *Each person gets $1\frac{1}{2}$ cookies and $\frac{1}{3}$ more.*

I explained to the girls, "You don't need the 'and' when there's a whole number and a fraction, but with two fractions next to one another, you have to include either 'and' or a plus sign." This information isn't lodged in conceptual understanding of fractions, but in the social conventions of the symbolization, and this is an example of when the only way to teach is by telling. The girls couldn't discover this; they had to learn about it from some source outside of themselves—me, a classmate or other person, or a book.

I then asked the girls what one fraction they could use to combine one-half and one-third so they didn't need the "and." They didn't have an immediate answer.

"Talk about it and I'll check back in a while," I told them. (When I checked back later, they were still stuck. "It's too hard," Delia said. I told them not to worry, that I'd be helping them with problems like that over the next several weeks.)

I checked back in with Sean and Joseph. The boys were now working on sharing ten cookies among six people. They were cutting out the ten cookies in preparation for sharing them.

"I think this is going to be hard," Sean said. Joseph didn't comment or react but kept on cutting. I left them to work, but Sean came in a bit and asked me for help. "I don't know how to explain what we did," he said.

I joined the boys and looked at what they had done. They had pasted one circle in each of the six sections. They then cut one-third out of each of the four remaining circles, leaving four pieces that were two-thirds of a circle and four pieces that were one-third of a circle. They pasted the two-thirds pieces in four of the sections, and pasted two of the one-third pieces in each of the other two sections. So far, they had written: *Each person gets 1 and $\frac{2}{3}$ cookies. First we put 1 cookie in each square and had three cookies left.*

"If you started with ten cookies, and put one in each of the six sections, then I don't understand why you would have three cookies left," I said.

"Oops," Joseph said. "We had four." Sean made the change on the paper.

"So how do I write about how we cut the rest?" he asked.

"Tell me what you did," I suggested.

"We drew on each circle to make thirds and then we cut out one third and then we pasted them down," Sean said.

"Start by writing that," I said. "Then read it aloud and see if you need to add more. Come and ask me to check and see if you need more details." As I always do, I tried to find ways to encourage them to talk about their ideas as a precursor to writing. It usually seems to help. (See Figure 11–4.)

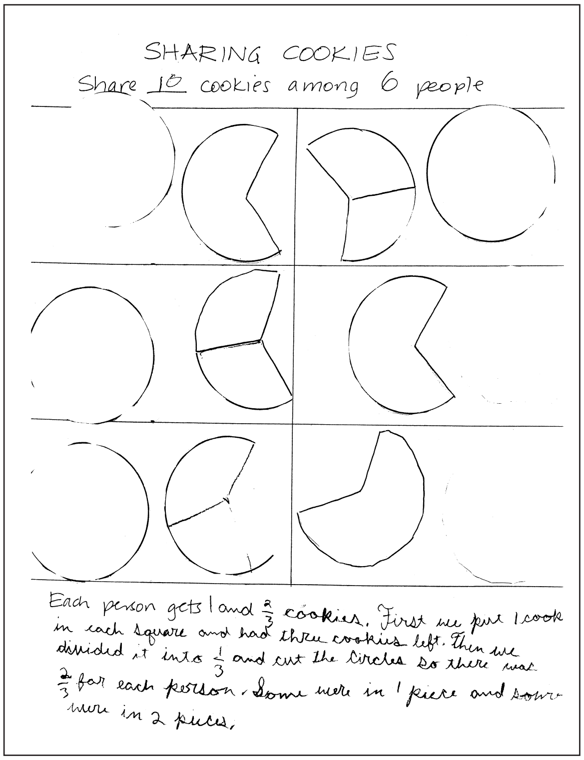


Figure 11–4 ***After sharing 9 cookies among 6 people, Sean and Joseph tackled sharing 10 cookies among 6 people.***

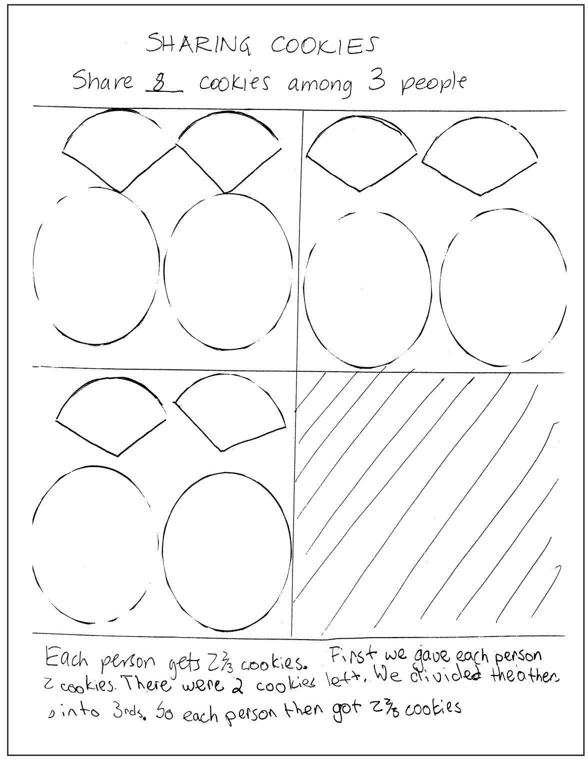


Figure 11–5 ***Lara and Delia were clear about how they shared 8 cookies among 3 people.***

At times I also encourage pairs to talk with other pairs when they're working on the same problem. For example, I noticed that when Josh and Robert shared two cookies among three people, in each section they pasted one-half of a cookie and one-sixth of a cookie. They wrote: *Each person gets $\frac{1}{2}$ and $\frac{1}{6}$ of a cookie. First we cut the cookies in half and their was one $\frac{1}{2}$ left so we cut it in thirds and each piece was $\frac{1}{6}$.* When Dan and Grant did the same problem, however, they had cut both cookies into thirds and pasted two of the thirds in each section. They wrote: *Each person gets $\frac{2}{3}$ of the cookie. If 3 people have 1 cookie they each get $\frac{1}{3}$ because we broke it into 3 parts. Then we did it again.*

I asked the four boys to look at the two papers together and let me know what they thought. They got into a huddle. A few minutes later, Robert came over to me.

"We did it," he said.

"What did you decide?" I asked.

"We're both right," Robert answered. "That was cool."

I did the same with two other pairs of students. Claudia and Libby had shared four cookies among six people by cutting all six cookies into thirds. That gave them twelve thirds. They pasted two of the thirds in each section. Jennifer and Davey, as I explained earlier, had solved the same problem by pasting one-half of a circle and one-sixth of a circle in each section. (See Figures 11–5 and 11–6 for two ways students solved the same problem.)

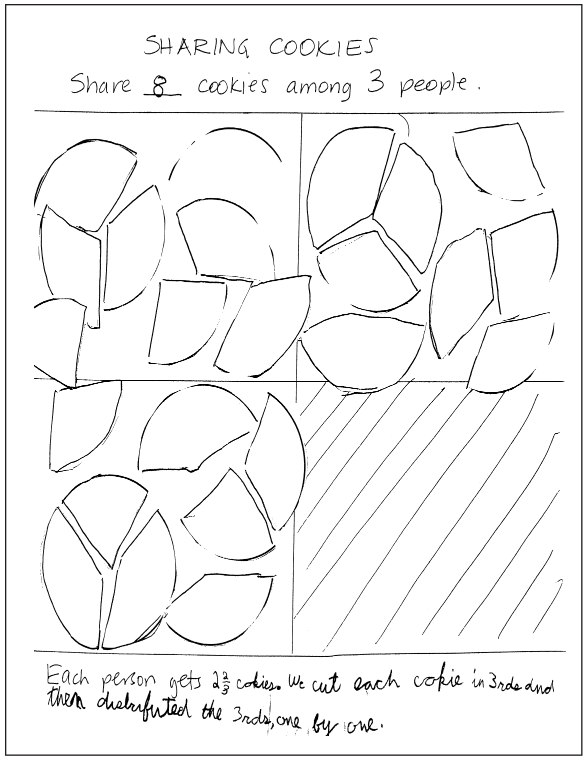


Figure 11–6 *Dan and Grant had a different solution for the same problem that Lara and Delia did.*

Questions and Discussion

▲ *What will you do differently when you teach this activity again?*

I liked the way this activity went. When I do it again, I might have students do the same problems and save the opportunity for them to choose their own numbers of cookies for the menu later. But I liked what resulted from this activity. It seemed accessible to all and at the same time challenging for those who were able.

▲ *Did you think about whether or not the fifth graders really needed to cut and paste the "cookies"? Did you consider making that optional for them?*

I've learned from experience not to worry about an activity being too babyish for the students because they have to cut, paste, or work with manipulatives. I've found that students don't object, unless the cutting, pasting, or working with materials seems like meaningless busywork. In this case, the circles provide a way for students to tackle the problems, and what students do with the circles gives me insights into their thinking. In this class, none of the students questioned whether or not they had to cut and paste circles. If they had, I would have explained my reasons—that I thought the circles would give them a way to think about the mathematical ideas and that how they cut the circles would give me important information about how they are thinking about fractions. I'd also tell the students that even as an adult, I need to "see" what I'm doing and rely heavily on sketches to help with math problems.

▲ ***Why did you have students choose the number of cookies they would share for the problems?***

I'm always on the lookout for ways to blend instruction and assessment in lessons so that when children are involved in learning activities, I have the opportunity to gain insights into how each of them is thinking. Giving students the chance to set parameters for problems gives me information about their comfort levels and the challenges they're willing to take on.

▲ ***Don't you think that if the students all did the same problems, you could more easily have a class discussion later?***

Yes, I guess that's true. As a matter of fact, when I circulated around the room, I looked for common problems that we could discuss. So, upon reflection, I could have assigned some problems in common, maybe giving them three specific ones to try, and then let them do two of their own choosing. This is one of those professional judgment calls for which there's no right answer and probably no wrong choice either.

▲ ***You asked students to solve problems of dividing cookies among three, four, and six people. How come you skipped five people?***

I find that it's too hard for the students to divide circles into fifths and then be able to discern the fifths from sixths. With circles, the shapes of halves and fourths are easily recognizable. Thirds are, also, with a little practice. Sixths are related to thirds, so students have a way to cut them. Fifths, however, are more difficult. I don't think that every material is suitable for all possible fraction situations. We'll get to fifths in another way. The context of money is more natural and suitable, I think, since a nickel is one-fifth of a quarter and twenty cents is one-fifth of a dollar.

▲ ***What do you talk about with students as you observe them working on this activity?***

When circulating around the room, I offer help when asked, always focusing my help on getting the students to reason for themselves. A good deal of the help I give is helping students get down in writing what they've done. Most important is that students can explain what they're doing and why it makes sense. Hearing students explain gives me insights into their understanding, and having the opportunity to talk about their ideas helps students confirm and often extend their thinking. With this activity, I first look at students' worksheets to check for correctness. If an answer is wrong, I talk with the students about it. Also, I look to see if there are ways I can challenge their thinking further, starting with what they've done.